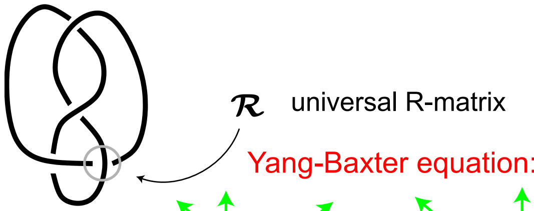
# Unification of Knot Homologies

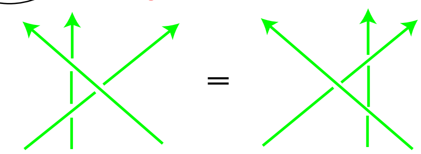
Sergei Gukov
2005 Simons Workshop in Mathematics and Physics

based on joint work [hep-th/0412243] with A.Schwarz and C.Vafa
[math.GT/0505662] with N.Dunfield and J.Rasmussen

## Polynomial Knot Invariants




Yang-Baxter equation:




$$\mathcal{R}_{12}\mathcal{R}_{13}\mathcal{R}_{23} = \mathcal{R}_{23}\mathcal{R}_{13}\mathcal{R}_{12}$$

quantum *sl(N)* invariant: $P_N(q) = \mathrm{Tr}\,(\mathcal{R}\otimes \ldots \otimes\mathcal{R})$

$$q^N P_N(\;) - q^{-N} P_N(\;) = (q^{-1} - q)\, P_N(\;)$$

*N=2:* Jones polynomial:

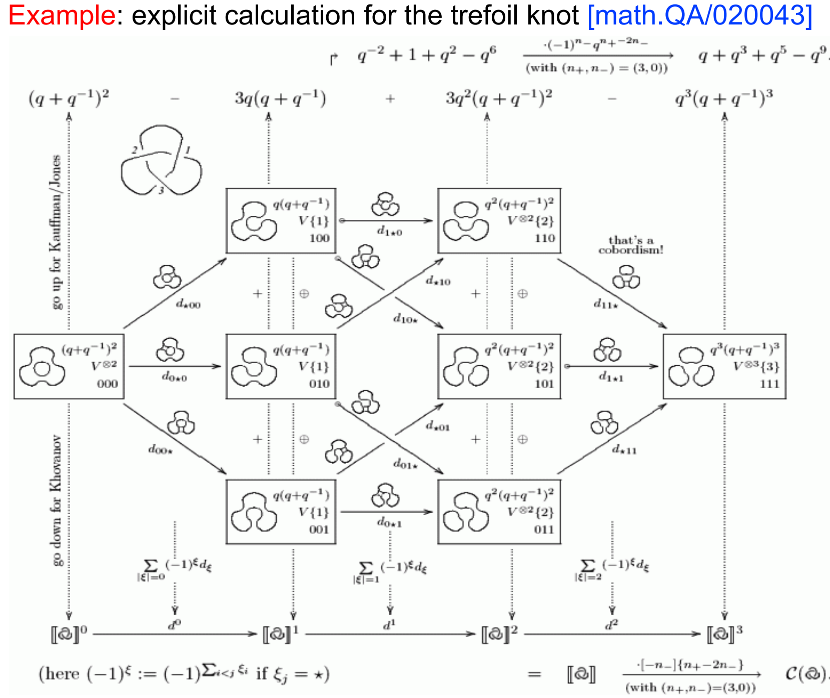
$$q^2 J(\;) - q^{-2} J(\;) = (q^{-1} - q)\, J(\;)$$

Example: $J(\;) = q + q^3 + q^5 - q^9$

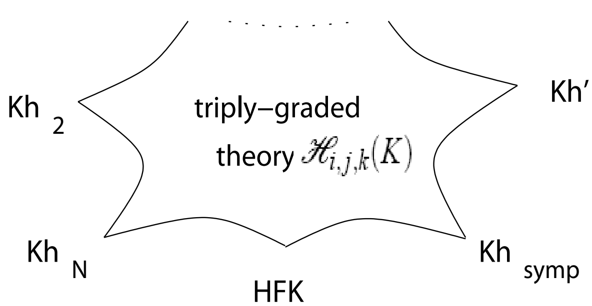
## General Picture of Knot Polynomials and Knot Homologies

| Polynomial Knot Invariant | Quantum Group | Homology |
|---|---|---|
| $\Delta(q)$ | $U_q(gl(1\|1))$ | ``knot Floer homology'' [Ozsvath-Szabo'02] [Rasmussen'03] |
| Jones | $U_q(sl(2))$ | ``Khovanov homology'' [Khovanov'99] |
| $P_N(q)$ | $U_q(sl(3))$ $U_q(sl(N))$ | [Khovanov'03] [Khovanov-Rozansky'04] |
| $Kh^*_{symp} = \oplus_{i-j=*} Kh^{i,j}$ | | ``symplectic Khovanov homology'' [Seidel-Smith'04] |
| $\dim Kh' = 2^l$ | $U_q(sl(1))$ | deformations [Lee'02;Gornik'04] |

Example: explicit calculation for the trefoil knot [math.QA/020043]



## Unification of Knot Homologies




Poincare polynomial = Superpolynomial

$$\mathcal{P}(K)(a,q,t) = \sum a^i q^j t^k \dim \mathcal{H}_{i,j,k}(K)$$

Conjecture: There exists a finite polynomial $\bar{\mathcal{P}}(K) \in \mathbb{Z}[a^{\pm1}, q^{\pm1}, t^{\pm1}]$ such that:

$$\overline{KhR}_N(q,t) = \frac{1}{q-q^{-1}}\bar{\mathcal{P}}(a=q^N, q, t)$$

for sufficiently large *N*.

Reduced Superpolynomial: $\mathcal{P}(K) \in \mathbb{Z}_{\geq 0}[a^{\pm1}, q^{\pm1}, t^{\pm1}]$ such that:

$$KhR_N(q,t) = \mathcal{P}(a=q^N, q, t) \quad \text{for sufficiently large } N.$$

Families of differentials: $d_N : \mathcal{H}_{i,j,k}(K) \to \mathcal{H}_{i-2,j+2N,k-1}(K)$

$$(\mathcal{H}, d_N) = \begin{cases} sl(N) \text{ knot homology}, & N>1 \\ \text{Lee's theory}, & N=1 \\ \text{knot Floer homology}, & N=0 \end{cases}$$

Example: trefoil knot