Hard thermal|dense loop photon/gluon propagator

Gauge-independent

leading-order high $T|\mu$ contributions to $\Pi^{\mu\nu}$ $(T, \mu \gg k_0, |\vec{k}|)$:

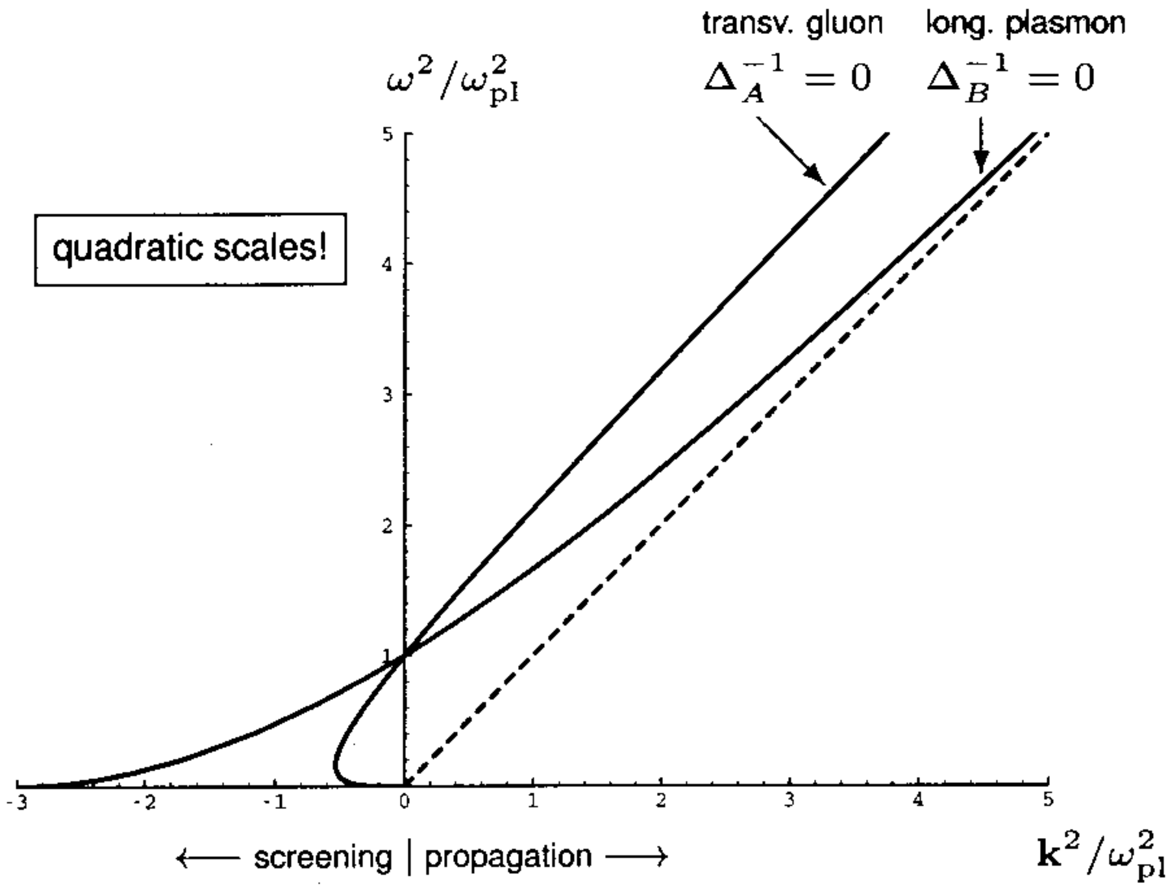
$$\Pi_A^{\mathsf{HTL}} = \tfrac{1}{2}({\Pi^{\mathsf{HTL}}}_\mu{}^\mu - \Pi_B^{\mathsf{HTL}}) \qquad \Pi_C^{\mathsf{HTL}} = 0$$

$$\Pi_B^{\mathsf{HTL}} = -\tfrac{k^2}{\mathbf{k}^2}\Pi_{00}^{\mathsf{HTL}} \qquad \Pi_D^{\mathsf{HTL}} = 0$$

$${\Pi^{\mathsf{HTL}}}_\mu{}^\mu = \frac{e^2T^2}{3}, \quad \Pi_{00}^{\mathsf{HTL}} = \frac{e^2T^2}{3}\left(1 - \frac{k^0}{2|\mathbf{k}|}\ln\frac{k^0+|\mathbf{k}|}{k^0-|\mathbf{k}|}\right)$$

$e^2 := g^2(N + N_f/2)$ in QCD ($\mathrm{SU}(N)$ with N_f flavors)

$\mu \neq 0$: $T^2 \to T^2 + 3\mu^2/\pi^2$ in terms $\propto N_f$ (all of it in QED)



Resummations in static limit

Matsubara frequencies $\omega_n = \begin{cases} 2\pi i n T & (B) \\ 2\pi i (n+\frac{1}{2})T & (F) \end{cases}$

hard, except for $n=0$ and (B)

static limit of $\Pi_{\mu\nu}$:

$$\Pi_{00}^{HTL}(0,k) = \frac{e^2T^2}{3} = \hat{m}_D^2 \quad \text{const.!}$$

$$\Pi_{ij}^{HTL}(0,k) = 0$$

dimensional reduction allows mass term for A_0

$$F^a_{i0} = \partial_i A^a_0 - \cancel{\partial_0 A^a_i} + g f^{abc} A^b_i A^c_0$$

$$\longrightarrow \left(D_i[\vec{A}]A_0\right)^a$$

effective 3+0-dim. field theory

for contributions from soft scales $\lesssim gT$

$$\mathcal{L}_E = \frac{1}{2}\mathrm{tr}\, F_{ij}^2 + \mathrm{tr}\left[D_i, A_0\right]^2 + m_E^2\, \mathrm{tr}\, A_0^2 + \frac{1}{2}\lambda_E\left(\mathrm{tr}\, A_0^2\right)^2 + \ldots$$

Braaten + Nieto 1996: use dim. reg. for calculating effective-field-theory parameters (removes IR contributions because scaleless integrals $=0$ in dim. reg.!)

$$\rightarrow \mathcal{L}_E = \frac{1}{2}\,\mathrm{tr}\,F_{ij}^2 + \mathrm{tr}\,[D_i, A_0]^2 + m_E^2\,\mathrm{tr}\,A_0^2 + \frac{1}{2}\lambda_E (\mathrm{tr}\,A_0^2)^2 + \ldots$$

with (SU(3) with N_f fermions)

$$g_E^2 = g^2 T \left[1 + \frac{g^2}{16\pi^2}\left[22\left(\ln\frac{\bar{\mu}}{4\pi T} + \gamma\right) + 1\right] + \ldots \right]$$

$$m_E^2 = \hat{m}_D^2 + \frac{g^4 T^2}{16\pi^2}\left[5 + 22\gamma + 22\ln\frac{\bar{\mu}}{4\pi T} + \frac{N_f}{3}\left(\frac{1}{2} - 8\ln 2 + 7\gamma + 7\ln\frac{\bar{\mu}}{4\pi T}\right) + \frac{N_f^2}{9}\left(1 - 2\gamma - 2\ln\frac{\bar{\mu}}{\pi T}\right)\right] + \ldots$$

$$\lambda_E^2 = \frac{9 - N_f}{12\pi^2}\, g^4 T$$

lowest loop orders: only $\hat{m}_D^2$ needed

e.g. $P = p^{hard} + p^{soft}$

$$p^{hard} = T^4 (c_1 + c_2 g^2 + c_3 g^4 + c_4 g^6 + \ldots)$$

$$p^{soft}/T = \frac{2}{3\pi} m_E^3 - \frac{3}{8\pi^2}\left(4\ln\frac{\bar{\mu}}{2m_E} + 3\right) g_E^2 m_E^2 - \frac{9}{8\pi^3}\left(\frac{89}{24} - \frac{11}{6}\ln 2 + \frac{\pi^2}{6}\right) g_E^4 m_E + \ldots$$

$$= \frac{2}{3\pi}\left(1 + \frac{N_f}{6}\right)^{\frac{3}{2}} g^3 T^3 + \# g^4 + \# g^5 \ldots$$

"plasmon effect"
(rather: Debye effect !)

Linde's problem (also: Polyakov)

in 3+0 dimensions (magnetostatic gluons)

$$\left(T\int d^3k\right)^{\ell} (gk)^{2\ell-2} \frac{1}{(k^2+\mu^2)^{3\ell-3}}$$

loop int. vertices propagators with IR reg. μ^2

$\ell=4$: $g^6 T^4 \ln\frac{T}{\mu}$

$\ell>4$: $g^6 T^4 \left(\frac{g^2 T}{\mu}\right)^{\ell-4}$

suggests: nonperturbative
magnetic screening mass $\mu \sim g^2 T$
(forbidden in QED [Fradkin 1965])

lattice: 3d YM (+"heavy" adjoint scalar A_0)
is confining
only mass scale is $g_E^2 = g^2 T$

NB: for 2 pt-fct  $|_{\omega=0}$
Linde's problem already at $\ell \geq 2$

Perturbative results for the QCD pressure @ $\mu = 0, T > T_c$

$$
\begin{aligned}
P \; = \; & \tfrac{8\pi^2}{45} T^4 \Big\{ (1 + \tfrac{21}{32} N_f) - \tfrac{15}{4}(1 + \tfrac{5}{12} N_f) \frac{\alpha_s}{\pi} + 30[(1 + \tfrac{1}{6} N_f)(\frac{\alpha_s}{\pi})]^{3/2} \\
& + \Big\{ 237.2 + 15.97 N_f - 0.413 N_f^2 + \tfrac{135}{2}(1 + \tfrac{1}{6} N_f) \ln[\frac{\alpha_s}{\pi}(1 + \tfrac{1}{6} N_f)] \\
& - \tfrac{165}{8}(1 + \tfrac{5}{12} N_f)(1 - \tfrac{2}{33} N_f) \ln \frac{\bar{\mu}}{2\pi T} \Big\} (\frac{\alpha_s}{\pi})^2 \\
& + (1 + \tfrac{1}{6} N_f)^{1/2} \Big[-799.2 - 21.96 N_f - 1.926 N_f^2 \\
& + \tfrac{495}{2}(1 + \tfrac{1}{6} N_f)(1 - \tfrac{2}{33} N_f) \ln \frac{\bar{\mu}}{2\pi T} \Big] (\frac{\alpha_s}{\pi})^{5/2} + \mathcal{O}(\alpha_s^3 \ln \alpha_s) \Big\}.
\end{aligned}
$$

Arnold & Zhai 1995

Zhai & Kastening 1995

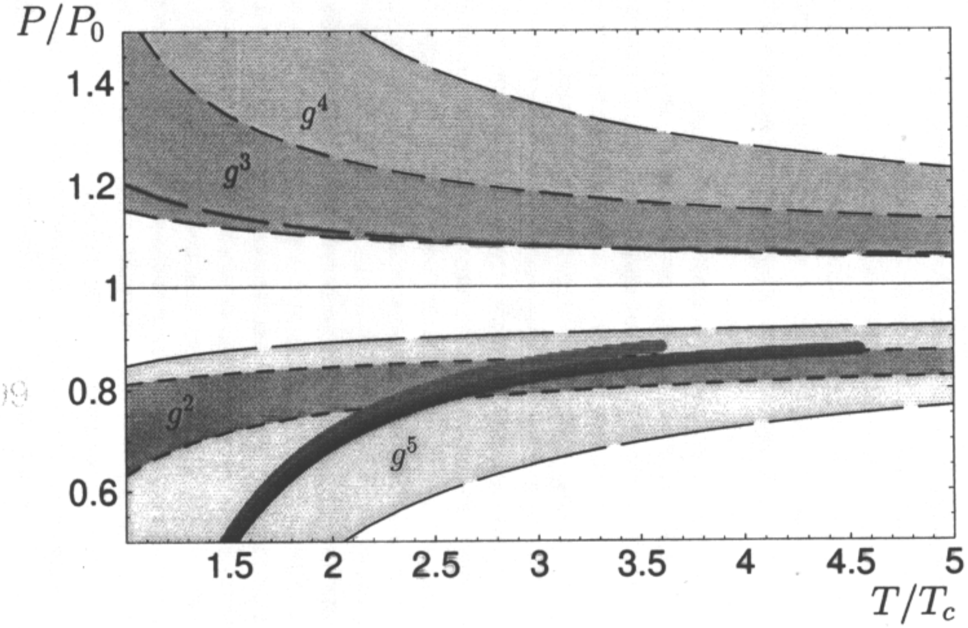
Braaten & Nieto 1996

No apparent convergence; steadily increasing renormalization scale ($\bar{\mu}$) dependence:

$N_f = 0$

Lattice data:

Boyd et al. (BI) 1996



$\bar{\mu} = \pi T \ldots 4\pi T$

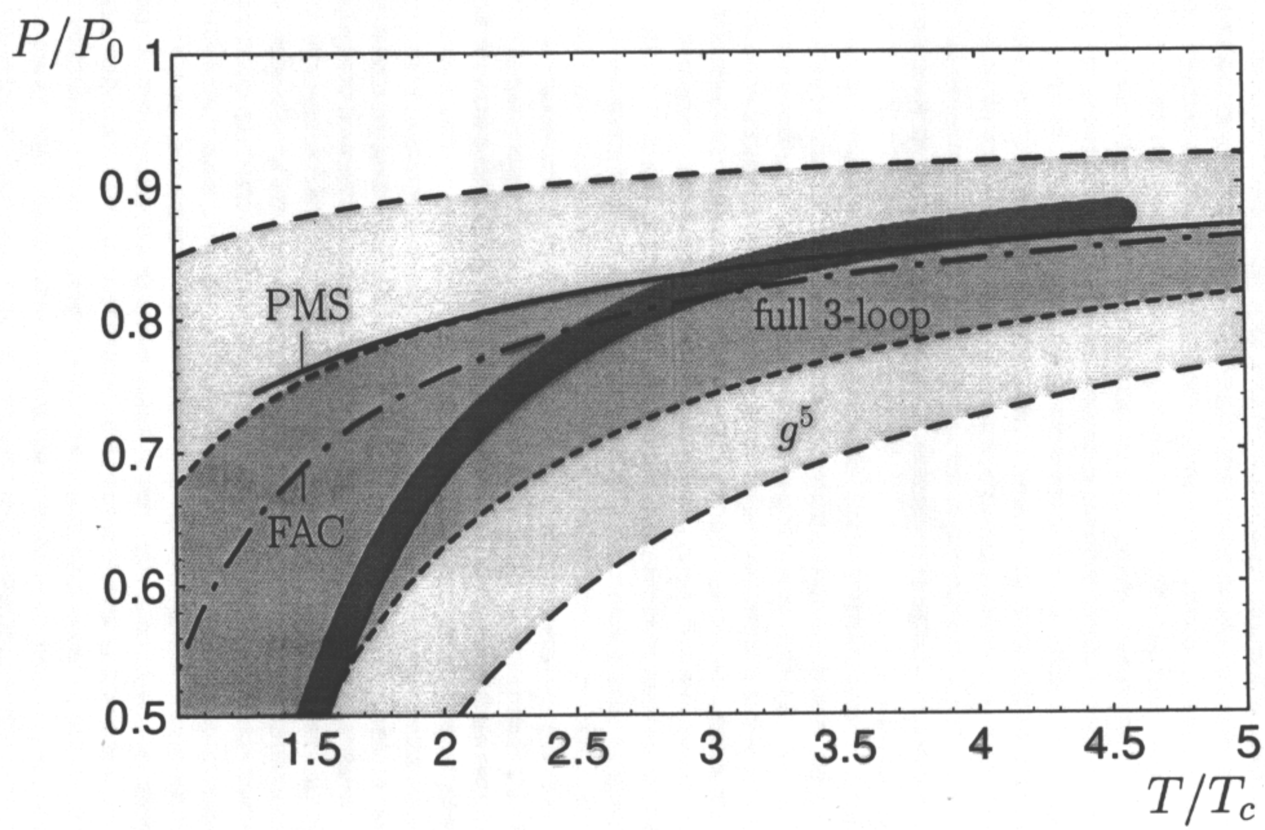
Improving apparent convergence in dimensional reduction

Expanding $P = P^{\rm hard} + P^{\rm soft}$ in powers (and log's) of g

$\rightarrow$ perturbative series with bad convergence

Not expanding $m_E^2(g)$, $g_E^2(g)$, $\ldots \rightarrow$ improved convergence for $T \gtrsim 3T_c$

J.-P. Blaizot, E. Iancu, AR, PRD68 (2003) 025011:



$\mu_{\rm MS} = \Lambda_E = \pi T \ldots 4\pi T$

$O(g^6 \ln(g))$-contribution

Last perturbatively calculable coefficient done by

Kajantie, Laine, Rummukainen & Schröder (2003):

$$P \ni N_g \frac{(N g_E^2)^3}{(4\pi)^4} \left[\left(\frac{43}{12} - \frac{157\pi^2}{768} \right) \ln \frac{\Lambda_E}{g_E^2} + \left(\frac{43}{4} - \frac{491\pi^2}{768} \right) \ln \frac{\Lambda_E}{m_E} + \boxed{\tilde{\delta}} \right]$$

$\tilde{\delta}$ determined by 3-d effective field theory #2

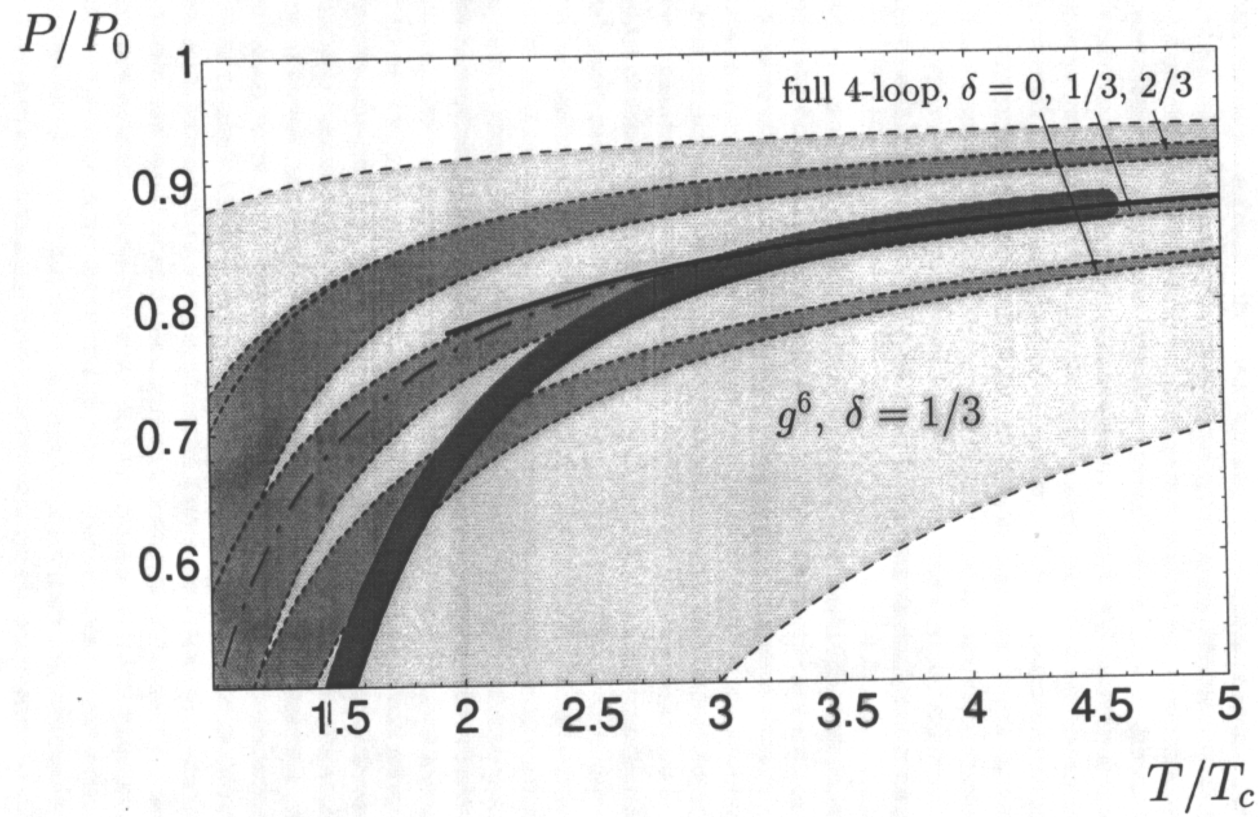
$\mathcal{L}_M = \frac{1}{2} \operatorname{tr} F_{ij}^2 + \ldots$, without adjoint scalar A_0

- nonperturbative mass gap $\sim g^2 T$, requires lattice calculation
  (and matching using 4-loop lattice perturbation theory) $\rightarrow$ contribution $\#(g^2T)^3T$

Improving apparent convergence in dimensional reduction

$\mathcal{P}^{\rm hard} + \mathcal{P}^{\rm soft}$ to order $g^6[\log(g) + \delta]$ with some $\delta \sim O(1)$: even stronger renormalization scale dependence

even greater improvement by not expanding out $m_E^2(g)$ and $g_E^2(g)$ and truncating:



Meanwhile, other improvement schemes
(applied separately to $\mathcal{P}^{\rm hard}$, $\mathcal{P}^{\rm soft}$; $\Lambda_E \neq \bar{\mu}_{\rm MS}$):

Padé : Cvetič & Kögerler, PRD70 (2004)

PMS: Inui, Niégawa & Ozaki, hep-ph/0501277