A. Rebhan Lect IV Gauge Theories

So far $\mathrm{Tr}[e^{-\beta H}\ldots]$
in a Hilbert space
for gauge theories will want to project to a physical Hilbert space

QED : ghosts decouple

$$\mathcal{L} = \frac{1}{2} A_\nu \Box A^\nu \quad \text{covariant gauge}$$

$$Z = \mathcal{N} \int_{\text{periodic}} \mathcal{D}A \, e^{i\int_0^{-i\beta} dt\, d^3x\, \mathcal{L}}$$

$$= (\det \Box)_{\text{periodic}}^{-\frac{1}{2}\times 4}$$

1 scalar DoF: $P = \frac{1}{\beta V} \log Z = \frac{\pi^2 T^4}{90}$

This × 4 is 2× to much

Restriction to physical $\mathcal{H}$ does this

Faddeev-Popov ghosts are decoupled (but so are physical modes)

Bernard 1974

insert $\prod_{a,x} \delta(F^a[A](x) - \xi^a(x)) \cdot$

$$\cdot \det \frac{\partial F^a}{\partial \omega_b}$$

$$\delta A^{\mu a} = D^{\mu ab}(A)\,\omega^b$$

Then insert

$$\int \mathcal{D}\xi \, e^{\frac{i}{2\alpha}\xi^a \xi^a} \longrightarrow \mathcal{L} = \mathcal{L}_{inv} + \frac{1}{2\alpha} F^a F^a$$

$$F^a(A) = \partial_\mu A^{\mu a}$$

for QED get $\det(\Box)_{\text{periodic}}$ from ghosts

$$(\det \Box)_{\text{periodic}} = \int_{\substack{\text{periodic} \\ \omega_n = 2\pi i n T}} \mathcal{D}b\, \mathcal{D}c\; e^{i\int b^a \frac{\partial F^a}{\partial A^b_\mu} D^{bc}_\mu(A)\, c^c}$$

α-independence built in from the beginning but details of proof come only in the calculation of the determinants

Covariant operator quantization
Hata-Kugo 1980
BRS method

$$\mathcal{L} = \mathcal{L}_{\text{inv.}} + \{ Q_{BRS},\, \underbrace{b}_{\text{antighost}} F[A] + \alpha b \underbrace{B}_{\text{auxiliary field}} \}$$

with
$$[iQ, A_\mu] = D_\mu c$$
$$[iQ, B] = 0$$
$$\{iQ, c\} = -\frac{g}{2}\, c \times c$$
$$\{iQ, b\} = i B$$

Spaces $V \to V_{\text{phys}}$ — "not yet "true" physical" space, $\mathcal{H}_{\text{phys}}$

$Q_{BRS}|\text{phys}\rangle = 0$ and 'not BRS-exact'

$N_c|\text{phys}\rangle = 0$ — ghost number

w/ Noether generator $N_c = \int d^3x \left[\partial_0 b\, c - b\, D_0 c \right]$

$$\mathcal{H}_{ephys} = \mathcal{V}_{phys} / \mathcal{V}_0$$

$\uparrow$ zero norm states

Projection:

$$\mathcal{H}_{ephys} = \mathcal{P}\, \mathcal{V}$$

$$\mathcal{P} = 1 - \{Q_{BRS}, \mathcal{R}\}$$

$\mathcal{R}$ given by Kugo & Ojima

Explicit form is no so important (& it's complicated)

Trick: use fact $[N_c, Q_{BRS}] = Q_{BRS}$

$$\Rightarrow \quad N_c Q = Q (N_c + 1)$$

$$\Rightarrow \quad N_c^n Q = Q (N_c + 1)^n$$

$$\Rightarrow \quad e^{i\pi N_c} Q = Q\, e^{i\pi(N_c+1)} = -Q\, e^{i\pi N_c}$$

$$\Rightarrow \quad \{e^{i\pi N_c}, Q\} = 0$$

Now will want

$$\mathrm{Tr}\left[\mathcal{P}\, e^{-\beta H} \cdots\right]$$

commute

$$= \mathrm{Tr}\left[\mathcal{P}\, e^{-i\pi N_c} e^{-\beta H} \cdots\right]$$

$$= \mathrm{Tr}\left[e^{-\beta H + i\pi N_c} \cdots\right]$$

$$- \mathrm{Tr}\left[\underbrace{\{Q, \mathcal{R}\}}_{Q\mathcal{R} + \mathcal{R}Q} e^{i\pi N_c - \beta H} \cdots\right]$$

$Q\mathcal{R}$: use cyclicity

For 2^d term given (assume "..." is gauge inv't)

$$\mathrm{Tr}\left[\mathcal{R} \underbrace{\{Q, e^{i\pi N_c}\}}_{=0\,!} e^{-\beta H} \cdots \right]$$

Result is that Trace is unrestricted but replace

$$-\beta H \to -\beta H + i\pi N_c$$

"imag chemical potential for ghosts"

$$\mu_c = i\pi/\beta$$

$$e^{-\beta(H-\mu_c N_c)}$$

In propagators

fermions $n_{BE} \longrightarrow -n_{FD} = -\frac{1}{e^{\beta k_0}+1}$

$n_{BE} = e^{\beta k_0} - 1$

chemical pot'l

$$\Rightarrow e^{\beta(k_0-\mu)} + 1$$

w/ imag. chemical pot'l

$$-n_{FD}(\mu = i\pi/\beta) = -\frac{1}{-e^{\beta k_0}+1} = n_{BE}\,!$$

$\Rightarrow$ change b.c.'s to periodic even while determinant remains in numerator for path integral

Another approach (RTF)
"freeze out the ghosts"
Landshoff & Rebhan

Varying the gauge-fixing

$$F^a \to F^a + \delta F$$

can be produced by ($\mathcal{G}$ is ghost prop.ʳ)

$$\delta F = \underbrace{\frac{\partial F}{\partial A_\mu} D_\mu(A)}_{\mathcal{G}^{-1}} \cdot \overbrace{\mathcal{G}\, \delta F[A]}^{\text{non-local } \delta A}$$

G.I. up to sources

But when $\mathcal{L} \to \mathcal{L} + j A$

$$\delta Z[j] = j_i \langle D^i_\alpha \, \mathcal{G}^\alpha{}_\beta \, \delta F^\beta \rangle$$

"DeWitt notation"

Same form finite & zero T

Use this formal property to study gauge-dependence of gluon propagator

Kobes, Kunstatter, AR 1990
Useful for analysis of HTLs.

HARD THERMAL LOOPS: Gauge boson self energy

universal!

calculation for massless scalar electrodynamics

$$\mathcal{L} = (D_\mu\phi)^* D^\mu\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad D_\mu = \partial_\mu + ieA_\mu$$

(ITF) polarization tensor

$$\Pi_{\mu\nu}(K) = e^2 \sum\!\!\!\!\!\!\int \left[\frac{(2P-K)_\mu(2P-K)_\nu}{P^2(P-K)^2} - \frac{2g_{\mu\nu}}{P^2}\right]$$

$K^\mu = (k^0, k^m)$

$K^\mu \Pi_{\mu\nu}(K) = 0$ (symmetric integration $P \to -P$)

$\to$ 2 independent components corresponding to 2 transverse tensors

$$A_{\mu\nu} = g_{\mu\nu} - \frac{K_\mu K_\nu}{K^2} - B_{\mu\nu}, \quad B_{\mu\nu} = \frac{\tilde{U}_\mu\tilde{U}_\nu}{\tilde{U}^2}, \quad \tilde{U}_\mu = K^2 U_\mu - K_\mu U\cdot K$$

$U^\mu = \delta^\mu_0$ in plasma rest frame

$\to$ calculate e.g. $\Pi^\mu{}_\mu$ and Π_{00}

$$\Pi^\mu{}_\mu(K) = e^2 \sum\!\!\!\!\!\!\int_P \left[\frac{-4}{P^2} - \frac{K^2}{P^2(P-K)^2}\right] = \frac{e^2T^2}{3} + O(T)$$

$(K_\mu \ll T)$

$$\sum\!\!\!\!\!\!\int_P \frac{-1}{P^2} = \sum\!\!\!\!\!\!\int \frac{1}{2p}\Big(\underbrace{\frac{1}{p_0+p}}_{\substack{-n(p)\\=1+n(p)}} - \underbrace{\frac{1}{p_0-p}}_{+n(p)}\Big) = \int_p \frac{2n+1}{2p} = \frac{T^2}{12}$$

$\to$ thermal mass for photons (gauge bosons

self-consistent for $eT \ll K_\mu \cancel{\ll T}$: $\Pi_A \sim \frac{1}{2}\Pi^\mu{}_\mu = \frac{e^2T^2}{6} = m_\infty^2$

because of $K^2 = 0$

$K^2 \neq 0$, $K_\mu \ll T$:

$$\Pi_{00}(K) = e^2 \sum\!\!\!\!\!\!\int_P \left[\frac{4p_0(p_0-k_0)}{P^2(P-K)^2} - \frac{2}{P^2} \right]$$

$$\frac{p_0}{P^2} = \frac{1}{2}\left(\frac{1}{p_0-p} + \frac{1}{p_0+p}\right), \quad \frac{p_0-k_0}{(P-K)^2} = \frac{1}{2}\left(\frac{1}{p_0-k_0-|\vec{p}-\vec{k}|} + \frac{1}{p_0-k_0+|\vec{p}-\vec{k}|}\right)$$

$$\frac{1}{p_0+X}\,\frac{1}{p_0+Y} = \frac{1}{X-Y}\left(\frac{1}{p_0+Y} - \frac{1}{p_0+X}\right)$$

$$\Big\downarrow \Sigma_{p_0}$$

$$\frac{1}{X-Y}\left[n(Y) - n(X)\right]$$ (dropping T=0 parts)

$$\Pi_{00}(K) = e^2 \int \frac{d^3p}{(2\pi)^3} \Big\{ \frac{1}{k_0 - p + |\vec{p}-\vec{k}|}\left[n(p) - n(|\vec{p}-\vec{k}|)\right]$$

$$+ \frac{1}{k_0+p-|\vec{p}-\vec{k}|}\left[n(|\vec{p}-\vec{k}|) - n(p)\right]$$

$$+ \cancel{\frac{1}{k_0-p-|\vec{p}-\vec{k}|}\left[n(p) + n(|\vec{p}-\vec{k}|)\right]}$$

$$+ \cancel{\frac{1}{k_0+p+|\vec{p}-\vec{k}|}\left[-n(p) - n(|\vec{p}-\vec{k}|)\right]} - \frac{2n(p)}{p} \Big\}$$

$K_\mu \ll p$: $n(p) \approx n(|\vec{p}-\vec{k}|)$, $|\vec{p}-\vec{k}| \approx p$

but in first two terms: $n(|p-k|) - n(p) \approx n'(p)\left[|p-k| - p\right]$

$$|p-k| - p \approx \vec{p}\cdot\vec{k}/p \equiv z k$$

$$\Pi_{00}^{HTL}(K) = 2e^2 \int \frac{p^2 dp}{(2\pi)^2}\, n'(p) \int_{-1}^{1} dz \left[-1 + \frac{k_0}{k_0 - zk}\right]$$

$$= \frac{e^2T^2}{3}\left[1 - \frac{k_0}{2k} \log\frac{k_0+k}{k_0-k}\right]$$

NB: $k_0 \to 0$: $\Pi_{00}^{HTL}(0,k) = \frac{e^2T^2}{3}$ but $\Pi_{00}^{HTL}(k_0,0) = 0$!