Double-trace operators in orbifold QFT

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Motivation: why non-SUSY CFT orbifolds? Orbifold theories at strong coupling

The initial idea is based on AdS/CFT which describes the strong coupling limit of a large N orbifolds of $\mathcal{N}=4$ SYM as a dual theory to the $AdS_5 \times S^5/\Gamma$ geometry. Naively this fact immediately implies conformal property since it is a geometrical symmetry of AdS_5. But dynamics may spoil this picture.

There are two different types of the action of the orbifold group Γ on S^5 -with and without fixed points. According to [2] in the first case the twisted sector of string theory in such a geometry contains a tachyon, and therefore the geometry $AdS_5 \times S^5/\Gamma$ is not stable and is not a valid description of QFT. So, we conclude that the orbifolds with fixed point are not conformal at strong coupling. The orbifolds without fixed points are tachyon free. We can understand it from the semi-classical analysis. The untwisted sector is inherited from $AdS_5 \times S^5$ and therefore doesn't contain tachyons. The twisted string starts at one point and ends at another one which is the image of the first point under the action of Γ. Since there is no fixed points the length of the string is of order R-the radius of S^5. But R is very large in the limit under consideration and therefore the string is very massive. So the twisted sector is also free of tachyons. So, we conclude that the freely-acting orbifold is indeed conformal at strong coupling. The next section is devoted to the weak-coupling analysis.

1 Weak coupling regime

The weak coupling analysis is based on the perturbative calculation on the field theory side. Before we turn to the technical details let us review the result obtained before. The first example, calculated by Tsetlyn and Zarembo [1], showed that the theory is not conformal due to renormalization of the double-trace operators. The studies of some other fixed-point examples [2] confirmed this result. The Coleman-Weinberg potential appeared to be non zero and the scalar field Φ gets a nontrivial vev which breaks conformal symmetry. This phenomenon was identified with tachyon condensation in the dual picture. The results presented below (based on [4]) generalized this consideration to a general orbifold with and without fixed points.

The starting point of our consideration is the inheritance theorem by Bershadsky and Johansen [3]. They showed that any planar diagram in the orbifold theory is inherited from $\mathcal{N} = 4$ in a sense that the result of orbifold calculation is just equal to the result of $\mathcal{N} = 4$ calculation after the rescaling of t' Hooft coupling constant $\lambda \to \lambda/|\Gamma|$. Therefore the beta-function for the gauge coupling constant is just zero for any orbifold. This doesn't immediately imply conformal symmetry since the word "planar" has to be clarified: only the planar diagrams with one (external) boundary i.e. those ones which correspond to the single-trace operators are inherited from $\mathcal{N} = 4$. But this argument can not be applied to the double-traces which appear at Coleman-Weinberg potential already at 1-loop level. One more technical comment is related to the representation γ of the orbifold group Γ inside the gauge group which we choose to be $SU(|\Gamma|N)$.

$$R^I_J(g) \in SO(6), \quad \gamma(g) \in SU(|\Gamma|N), \quad g \in \Gamma \tag{1}$$

$$\Phi^I \to R^I_J \gamma \Phi^J \gamma^+ \tag{2}$$

This representation must be regular i.e.

$$Tr(\gamma(g)) \neq 0 \Leftrightarrow g = 1 \in \Gamma \tag{3}$$

Otherwise we do not have any inheritance even for single-trace.

Comment by Vafa: This is a QFT analog of what happens with stringy amplitudes when we sum over all topologies.

With the inheritance in hand we proceed with $\Gamma = Z_n$ orbifold which is specified by the embedding $r : Z_n \to SU(4)$.

$$r = \begin{pmatrix} e^{in_1\alpha} & 0 & 0 & 0 \\ 0 & e^{in_2\alpha} & 0 & 0 \\ 0 & 0 & e^{in_3\alpha} & 0 \\ 0 & 0 & 0 & e^{-i(n_1+n_2+n_3)\alpha} \end{pmatrix} \qquad \alpha = \frac{2\pi k}{n} \tag{4}$$

The matrix R can be easily constructed from r. The representation γ is chosen to be

$$\gamma = diag(1, e^{i\alpha}, e^{2i\alpha}, ..., e^{i(n-1)\alpha}) \tag{5}$$

where 1 is a $N \times N$ unit matrix.

As we mentioned before double-trace operators of the form $Tr(g\Phi^2)Tr(g^+\Phi^2)$ (with $SO(6)$ indexes contracted in all possible ways) will be generated at one loop. So, we need to introduce these operators at tree level with some coupling in front. Then if the theory admits a fixed line when all corresponding beta-functions vanish we denote it as a conformal theory.

We would like to make a comment here. One might be interested in considering the orbifold theory with gauge group $SU(N)^n$ rather then $U(N)^n/U(1) = SU(N)^n \times U(1)^{n-1}$. In other words, one may want to remove unwanted $U(1)$ factors as they require their own coupling constant e. Generally e will renormalize independently of g of $SU(N)$. As a result of this fact there is no reason why β_e has to be zero and one indeed has to remove $U(1)'s$ if wants to find a fixed line. A detailed analysis shows that such a procedure is equivalent to the addition of some double-traces at tree level. Therefore our logic from the last paragraph is not altered: we are looking for a fixed line by adding all possible double-traces at tree level.

The beta function for the coupling f introduced in front of $\frac{1}{N}Tr(g\Phi^2)Tr(g^+\Phi^2)$ will have the following structure:

$$\beta_f \sim \frac{1}{16\pi^2 n}(a\lambda^2 + b\lambda f + cf^2) \tag{6}$$

where a, b, c are some numbers. If we have more than one coupling f, then f will be a matrix as well as a, b, c. Here the $a\lambda^2$ term stands for the Coleman-Weinberg, $b\lambda f$ corresponds to the anomalous dimension of one of $Tr(g\Phi^2)$ and cf^2 is a result of fusion of two traces.

If the action of Z_n leaves one point fixed ($n_1 = -n_2 = n'$,$n_3 = -n_4 = n''$) then there is always an even dimensional space invariant under Z_n. One can consider an operator

$$O_g^{\mu\nu} = Tr(g\Phi^\mu\Phi^\nu) \tag{7}$$

$$O_g^{<\mu\nu>} = O_g^{\mu\nu} - \frac{\delta^{\mu\nu}}{d}O_g^{\rho\rho} \tag{8}$$

where d is the number of invariant dimensions and $\mu.\nu, \rho$ are the indexes along them. This operator has zero anomalous dimension and therefore the corresponding beta-function

$$\beta \sim \sin(n'\alpha/2)^2 \sin(n''\alpha/2)^2 \lambda^2 + f^2 \tag{9}$$

is not zero unless $n' = 0$ and $\mathcal{N} = 2$ supersymmetry is restored. So, we conclude that a general non-supersymmetric orbifold with fixed point is not conformal. This result generalizes the observation made in [2].

A similar story occurs with the free-acting Z_n orbifolds. Several families of orbifold theories were studied in [4] and it was shown that there is no fixed line either.

Finally we have the following table of results:

	Non − susy w. fixed point	Non − susy freely acting	SUSY orbifold
Strong coupling	Not a CFT (tachyon)	CFT (no tachyon)	CFT (no tachyon)
Weak coupling	Not a CFT (see (9))	Not a CFT (see [4])	CFT (inheritance theorem)

The most interesting case is the case of freely acting nonsupersymmetric orbifold at large N. It is conformal at strong coupling regime as follows from AdS description. But there is no fixed line at weak coupling as follows from the one-loop analysis. This means that the "planar" theory undergoes a phase transition at $\lambda \sim 1$ when the fixed line appears. It would be interesting to investigate the behavior at this point in more detail. But unfortunately this is out of scope now since both gravity-dual and perturbative descriptions break down at finite lambda.

Comment by Vafa[1] : The full theory (which includes all $1/N$ terms) is not conformal even at very large coupling due to Casimir energy which is non-zero in the curved background of $AdS \times S^5$. But the "planar" (leading in $1/N$ limit) theory is indeed scale-invariant and the phase transition argument may be applied to it.

References

[1] A. A. Tseytlin and K. Zarembo, "Effective potential in non-supersymmetric SU(N) x SU(N) gauge theory and interactions of type 0 D3-branes," Phys. Lett. B **457**, 77 (1999) [arXiv:hep-th/9902095].

[2] A. Adams and E. Silverstein, "Closed string tachyons, AdS/CFT, and large N QCD," Phys. Rev. D **64**, 086001 (2001) [arXiv:hep-th/0103220].

[3] M. Bershadsky and A. Johansen, "Large N limit of orbifold field theories," Nucl. Phys. B **536**, 141 (1998) [arXiv:hep-th/9803249].

[4] A. Dymarsky, I. R. Klebanov and R. Roiban, "Perturbative search for fixed lines in large N gauge theories," [arXiv:hep-th/0505099].

[1]This is not the actual comment maid during the presentation but the result of further discussion.