Quasi-particles at NLO
in HTL perturbation theory

Leading order: **Hard Thermal Loops**

(because generated by **hard** loop momenta $\sim T$)

need to be resummed for soft momenta $k \sim gT$:

$\Gamma^{\text{HTL}}_{,N} \sim g^N T^2 k^{2-N} \sim g^{N-2} k^{4-N} \sim \left.\frac{\partial^N \mathcal{L}}{\partial A^N}\right|_{k\sim gT}$

$\rightarrow$ effective (Wilson-renormalized) theory **for $k \sim gT$**

$$\mathcal{L} \rightarrow \mathcal{L} + \mathcal{L}^{\text{HTL}}$$

Explicit gauge-invariant integral representation:

Taylor-Wong Braaten-Pisarski 1990

$$\begin{aligned}\mathcal{L}^{\text{HTL}} &= \hat{M}^2 \int \frac{d\Omega_p}{4\pi} \bar{\psi} \gamma^\mu \frac{\hat{P}_\mu}{\hat{P}\cdot D(A)} \psi \\ &\quad - \frac{3}{2}\omega_{pl.}^2 \,\mathrm{tr} \int \frac{d\Omega_p}{4\pi} F^{\mu\alpha} \frac{\hat{P}_\alpha \hat{P}^\beta}{(\hat{P}\cdot D_{adj.}(A))^2} F_{\mu\beta}\end{aligned}$$

where $\hat{P} = (1, \vec{p})$, $\vec{p}^2 = 1$, i.e. $\hat{P}^2 = 0$, whose spatial components are averaged over by $d\Omega_p$

$\hat{P}$ ... remnant of hard plasma constituents' momenta $P = T\hat{P}$

HTL fermionic quasiparticles

$$\Sigma(\omega,\vec{k}) = a(\omega,k)\gamma^0 + b(\omega,k)\frac{\vec{k}\cdot\vec{\gamma}}{|\vec{k}|}$$

Klimov 1981

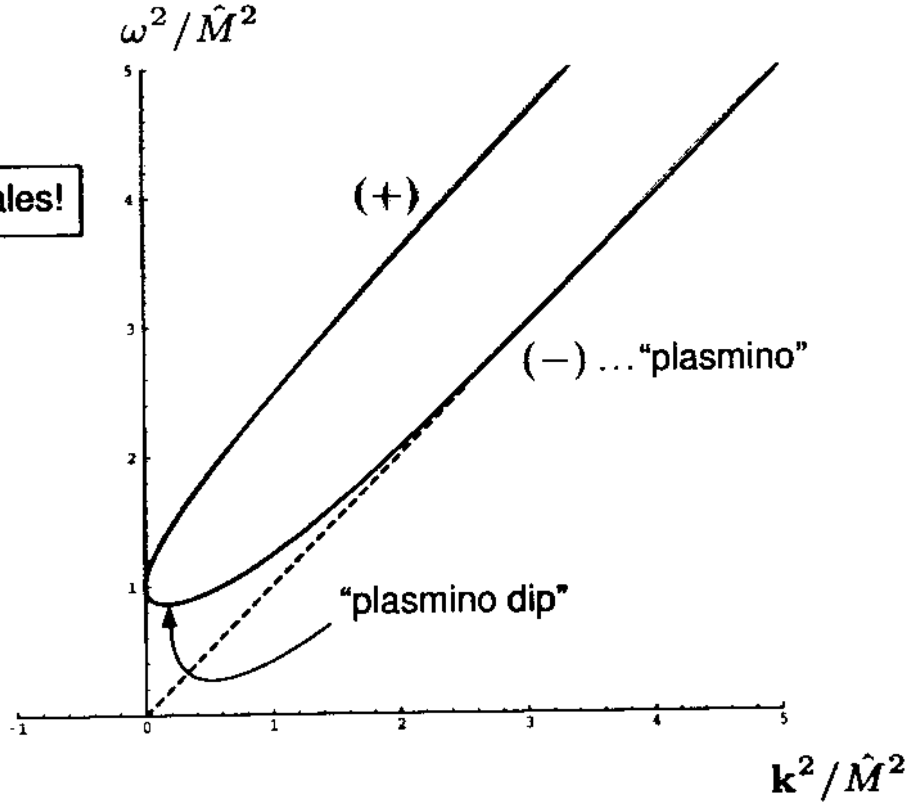
$$\Sigma_\pm(\omega,k) \equiv b(\omega,k) \pm a(\omega,k)$$

$$\Sigma_\pm^{\rm HTL}(\omega,k) = \frac{\hat{M}^2}{k}\left(1 - \frac{\omega\mp k}{2k}\log\frac{\omega+k}{\omega-k}\right)$$

$$\hat{M}^2 = \frac{g^2 C_f}{8}\left(T^2 + \frac{\mu^2}{\pi^2}\right)$$

$C_f = (N^2-1)/2N$ in $SU(N)$; $g^2 C_f \to e^2$ in QED

quadratic scales!



Properties of HTL effective action

$$\hat{M}^2 \int \frac{d\Omega_p}{4\pi} \bar{\psi}\gamma^\mu \frac{\hat{P}_\mu}{\hat{P}\cdot D(A)} \psi - \frac{3}{2}\omega_{\rm pl.}^2 \,{\rm tr} \int \frac{d\Omega_p}{4\pi} F^{\mu\alpha} \frac{\hat{P}_\alpha \hat{P}^\beta}{(\hat{P}\cdot D_{\rm adj.}(A))^2} F_{\mu\beta}$$

- manifestly gauge invariant; gauge independent, too
- non-local
- Hermitian only prior to analytic continuation to real time/frequencies (Landau damping)
- infinitely many (nonlocal) vertices

e.g. $\Gamma^{abc\ \text{cl+HTL}}_{\mu\nu\rho} = igf^{abc}\Big\{ g_{\mu\nu}(k-q)_\rho$ + cycl.

$$+3\omega_{\rm pl.}^2 \int \frac{d\Omega_p}{4\pi} \hat{P}_\mu \hat{P}_\nu \hat{P}_\rho \left[\frac{r_0}{k\cdot\hat{P}\, r\cdot\hat{P}} - \frac{q_0}{k\cdot\hat{P}\, q\cdot\hat{P}} \right] \Big\}$$

QCD: HTL vertices for any number of external gluons

QED: $\hat{P}\cdot D_{\rm adj.}(A)) \to \hat{P}\cdot\partial$:

"only" HTL photon self-energy $\Pi_{\mu\nu}$

fermionic self-energy Σ,

and vertices 2 fermions+any number of photons

NLO corrections to gluonic quasiparticle dispersion laws:

1-loop HTL resummed gluon self-energy:

$\delta \Pi_{\mu\nu} =$ 

1) long wavelength plasmons:

$\vec{k}=0: \quad \omega_{pl.} = \omega_{pl.}^{HTL} + \delta\omega_{pl.} - i\gamma(\vec{k}=0)$

Braaten + Pisarski 1990: $\gamma \approx 0.26\sqrt{N}\, g\, \omega_{pl.}^{HTL}$

Schulz 1993: $\delta\omega_{pl.} = -0.09\sqrt{N}\, g\, \omega_{pl.}^{HTL}$

2) damping of moving plasmons: Pisarski 1993

$$\gamma_{t,l}(k) = \frac{g^2 NT}{4\pi}\, v_{t,l}^{(g)}(k) \ln\frac{\#}{g}$$

$\nwarrow \frac{m_D}{m_m}$!

3) enhanced (dynamical) screening

$\varkappa^2 \equiv -\vec{k}^2 \qquad \delta\varkappa_{t,l}^2(\omega) = \frac{g^2 NT}{2\pi}\, \varkappa_{t,l}^{HTL}(\omega) \ln\frac{\#}{g}$

AR 1993

Flechsig, AR, Schulz '94

$\varkappa_l(0) = m_D$

$\varkappa_t(0) = 0$

4) asymptotic (energetic) quasiparticle masses

$\delta m_\infty^2(k)$

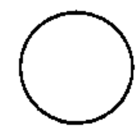
Blaizot, Iancu, AR (2000)

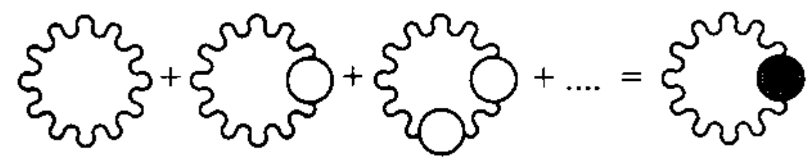
Blaizot, Ipp, AR, Reinosa (2005)

Large N_f limit of QCD and QED

G. D. Moore, JHEP 10 (2002) 055: $N_f \to \infty$, $N_c \sim 1$, $g^2 N_f \sim 1$
as testing ground for weak-coupling techniques at high T

Much simpler than large-N_c:

order N_f^1: 

order N_f^0: 

dressed gluon propagator contains typical gauge-theory phenomena such as

- Debye screening for electrostatic modes
- unscreened magnetostatic modes
- complicated dispersion laws, Landau damping, plasmon damping

and can be solved exactly (nonperturbative w.r.t. $g_{\rm eff}^2 \propto g^2 N_f$)

Large N_f limit of QCD and QED

Effective coupling constant $g_{\rm eff}^2 = \begin{cases} \dfrac{g^2 N_f}{2}, & \text{QCD}, \\ g^2 N_f, & \text{QED}. \end{cases}$

One-loop beta function exact: $\dfrac{1}{g_{\rm eff}^2(\mu)} = \dfrac{1}{g_{\rm eff}^2(\mu')} + \dfrac{\ln(\mu'/\mu)}{6\pi^2}$.

No asymptotic freedom — instead: Landau singularity at exponentially large

$$\Lambda_{\rm L} = \bar\mu_{\rm MS} e^{5/6} e^{6\pi^2/g_{\rm eff}^2(\bar\mu_{\rm MS})}.$$

Theory only exists as cutoff-theory with $\Lambda_{\rm Cutoff} < \Lambda_L$

But thermodynamic potential insensitive to cutoff as long as $T, \mu \ll \Lambda_{\rm L}$

Technicality: cutoff needs to be imposed in Euclidean invariant manner, otherwise spurious singularities

Thermodynamic potential of large-N_f QCD and QED

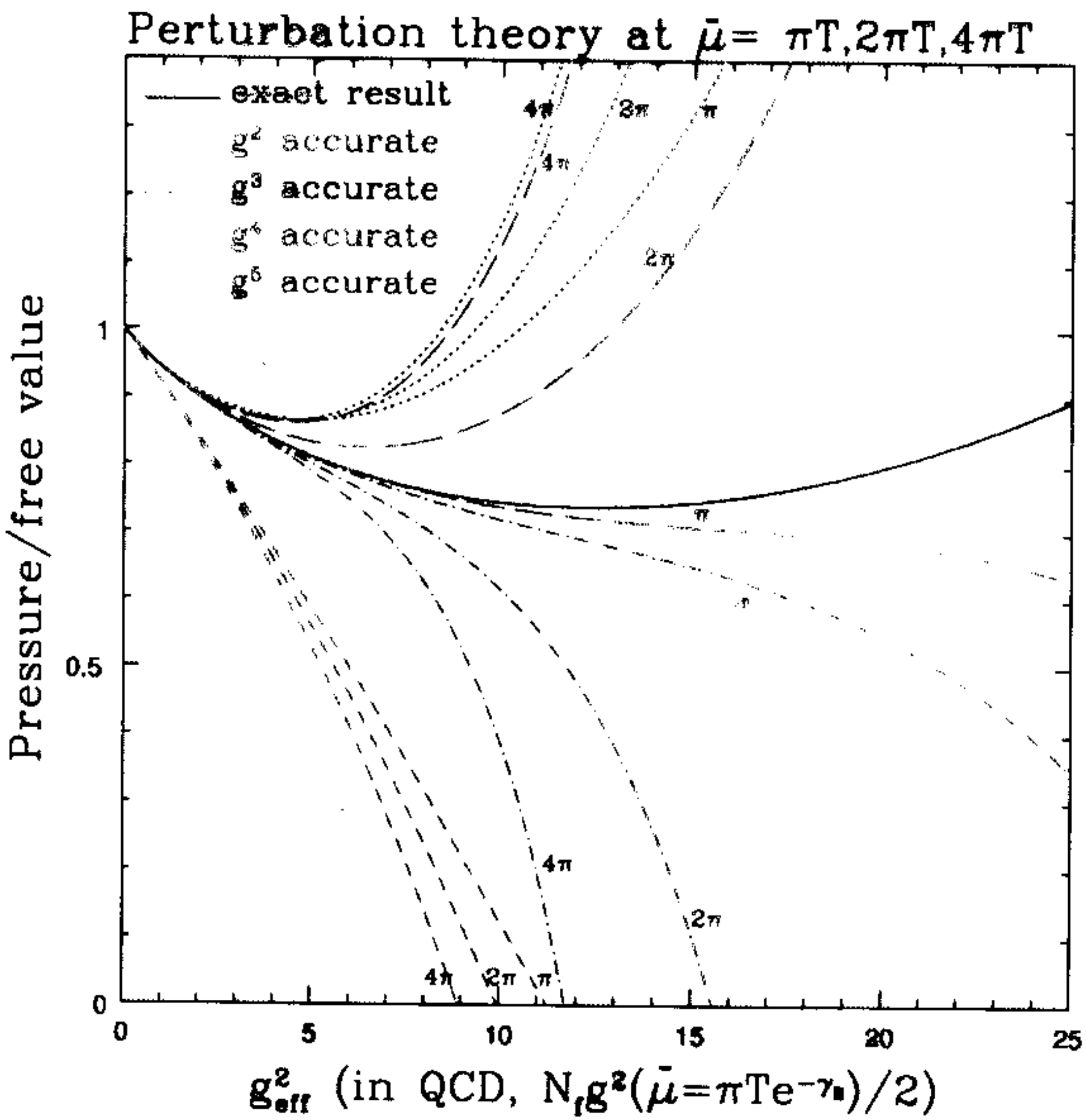
$$
\begin{aligned}
P \;=\;& N N_f \left(\frac{7\pi^2 T^4}{180} + \frac{\mu^2 T^2}{6} + \frac{\mu^4}{12\pi^2} \right) \\
+\;& N_g \int \frac{d^3q}{(2\pi)^3} \int_0^\infty \frac{d\omega}{\pi} \Bigg[2 \Big\{ \left[n_b + \tfrac{1}{2}\right] \,\mathrm{Im}\ln\left(\mathrm{q}^2 - \omega^2 + \Pi_\mathrm{T} + \Pi_\mathrm{vac}\right) \\
& \qquad\qquad - \tfrac{1}{2}\,\mathrm{Im}\ln\left(\mathrm{q}^2 - \omega^2 + \Pi_\mathrm{vac}\right) \Big\} \\
& + \Big\{ \left[n_b + \tfrac{1}{2}\right] \,\mathrm{Im}\ln \frac{\mathrm{q}^2 - \omega^2 + \Pi_\mathrm{L} + \Pi_\mathrm{vac}}{\mathrm{q}^2 - \omega^2} - \tfrac{1}{2}\,\mathrm{Im}\ln \frac{\mathrm{q}^2 - \omega^2 + \Pi_\mathrm{vac}}{\mathrm{q}^2 - \omega^2} \Big\} \Bigg] \\
& + O(N_f^{-1})
\end{aligned}
$$

with $\Pi^{\mu\nu} = \Pi^{\mu\nu}_\mathrm{vac} + \Pi^{\mu\nu}_{mat}$, $\Pi^{\mu\nu}_{mat} \ni \Pi_T, \Pi_L$, 2 distinct structure functions

$P - P_0$ finite as $N_f \to \infty$

Pressure of large-N_f QCD and QED @ $\mu = 0$

G. D. Moore, JHEP 10 (2002) 055 E: hep-ph/0209190, A. Ipp, G. D. Moore, AR, JHEP 01 (2003) 037



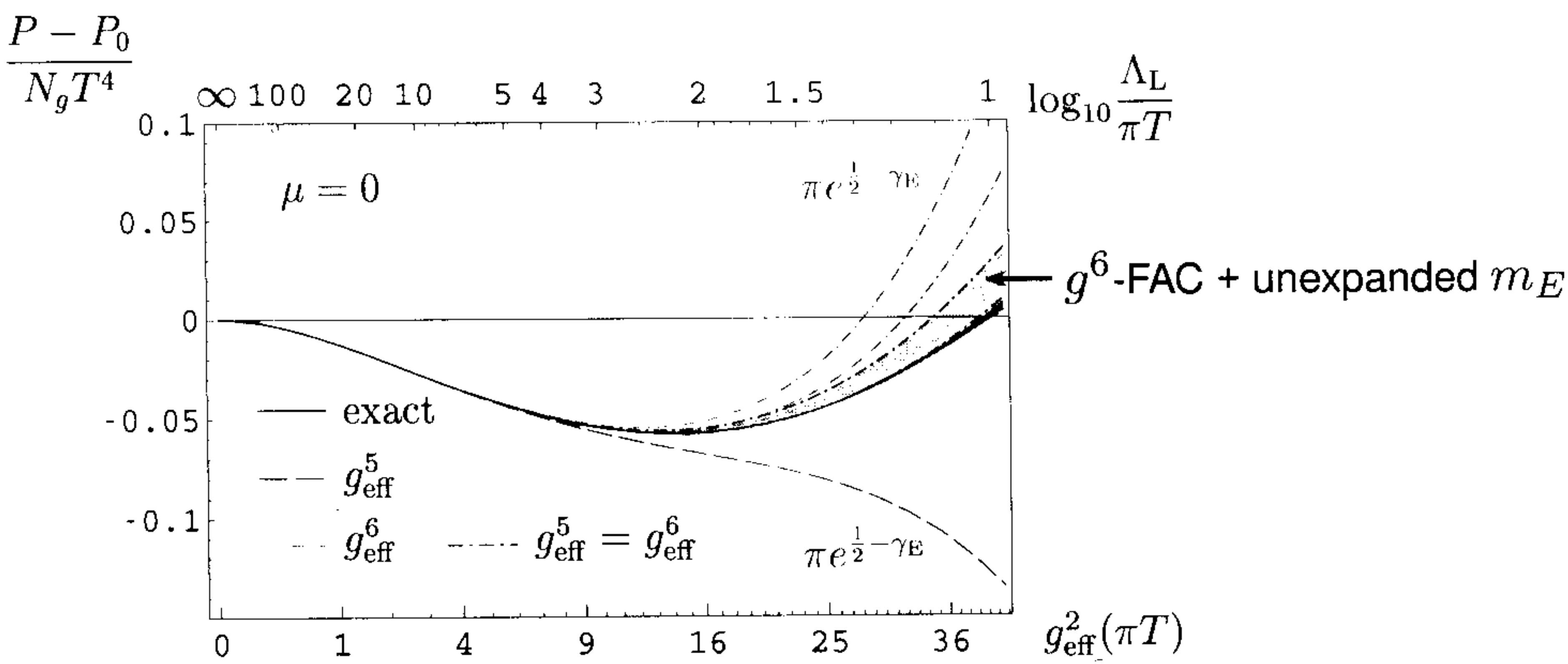
Comparison with
strict perturbation theory

Pressure of large-N_f QCD and QED @ $\mu = 0$

Numerical result sufficiently accurate to verify perturbative results through order g_{eff}^5 and to extract g_{eff}^6 term (no log here!)

$$P\Big|_{g_{\text{eff}}^6,\,\mu=0,\,\bar\mu_{\overline{\text{MS}}}=\pi T} = +20(2)\,N_g\left(\frac{g_{\text{eff}}}{4\pi}\right)^6 T^4$$

Strict perturbative $O(g^6)$-result vs. unexpanded m_E:

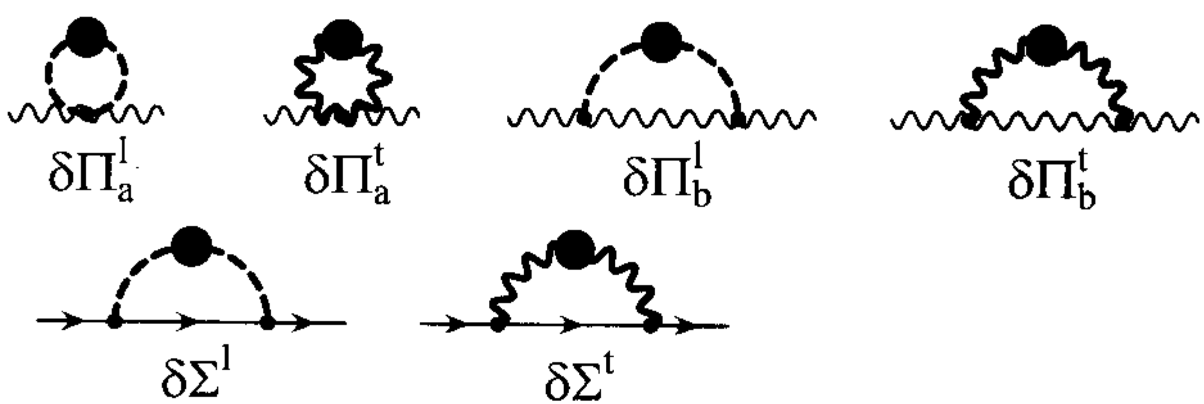


HTL-resummed entropy at high T, $\mu = 0$

Blaizot, Iancu & AR (1999):
HTL resummation through 2-loop Φ-derivable (2PI) entropy expression:

$$\begin{aligned} \mathcal{S} \; = \; & -\mathrm{tr} \int_K \frac{\partial n(k_0)}{\partial T} \left[\Im m \log G^{-1} - \Im m \Pi \, \Re e G \right] \\ & -2\, \mathrm{tr} \int_K \frac{\partial f(k_0)}{\partial T} \left[\Im m \log S^{-1} - \Im m \Sigma \, \Re e S \right], \end{aligned}$$

- nontrivial reorganization of perturbation theory:
  3/4 of awkward g^3 contribution contained in NLO correction to m_∞^2, M_∞^2:

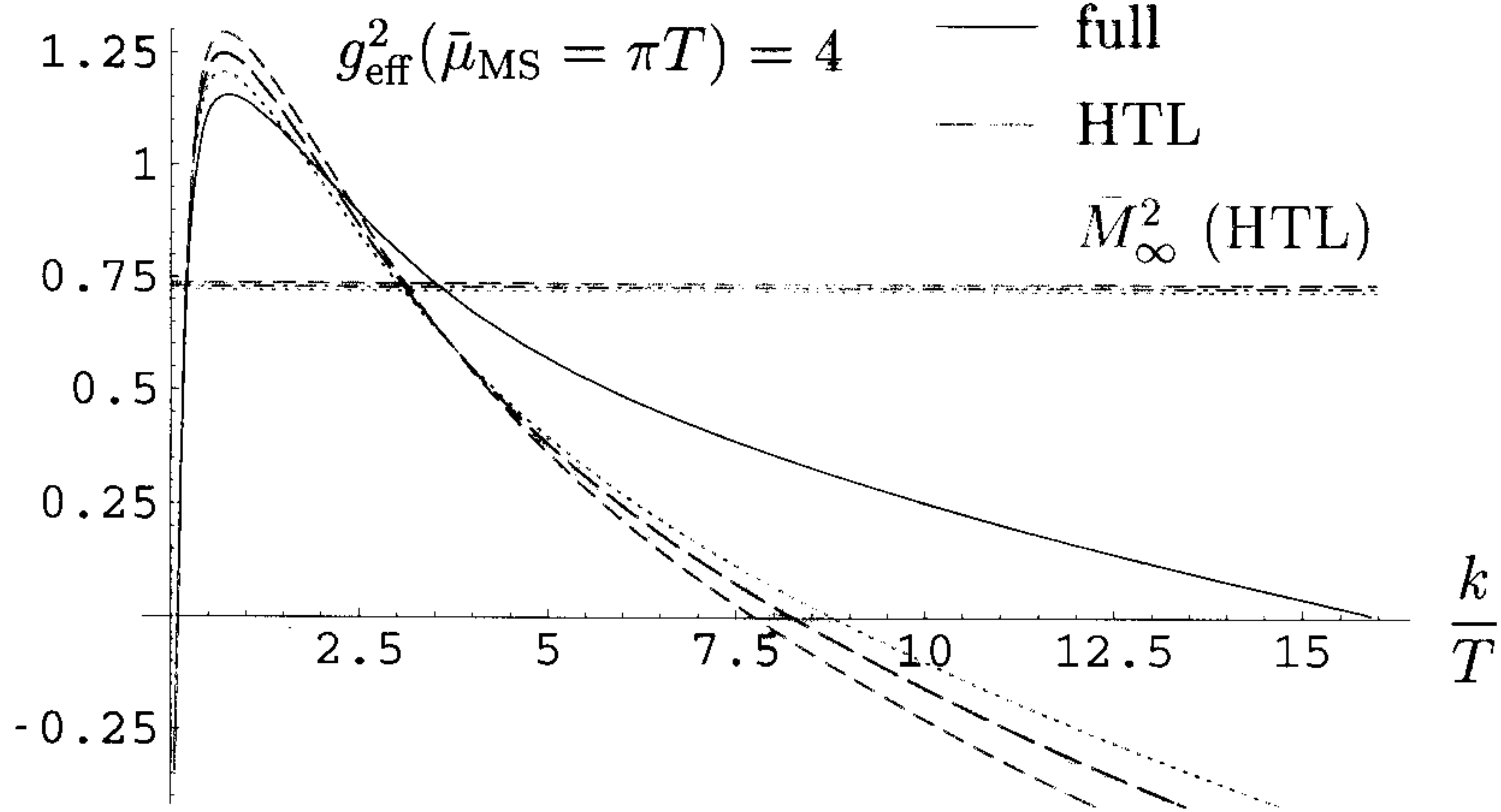


(momentum-dependent even for $k \gg gT$)

Testing HTL-resummed entropy in large-N_f limit

requires NLO asymptotic mass of fermions:

$$M_\infty^2(k) = 2k\,\mathrm{Re}\Sigma_+(\omega = k)N_f/(T^2 C_f)$$



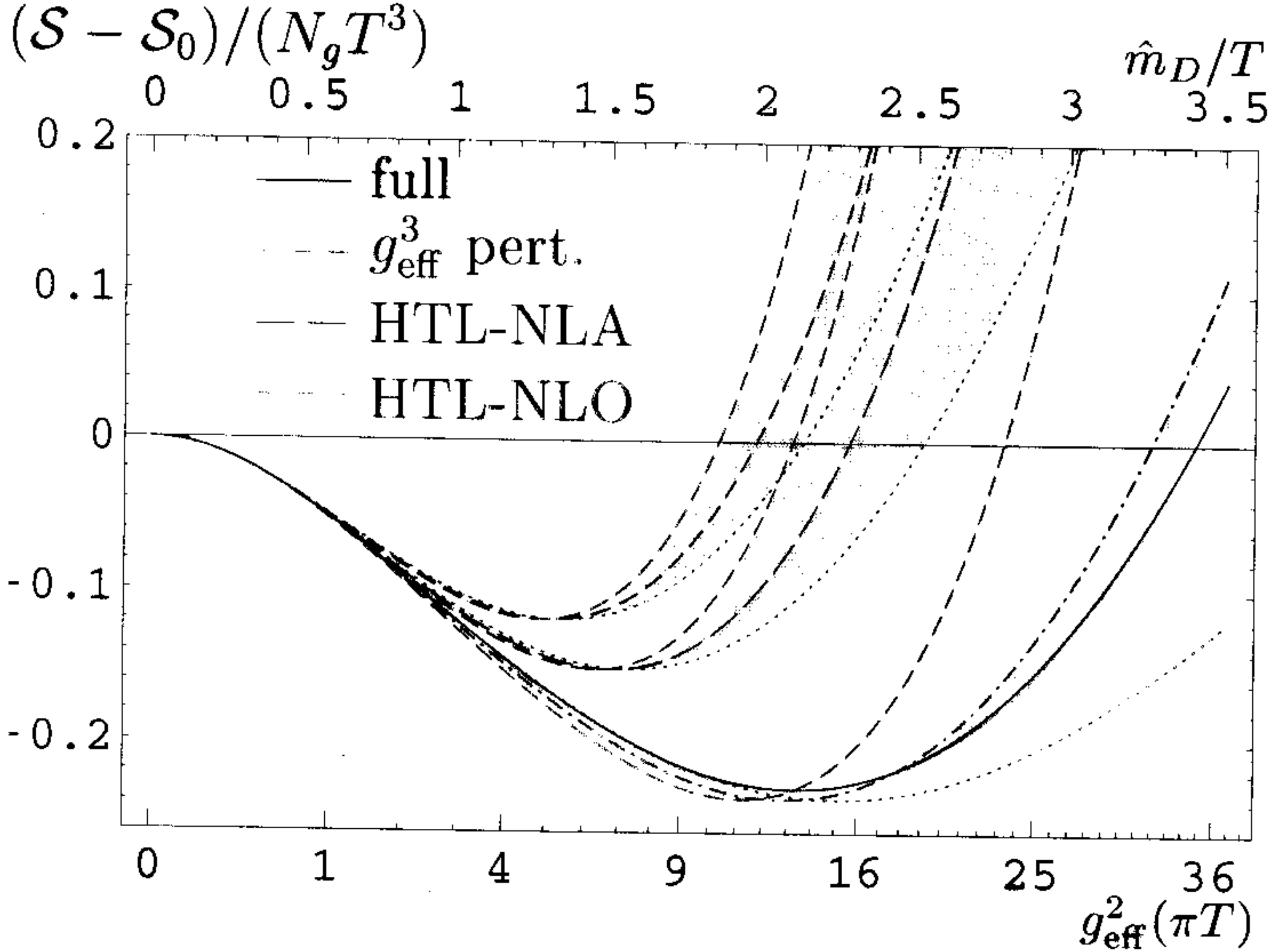
Blaizot, Ipp, AR + Reinosa hep-ph/0509052

$\bar\mu_{\rm MS} = \frac{1}{2}\bar\mu_{\rm FAC-m} \dots 2\bar\mu_{\rm FAC-m}$

-p

Testing HTL-resummed entropy in large-N_f limit

Comparison of large-N_f entropy with perturbative and HTL-resummed results



pert. = strict pert.th. truncated at g^3

HTL-NLA: full LO HTL $+\delta M_\infty^2(k)|_{g^3}$

HTL-NLO: $\delta M_\infty^2(k)$ unexpanded HTL resummat.

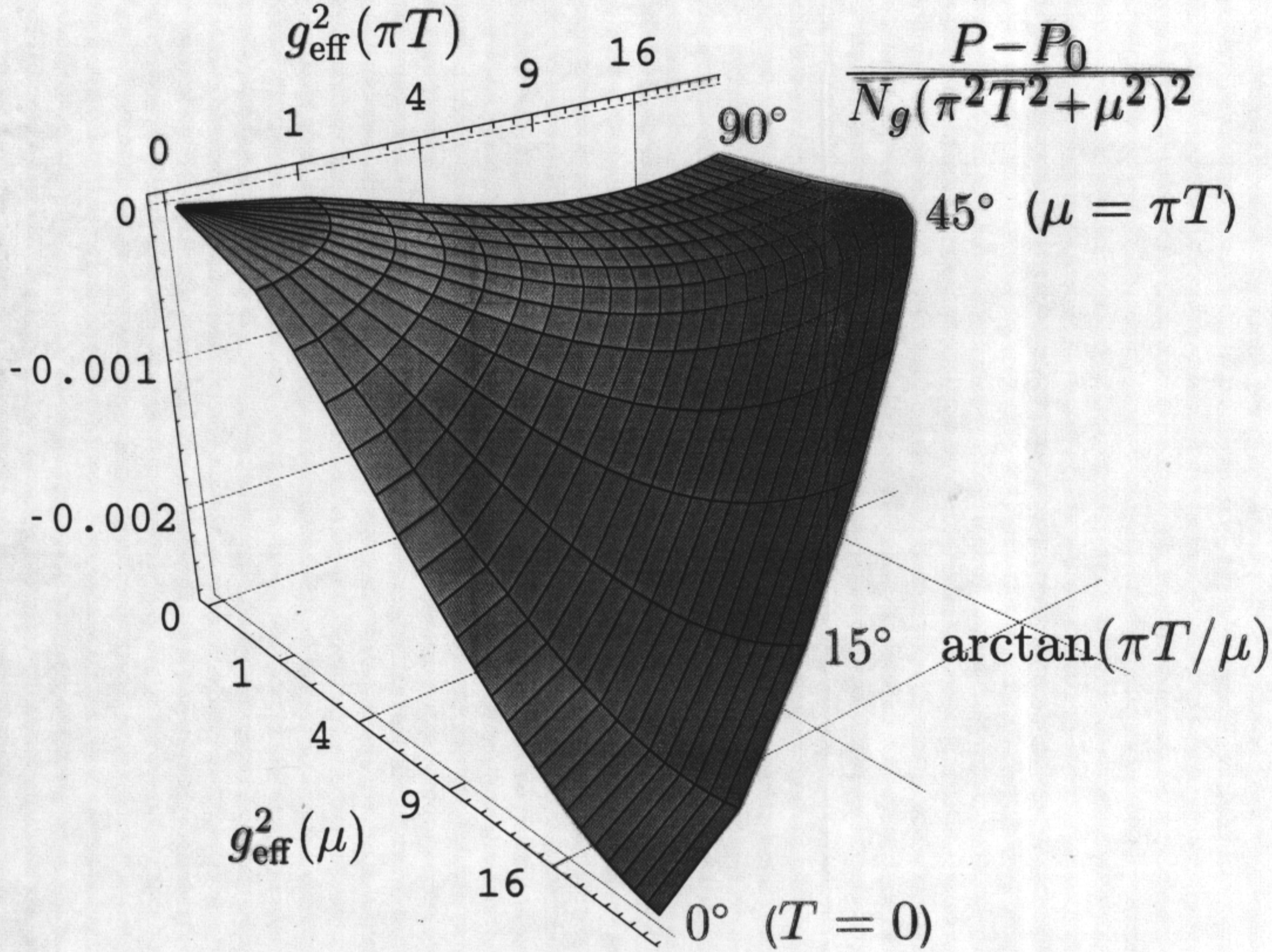
$(\bar\mu_{\rm MS} = \frac{1}{2}\bar\mu_{\rm FAC-m} \ldots 2\bar\mu_{\rm FAC-m})$

Blaizot, Ipp, AR + Reinosa hep-ph/0509052

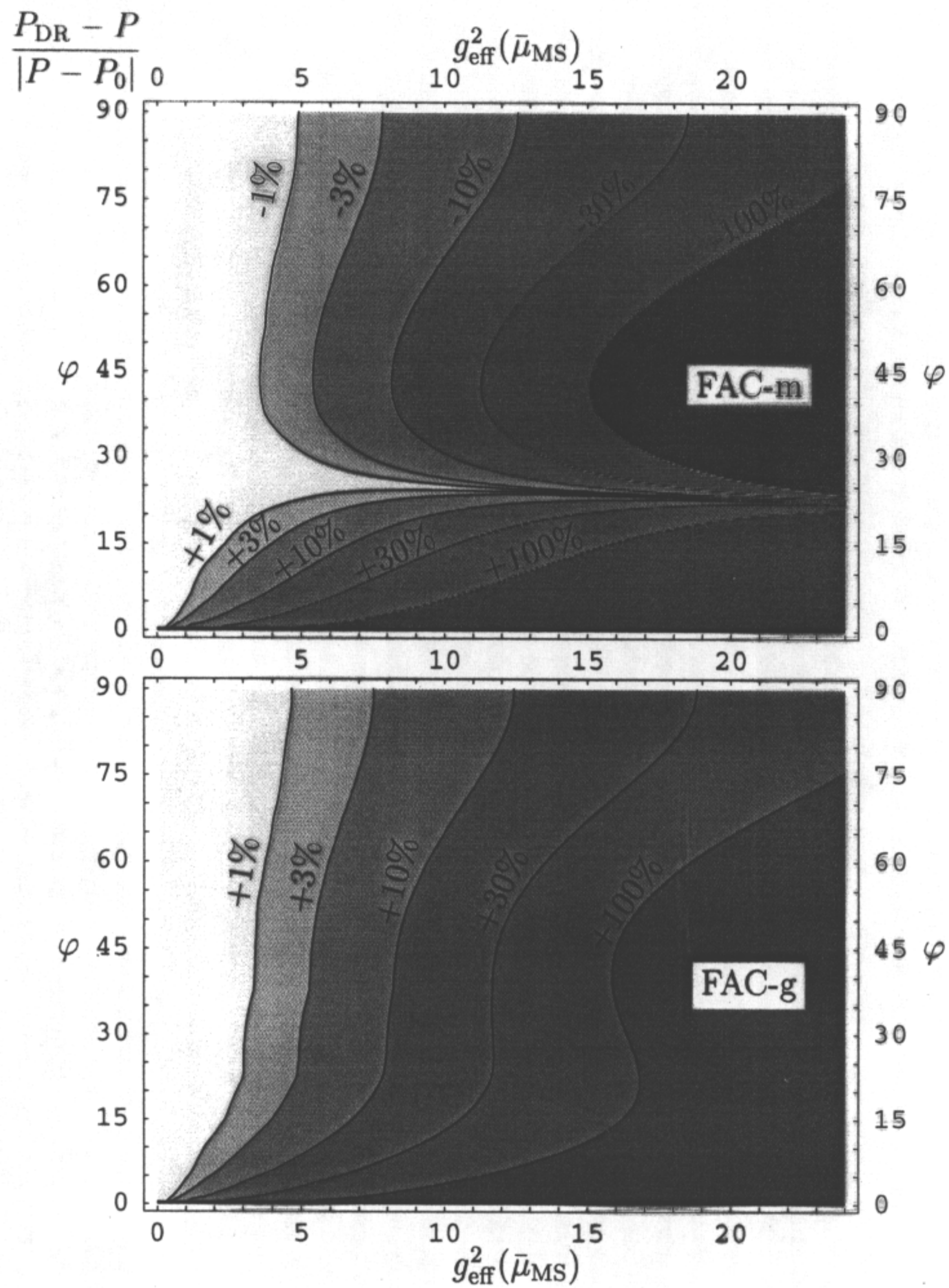
Large-N_f pressure at finite chemical potential μ

Straightforward generalization to finite chemical potential by evaluating $\Pi_{T,L}$ with

$$n_f(k,T,\mu) = \frac{1}{2}\left(\frac{1}{e^{(k-\mu)/T}+1} + \frac{1}{e^{(k+\mu)/T}+1}\right)$$



Comparison with complete dimensional reduction results for all μ, T



Ipp, AR & Vuorinen, PRD69 (2004) 077901

Comparison for two **F**astest **A**pparent **C**onvergence scales (for m_E^2 and g_E^2, resp.)

Breakdown of Dim.Red. for $\varphi \equiv \arctan(\pi T/\mu) \lesssim 18°$, i.e. $T \lesssim 0.1\mu$

Work in progress:

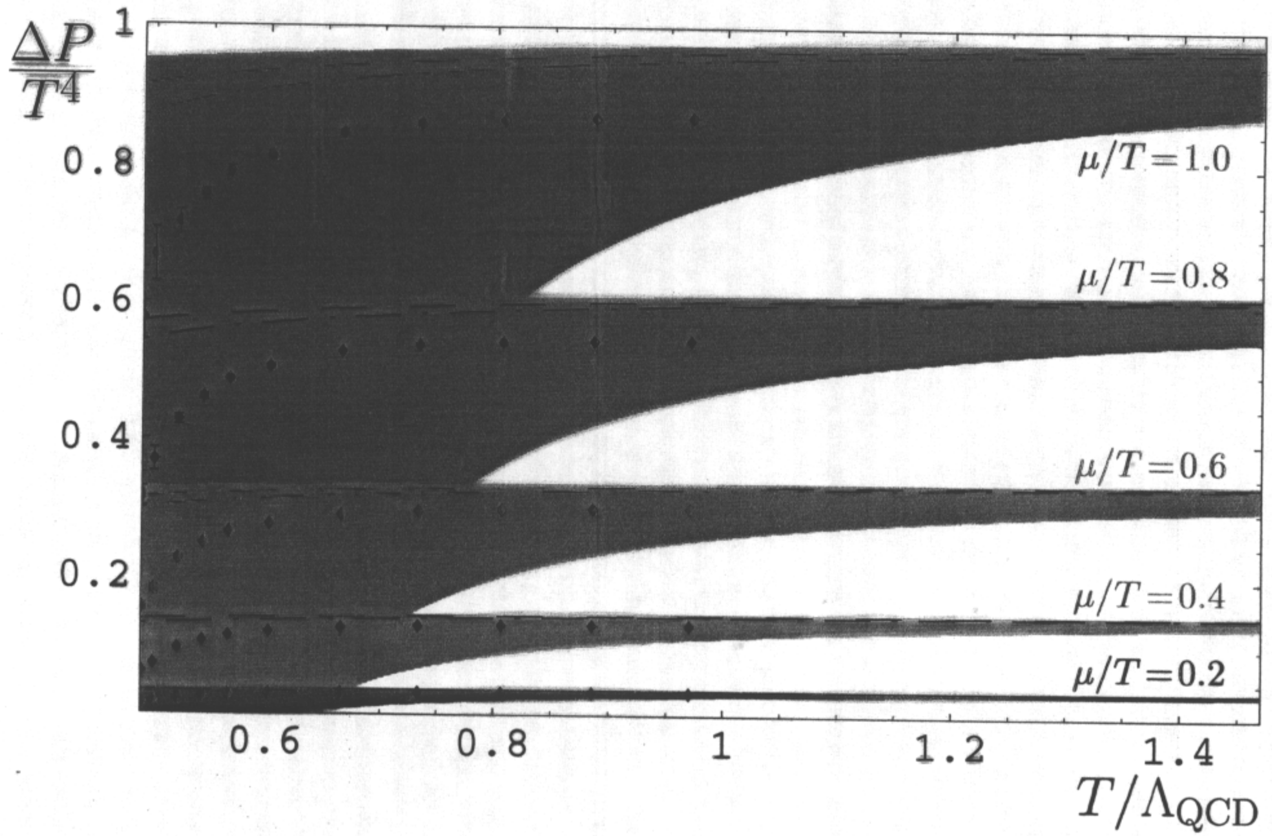
Kajantie, Ipp, AR, Vuorinen

IR-safe part of Dim.Red. together with numerical evaluation of resummed ring diagrams should cover also $T \lesssim 0.1\mu$

Improving apparent convergence in dimensional reduction

Works also at finite chemical potential $\mu \lesssim T$:

→Vuorinen, PRD68 (2003) 054017; Ipp, AR & Vuorinen, PRD69 (2004) 077901



$\Delta P = P(T,\mu) - P(T,0)$ for $N_f = 2$, unexpanded 3-loop results with $\bar{\mu}_{\rm MS}$ varied by a factor of 4 and two FAC schemes (dashed)

vs. lattice data from Allton et al, PRD68 (2003) 014507 (not yet continuum extrapolated!)

Non-Fermi-Liquid Behavior at small $T \neq 0$

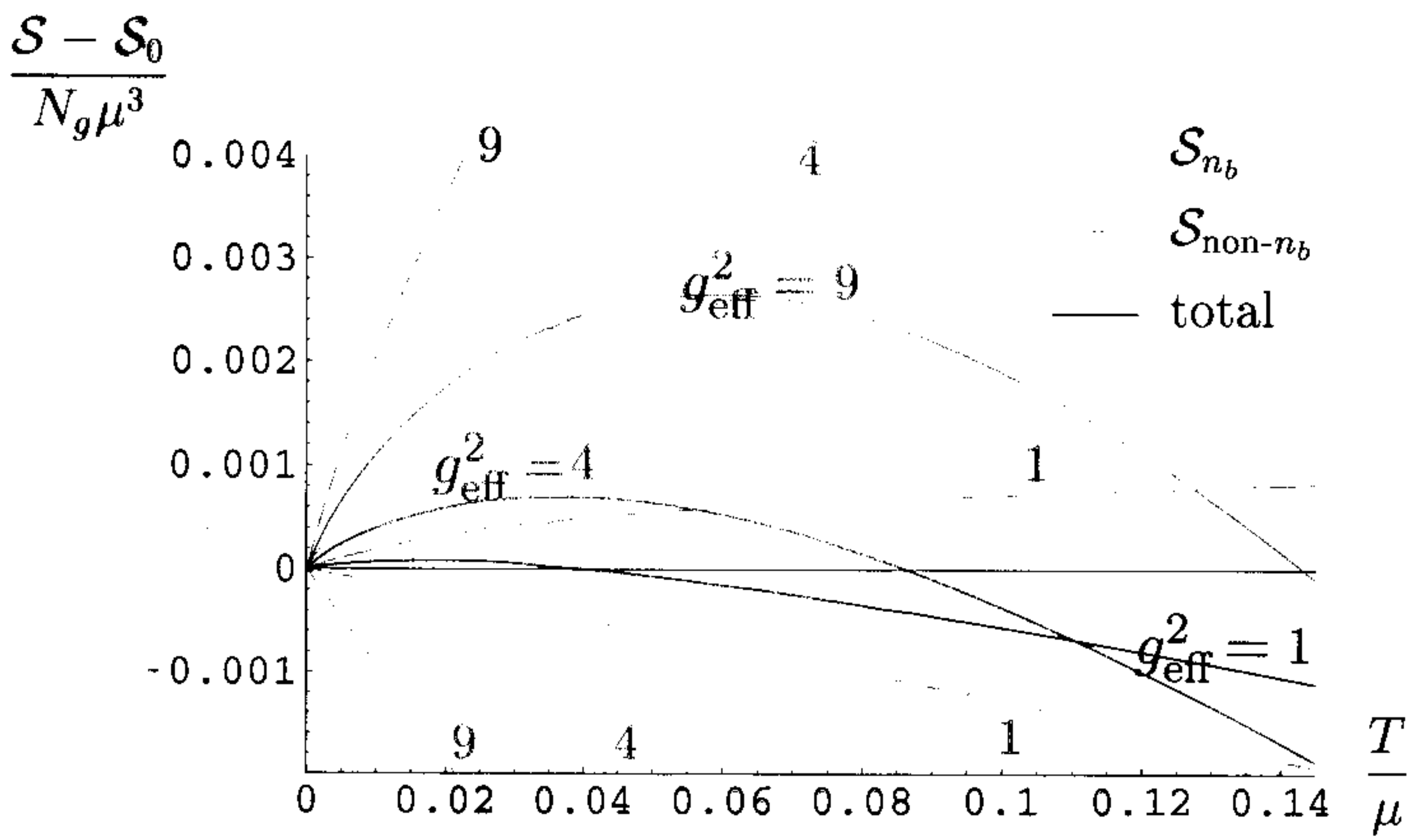
LO (2-loop) term in interaction pressure gives

$$P - P_0 = -N_g \left[\tfrac{5}{9}T^4 + \tfrac{2}{\pi^2}\mu^2 T^2 + \tfrac{1}{\pi^4}\mu^4\right] \frac{g_{\rm eff}^2}{32} + \dots$$

Entropy $\mathcal{S} = \left(\frac{\partial P}{\partial T}\right)_\mu$ at small T should start as

$$\mathcal{S} - \mathcal{S}_0 = \bigcirc\!\!\!\!-\; N_g \frac{g_{\rm eff}^2}{8\pi^2}\mu^2 T + \dots$$

BUT:



Origin of nonanalytic terms in low-T entropy / specific heat:

$$P_{t,n_b} = -N_g \int \frac{d^3q}{(2\pi)^3} \int_0^\infty \frac{dq_0}{\pi} 2 n_b(q_0) \, \mathrm{Im} \ln D_t^{-1}$$

low T, $q_0 \lesssim T$ $\quad \mathrm{Re}\, D_t^{-1} = q^2 + O(q_0^2)$

dyn. scr.

$$\mathrm{Im}\, D_t^{-1} = -\frac{g^2 N_f (4\mu^2 + q^2)}{32\pi} \frac{q_0}{q} \theta(2\mu - q) + \ldots$$

$$\int_0^{2\mu} dq \, q^2 \,\mathrm{atan} \frac{c q_0 (4\mu^2 + q)}{q^3} = \frac{4\mu^2}{3} c q_0 \left(\ln \frac{2\mu}{c q_0} + \frac{5}{2} \right) + O(q_0^{5/2})$$

regular, but $\partial/\partial q_0$ log. div.

Low-temperature expansion of anomalous specific heat

Dynamical magnetic screening scale $\kappa = [\pi m_{\rm D}\omega/4]^{1/3}$
$\longrightarrow$ low-T entropy with log's and fractional powers in T:

T. Holstein, R E. Norton & P. Pincus, PRB8 (1973) 2649; Chakravarty, Norton & Syljuasen, PRL 74 (1995) 1423

A. Ipp, A. Gerhold & AR, PRD69 (2004) 011901R; PRD70 (2004) 105015

$$\frac{S-S_0}{N_g} = \frac{g_{\rm eff}^2\mu^2 T}{36\pi^2}\left(\ln\frac{4g_{\rm eff}\mu}{\pi^2 T} - 2 + \gamma_E - \frac{6}{\pi^2}\zeta'(2)\right)$$
$$-\frac{8\,2^{2/3}\Gamma(\frac{8}{3})\zeta(\frac{8}{3})}{9\sqrt{3}\pi^{11/3}}(g_{\rm eff}\mu)^{4/3}T^{5/3} + \frac{80\,2^{1/3}\Gamma(\frac{10}{3})\zeta(\frac{10}{3})}{27\sqrt{3}\pi^{13/3}}(g_{\rm eff}\mu)^{2/3}T^{7/3}$$
$$+\frac{2048-256\pi^2-36\pi^4+3\pi^6}{540\pi^2}T^3\left[\ln\frac{g_{\rm eff}\mu}{T} - 4.3493485\ldots\right] + O(T^{11/3})$$

- Systematic expansion for $T/\mu \sim g_{\rm eff}^{1+\delta}$ with $\delta > 0$:

$$\frac{S-S_0}{N_g\mu^3} \sim g_{\rm eff}^{3+\delta}\ln\frac{c}{g_{\rm eff}} + g_{\rm eff}^{3+(5/3)\delta} + g_{\rm eff}^{3+(7/3)\delta} + g_{\rm eff}^{3+3\delta}\ln\frac{c}{g_{\rm eff}} + g_{\rm eff}^{3+(11/3)\delta} + \ldots$$

Extension to all $T \ll \mu$

Anomalous low-temperature series is **applicable** only for $T \ll g_{\rm eff}\mu$

complete infinite low-temperature series is contained in

HDL-resummed expression Gerhold, Ipp & AR, PRD70 (2004)

$$\frac{1}{N_g}(\mathcal{S} - \mathcal{S}^{0}) = -\frac{g_{\rm eff}^2\mu^2 T}{24\pi^2} - \frac{1}{2\pi^3}\int_0^\infty dq_0\,\frac{\partial n_b(q_0)}{\partial T}\int_0^\infty dq\,q^2\Bigg[2\,{\rm Im}\,\ln\left(\frac{q^2 - q_0^2 + \Pi_T^{\rm HDL}}{q^2 - q_0^2}\right) + {\rm Im}\,\ln\left(\frac{q^2 - q_0^2 + \Pi_L^{\rm HDL}}{q^2 - q_0^2}\right)\Bigg] + O(g_{\rm eff}^4\mu^2 T)$$

- full leading-order result $\forall\, T \ll \mu$

$g_{\rm eff}\mu \ll T \ll \mu$:

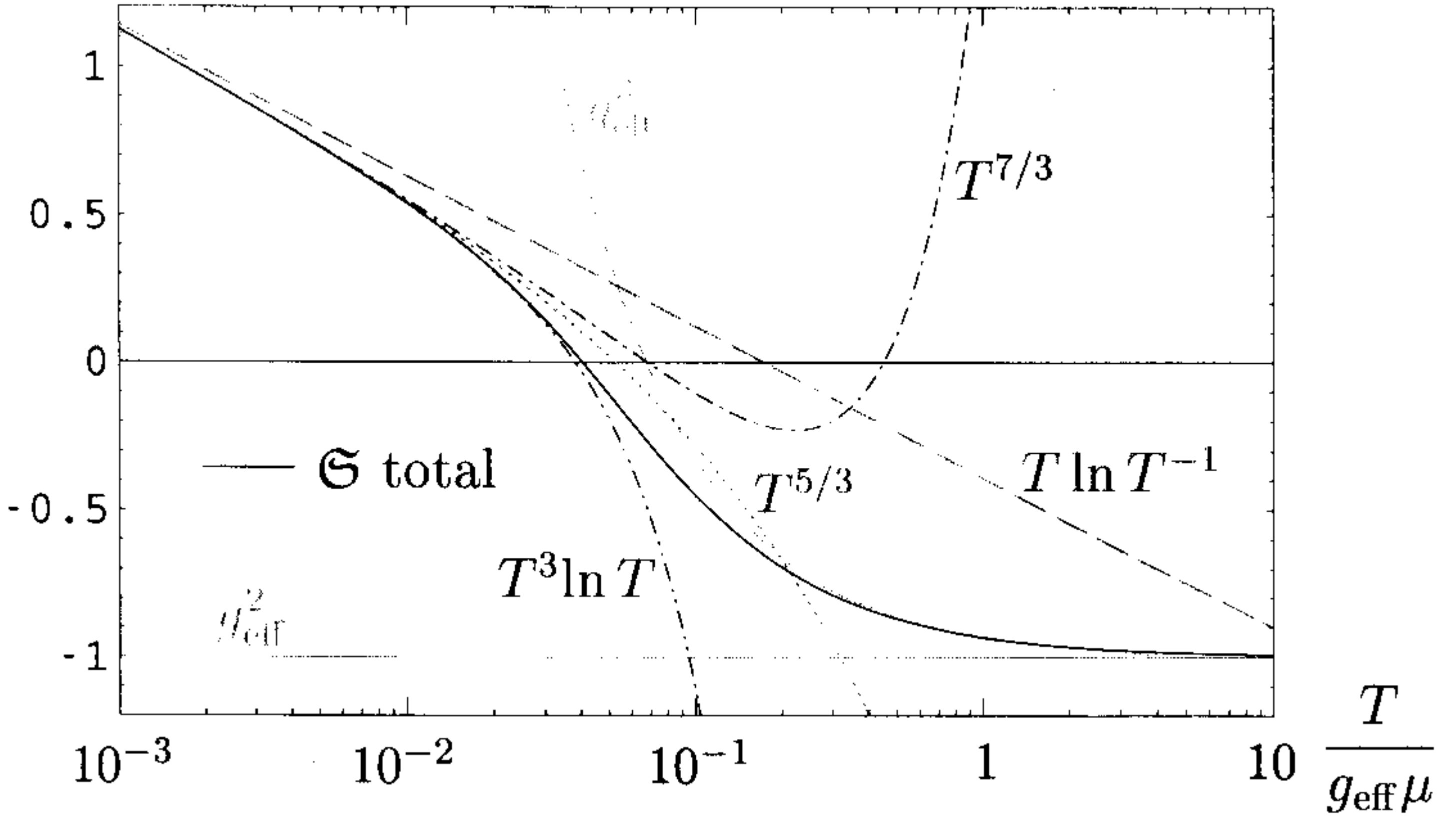
dominant resummation effect now *longitudinal plasmon effect* (Debye screening)

$$\frac{1}{N_g}(\mathcal{S} - \mathcal{S}_0) \simeq -\frac{g_{\rm eff}^2\mu^2 T}{8\pi^2} + \frac{g_{\rm eff}^3\mu^3}{12\pi^4}$$

$\leftarrow$ also from dimensional reduction

HDL-resummed low-T entropy

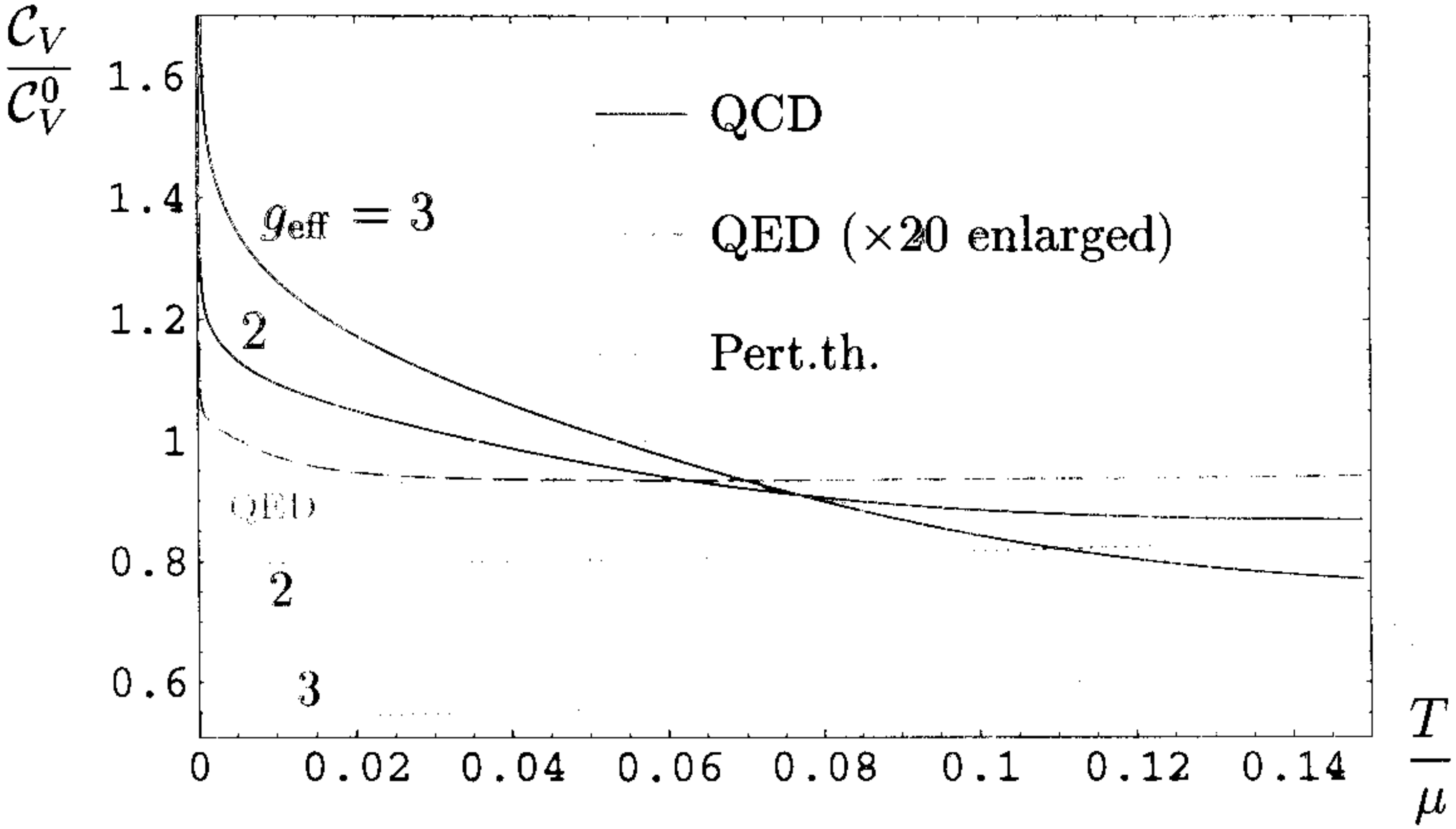
$$\mathfrak{S} = \frac{\mathcal{S} - \mathcal{S}_0}{N_g (g_{\rm eff}\mu)^2 T/(8\pi^2)}$$



low-temperature expansion to order $T \ln T$, $T^{5/3}$, $T^{7/3}$, $T^3 \ln T$, resp.

$g_{\rm eff}$, $g_{\rm eff}^2$: **perturbative result** for $g_{\rm eff}\mu \ll T \ll \mu$ (dim.red.)

HDL-resummed result for the specific heat



$N_f = 2$ QCD: $g_{\rm eff} = 2$ and 3 correspond to $\alpha_s \approx 0.32$ and 0.72

significant deviations from naive perturbative result for low-T specific heat in QCD for $T/\mu \lesssim 0.05$

→ cooling of (proto-)neutron stars with normal quark matter component

Non-Fermi-liquid effects also in neutrino emissitivity (T. Schäfer & K. Schwenzer, PRD70 (2004) 114037)