Rebhan Thermal Field Theory III

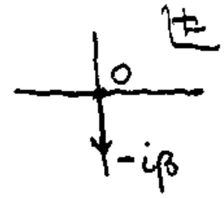
Real-time formalism (RTF)

$$Z[j] = \mathcal{N} \int_{\varphi(t_0 - i\beta) = \varphi_0(t_0)} \mathcal{D}\varphi \exp i \int_{\mathcal{C}} d^4x \left(\mathcal{L}(x) + j(x)\varphi(x)\right)$$

$\mathcal{C}$ path $t_0 \to t_0 - i\beta$ w/ Im. part monot. decreasing

(anti)periodic b.c.'s for (fermions) boson

Last time: ITF $t_0 = 0$



$\Rightarrow$ (imaginary) discrete Matsubara frequencies $k_0 = i\omega_n$

bosons: $i\omega_n = i 2\pi n T + \mu$

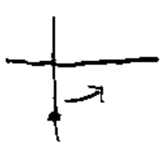
μ ← chemical potential

fermions : $i 2\pi (n + \tfrac{1}{2}) T + \mu$

Vary the contour time (t) along path
T_c orders t_i's along the path

$\langle T_c \, \phi(t_1) \cdots \phi(t_n) \rangle$

$\longleftarrow$ contour time



ambiguity in continuation:
can always mutiply by
$e^{i l k_0 / T}$ where l chosen
as a Matsubara ω

$= 1$ on original path

Baym & Mermin (1961) Ambuities fixed by requiring falloff/finiteness when $|k_0| \to \infty$

Issue clarified by Weldon "Mishaps..."

Esp. IR problems
nonalyticity for $k_0, |\vec{k}| \to 0$
(need not commute) related
to rest frame of the heat bath

Study the propagator

$$D_c(t,t') = \underset{\text{along } c}{\theta_c(t-t')}\, D^>(t,t') + \theta_c(t'-t)\, D^<(t,t')$$

$$D^>(t,t') = \langle \phi(t)\, \phi(t') \rangle$$

$$D^<(t,t') = \langle \phi(t')\, \phi(t) \rangle$$

$$= \frac{1}{Z} \mathrm{Tr}\left(e^{-\beta H} \phi(t') \underset{e^{\beta H} e^{-\beta H}}{\uparrow} \phi(t) \right)$$

as last lecture $\longrightarrow$

$$= D^>(t - i\beta, t') \qquad \text{KMS condition (free or interacting)}$$

F.T. $$\tilde{D}^>(k_0) \equiv \int_{-\infty}^{\infty} dt\, e^{ik_0 t} D^>(t)$$

KMS $\Rightarrow$ $$\tilde{D}^<(k_0) = \int_{-\infty}^{\infty} dt\, e^{ik_0 t} \underbrace{D^<(t)}_{D^>(t-i\beta)}$$

$$= e^{-\beta k_0} \tilde{D}^>(k_0)$$

Spectral density

$$\rho(k_0) = \int_{-\infty}^{\infty} dt\, e^{ik_0 t} \langle [\phi(t), \phi(0)] \rangle$$

$$= \tilde{D}^{>}(k_0) - \tilde{D}^{<}(k_0)$$

KMS:

$$= \tilde{D}^{>}(k_0)(1 - e^{-\beta k_0})$$

$$= \tilde{D}^{<}(k_0)(e^{\beta k_0} - 1)$$

$$\Rightarrow \tilde{D}^{>} = (1 + f(k_0))\rho(k_0)$$

$$\tilde{D}^{<} = f(k_0)\rho(k_0)$$

$$f(k_0) = \frac{1}{e^{\beta k_0} - 1}$$

(for $k_0 < 0$, recall # density $n(k_0) = f(|k_0|)$)

Also:

$$D_c(t, t') = \Theta_c(t - t')[D^{>} - D^{<}] + D^{<}$$

a useful form : the Mills propagator

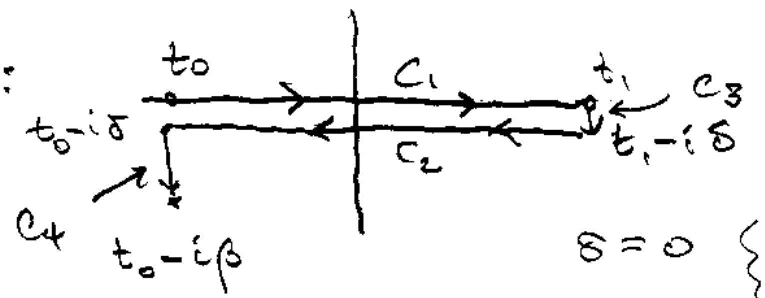
To 4-dim. F.T. :

$$D_c(x - x') = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} [\Theta_c(t-t') + f(k_0)]\rho(k_0, \vec{k})$$

For pert. theory, use

free: $\rho_0 = 2\pi\,\varepsilon(k_0)\,\delta(k^2 - m^2)$

RTF:



$\delta = 0$ { Schwinger-Keldysh

$\delta = \beta/2$ TFD (thermo-field dynamics; Umezawa & others)

Keldysh: adiabatic switching-off of intn. simplifies large-time

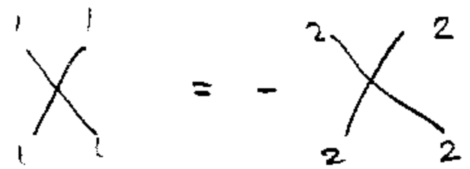
See: F. Gelis papers for proper justification of omitting vertical path.

result: $Z[j] = Z_{12}[j]\, Z_{34}$

$$\Rightarrow \int_C = \int_{-\infty}^{\infty} dt + \int_{+\infty - i\delta}^{-\infty - i\delta} = \Big(\overset{\varphi^{(1)}}{\int_{-\infty}^{\infty}} - \overset{\varphi^{(2)}}{\int_{-\infty - i\delta}^{\infty - i\delta}} \Big)\, dt$$

label fields by their location on path: 1 or 2

Vertices: same except for an overall sign



For kinetic part: 2×2 matrix

$$(D_c)_{ab} \equiv \langle T_c\, \phi_a(t)\, \phi_b(t') \rangle$$

$$[\Theta_c(t-t')]_{ab} = \begin{pmatrix} \theta(t-t') & 0 \\ 1 & \theta(t'-t) \end{pmatrix}$$

see p (3): f term present for all a,b: $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ matrix

using $\quad \Theta(t) = \int \frac{d\omega}{2\pi}\, e^{-i\omega t} \frac{i}{\omega + i\eta}$

find: (exercise)

$$D_{11} = \frac{i}{k^2 - m^2 + i\eta} + n(k_0)\, 2\pi\, \delta(k^2 - m^2)$$

(non-zero δ)

$$D_{12} = e^{\delta k_0} [n(k_0) + \Theta(-k_0)]\, 2\pi\, \delta(k^2 - m^2)$$

$$D_{21} = e^{-\delta k_0} [n(k_0) + \Theta(k_0)]\, 2\pi\, \delta(k^2 - m^2)$$

$$D_{22} = D_{11}^*$$

$$\mathbb{D} = \begin{pmatrix} \frac{i}{k^2-m^2+i\eta} & 2\pi\delta^-(k^2-m^2)\,e^{\delta k_0} \\ 2\pi\delta^+(k^2-m^2)\,e^{-\delta k_0} & \frac{-i}{k^2-m^2-i\eta} \end{pmatrix} + 2\pi\delta(k^2-m^2)\,n(k_0)\begin{pmatrix} 1 & e^{\delta k_0} \\ e^{-\delta k_0} & 1 \end{pmatrix}$$

$$\delta^{\pm} = \theta(\pm k_0)\,\delta$$

Can write in the following diagonal form

$$\mathbb{D} = M(k_0)\begin{pmatrix} \frac{i}{k^2-m^2+i\eta} & \\ & \frac{-i}{k^2-m^2-i\eta} \end{pmatrix} M^{+}(-k_0)$$

where $M(k_0) = \sqrt{n(|k_0|)}\begin{pmatrix} e^{\beta\frac{|k_0|}{2}} & e^{(\delta-\frac{\beta}{2})k_0} \\ e^{(\frac{\beta}{2}-\delta)k_0} & e^{\beta\frac{|k_0|}{2}} \end{pmatrix}$,

can write $\frac{-i}{k^2-m^2-i\eta}$ as $\langle 0 | T_c[\phi_c'\,\phi_d']|0\rangle$

type 1 time ordered
type 2 anti time ord.

new fields

$$\mathbb{D}_{ab} = M_{ac}(k)\,M_{bd}(-k)\,\langle 0|T_c[\phi_c'\,\phi_d']|0\rangle$$

$$M_{ac}\,\phi'_c = \phi_a^{TFD} = \begin{pmatrix} \phi \\ \phi^{+} \end{pmatrix}$$

$\uparrow$ type 2

the creation and annihilation operators become

$$\alpha'_a = \underbrace{M^{-1}\,\alpha_a^{TFD}}_{e^{iG}\,a\,e^{-iG}} \quad \text{with } G = G^{+} \text{ when } \delta = \frac{\beta}{2}$$

they act on the thermal vacuum $|0(\beta)\rangle$

Now for some operator A

$$\langle A \rangle = Z^{-1}(\beta)\sum_n e^{-\beta E_n}\langle n|A|n\rangle \neq \langle 0(\beta)|A|0(\beta)\rangle$$

Need <u>doubling</u> for the Hilbert Space $\quad A \longrightarrow A \otimes \mathbb{1}$

$$|0(\beta)\rangle = \sum_n Z^{-1/2}\, e^{-\beta E_n/2}\,|n\rangle \otimes |\tilde{n}\rangle$$