# The Tachyon at the End of the Universe

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*Notes by Ari Pakman*

Although we are not on the beach as yesterday in Cumrun's talk, I still hope to entertain you with some amusing physics. I am going to describe you some work I have done with John McGreevy [1], and ongoing work with him and Gary Horowitz, about a way string theory can resolve space-like singularities, like those appearing in black holes and in FRW cosmologies. See the references in [1] for background material and other related works.

There exists a long-standing hope that if we extrapolate GR to regions of strong curvature, string theory makes new physics kick in to resolve the singularity. We will see that in some simple examples, even *perturbative* string theory has some power in this regard.

Let us consider first flat spatial slices in a FRW setting (given time we will come back to black holes later), with a metric

$$ds^2 = -dt^2 + R^2(t)\, d\vec{x}^2 + ds_\perp^2 \tag{1}$$

The radius $R(t)$ expands in time, and as we evolve the metric backwards it shrinks, passing through the string scale $l_s$ at time $t_s$ . In this metric we will require the following

- Some of the $\vec{x}$ directions are periodic $x_i \sim x_i + 2\pi$.
- The above circles should be of the so-called Scherk-Schwarz type, namely, spacetime fermions will be anti-periodic around them, so supersymmetry is broken from the beginning.
- We will assume that the matter source in this FRW is weak enough, so that the velocity of the shrinking radius is small, $\dot{R}(t_s) \ll 1$ at the time $t_s$ when the radius reaches the string scale, $R(t_s) \sim l_s$.

The spectrum of strings winding around the Scherk-Schwarz circle goes like (up to order 1 constants)

$$m^2 = -\frac{1}{l_s^2} + \frac{R^2(t)}{l_s^4}\,, \tag{2}$$

where the first term comes from the Casimir energy on the string, and the second from the tension of the string winding the circle. So when $R(t < t_s) < l_s$, we get a tachyon. We are ignoring kinetic contributions to the string energy, and that is one of the reasons to require $\dot{R}(t_s) \ll 1$.

At the scale $l_s$ GR is no longer reliable, and computations diverge and are ambiguous. To illustrate this point, note that both in a worldline or in a worldsheet description, an amplitude will go like

$$Z \sim \int [\mathcal{D}X]\, e^{i\int d^2\tau\, G_{\mu\nu}\partial X^\mu \partial X^\nu}\,. \tag{3}$$

When components of the metric go to zero, say $G_{xx} \sim 0$, the amplitude is no longer dominated by the classical solution, since fluctuations around the latter are no longer supressed. The physics appears to be strongly coupled, both from the worldline/worldsheet point of view, as well as from the spacetime. But in our setting, a new string theoretic degree of freedom appears in the form of the winding tachyon which condenses, so the worldsheet action becomes,

$$Z \sim \int [\mathcal{D}X]\, e^{i\int d^2\tau\, G_{\mu\nu}\partial X^\mu \partial X^\nu + \mu e^{-\kappa X^0} cos(w\Theta)}\,, \tag{4}$$

with $\Theta$ one of the periodic directions and $w$ the winding energy $R/l_s^2$. As is suggested by the form of the path integrand, the effect of the tachyon will be to cure the singularity by suppressing fluctuations that would have been uncontrolled in the pure GR singularity as $G_{xx} \to 0$. The analysis of this effect has close ties to Liouville theory; in a Euclidean continuation the tachyon introduces a Liouville-type wall that regulates the strong coupling problem before the $X^0$ time reaches the singular region. Roughly speaking, this happens because as $X^0$ decreases, the tachyon term increases quicker than $R$ decreases, an effect we will enforce by virtue of the third condition listed above $\dot{R} \ll 1$.

**Question**: Are there other situations, for which the tachyon is not a winding mode?
**Answer**: One interesting generalization would be having a shrinking sphere instead of a circle. In that case, the winding mode is unstable, but there is a mass gap and there is state playing the same tachyonic role. The problem in this case is that we cannot tune the velocity of the shrinking. You can see this from the Friedman equation for positive curvature. This case is under investigation–the rapid velocity introduces some interesting string production that may play an important role.

**Remark (by C. Vafa)**: In the GR side, you have a singularity, whether or not the spacetime fermions are periodic. Whereas the appearance of tachyons depends on having chosen anti-periodic boundary conditions. Now, even if you have had periodic fermions, susy would have been eventually broken at the string scale, but no tachyon would have appeared to resolve the singularity.

- That's right. This is one of the cases where breaking supersymmetry makes things easier. Sometimes enforcing unbroken supersymmetry leads to cancellation of generic effects which, if present, would smooth out the physics. This seems to provide an example of that for which we can controllably analyze the smoothing effect induced by the non-supersymmetric ingredients. Note that for time-like singularities, there is a whole zoo of them, with different quantum numbers, etc., and they get resolved in ways that differ very much in their details. So I don't anticipate a universal resolution of space-like singularities either. Let me add that the case where susy is broken as we did seems to me the case of more physical interest, as supersymmetry breaking in the early universe is generic. Moreover, in higher dimensional Schwarzchild black holes, the susy breaking occurs at the Kaluza-Klein scale of the spatial slices, so it is the type of susy breaking we are considering.

As we said, the tachyon degree of freedom behaves pretty much as a timelike Liouville action, and we will use Liouville-type techniques in our case. But we do not have a linear dilaton, since we are in a theory at the critical dimension. As in Liouville theory, the form of the action (4) is only valid semiclassically.

**Question**: Don't you need a dilaton linear in $X^0$?
**Answer**: When we Wick-rotate in the worldsheet and in spacetime to an Euclidean background, it will be useful to make the dilaton linear in order to obtain a conformal theory at this intermediate step of the analysis of the 2d field theory on the gauge-fixed worldsheet. But when we Wick-rotate back, the linear dilaton we had had on the Euclidean side does not affect those amplitudes we are interested in. And again, in the physical theory (Lorentzian signature, where $X^0$ is time and we are in the critical dimension) there is no linear dilaton (indeed no time dependence in the dilaton) semiclassically.

We often think of tachyons as signaling an instability of the vacuum, but in our approach to this subject, the tachyon is emerging with a more positive role. The tachyon is playing a role in the string version of quantum gravity that was being played previously by Euclidean quantum gravity processes. We see this also in some topology changing processes that are mediated by tachyons [2]. (Note added: the latter examples may have some interesting connections to mathematics, in that they provide a natural string theoretic surgery process that appears automatically as the culmination of Ricci flow.)

**Question**: Should one not be worried by the instability that the tachyon usually represents?
**Answer**: Our approach is to check whether the string perturbative amplitudes form a self consistent set with the right properties such as singularity structure, physical interpretation, and so on. We start with the full perturbative (first quantized) string path integral, with the integrand determined by the semiclassical forms of the physical quantities. Then the $\alpha'$ corrections will be generated by the path integral. In analyzing this system, if it is true that perturbation theory does not break down, then we may not need to worry about extra quantum effects which are non-perturbatively suppressed.

There are many heuristic arguments suggesting that Tachyon condensation in general corresponds to a loss of degrees of freedom, and part of our point is to confirm (or rule out) these heuristic ideas via string theoretic calculations without extra simplifying assumptions. This loss of degrees of freedom has been concretely seen in localized orbifold tachyons and in brane-antibrane annihilation. More generally, this idea stems from various sources:

- Firstly, for an operator such as

$$e^{-\kappa X^0}\, cos(w\Theta) \tag{5}$$

  the matter part $cos(w\Theta)$ is a relevant operator, and the time dependent exponential behaves, at least semi-classically, as an RG flow in which one goes closer to the IR as time goes on (toward the past in the big bang case), and induces in cases like (5) a Kosterlitz-Thouless transition. But this is only an heuristic argument, since we are ignoring possibly important interactions between the matter and the gravity sectors. In other words, given the technical challenges of treating the full time dependence, in such an argument (widely adopted for localized tachyon condensation) one presumes that RG flow gives an accurate trajectory in the string configuration space, without necessarily following the full time dependent evolution conserving energy.

- Secondly, the analogue in quantum field theory involves a manifest massing up of particles, by an analysis of the quantum field theory on the worldline of a particle whose mass has an exponential dependence with time, as we will review below.

- Finally, if we look at the compact circle in (1) in another regime of parameters, at large radius, the KK theory can be formally be analyzed using semi-classical Euclidean quantum gravity techniques. It has an instability studied by Witten [3], and decays into a space-time containing an expanding *bubble of Nothing* which eats the space. So combining this with the above arguments, it is not unreasonable that if we go to the opposite regime, with small radius, the tachyon drives the system towards a *Nothing* phase as well, by losing degrees of freedom. So for those of you tired of string theory being a theory of more than everything, the tachyon condensation leads to a *Theory of Nothing.*

**Question**: Is this final state related to a big-bang or a big-crunch?
**Answer**: For now I will consider a specific ground state, which is the analogue of the Hartle-Hawking state, where you have no excitations on the past and you ask what happens in the future. We are in the processes of assessing the status of excited states using similar techniques. One idea we are pursuing is a possible connection between the state(s) arising here and that required for the black hole final state proposal of Horowitz-Maldacena.

### One loop computation

We would like to go beyond the heuristic arguments exposed above, by using methods that are faithful to the string framework, without neglecting gravitational interactions, or appealing the the quantum field theory analogy (a.k.a. reducing to mini-superspace). This is where the Liouville methods turn out to be useful. We will show first that the tachyon effects can kick in and lift the string modes before we hit a crunch as we go back in time. We consider first the vacuum amplitude at one loop. This encodes a trace over the spectrum of the theory, and we wish to read from it its the region of support in time. So we start with

$$Z_1 = \int [\mathcal{D}(X^\mu, ghosts, moduli)]\, e^{-\int d^2\sigma \left[-(\partial X^0)^2 + (\partial \vec{X})^2 + \mu e^{-\kappa X^0} \hat{T}\right]}\,, \tag{6}$$

where $\hat{T}$ contains fields from the spatial directions. Now we will use techniques borrowed from Liouville theory [4]. I enjoy telling you about these methods, which I realize are old hat to many, but to which I am a new convert. Let us first decouple the $X^0$ field into a zero mode term $X_0^0$ and oscillators terms,

$$X^0 = X_0^0 + \hat{X}^0\,, \tag{7}$$

and consider instead of $Z_1$, its derivative with respect to $\mu$

$$\frac{\partial Z_1}{\partial \mu} = \int \left[\mathcal{D}(\vec{X}, \hat{X}^0, ghosts, moduli)\right] dX_0^0\, e^{-S_{kinetic}} \frac{1}{\kappa} \frac{\partial (e^{-\kappa X_0^0})}{\partial X_0^0} C e^{-\mu e^{-\kappa X_0^0} C}\,, \tag{8}$$

where

$$C \equiv \int d^2\sigma\, \mu e^{-\kappa \hat{X}^0} \hat{T}\,. \tag{9}$$

Now calling $y \equiv e^{-\kappa X_0^0}$, the integral of $X_0^0$ over $(-\infty, +\infty)$ becomes

$$\int_0^\infty dy\, \frac{C}{\kappa} e^{-\mu C y} = \frac{1}{\mu \kappa}\,, \tag{10}$$

so we got rid of $C$! The above integral is well defined for $C > 0$. Otherwise one could define it by analytical continuation, but physically there are IR divergences from regions of negative potential; for this reason we will find it simplest to stick to the heterotic case where such negative regions are absent (in type II we presented their interpretation in the paper). Plunging this into (8), the dependence on $\mu$ is just $1/\mu$, and integrating we get

$$Z_1 = -\frac{\ln(\mu/\mu_*)}{\kappa}\, \hat{Z}_{free} \tag{11}$$

where $\hat{Z}_{free}$ does not include the $X_0^0$ mode, and $\mu_*$ is an integration constant. Now, a similar computation in a compactified *flat* spacetime would have yielded

$$Z_1^{flat} = \delta(0)\, \hat{Z}_{free}^{flat}\,, \tag{12}$$

where the delta function comes from the $X_0^0$ zero mode integral over all time. In our case, even though we integrated over an infinite of interval of time, because of the tachyon term we got

$$\Delta X_0 \sim -\frac{\ln(\mu/\mu_*)}{\kappa} \tag{13}$$

and this is the time range where our amplitudes have support. It corresponds to the range in which the tachyon condensate is absent. We wish this quantity to be much smaller that the time range it takes for the radius of the shrinking circle to reach the string scale, namely

$$\frac{l_s}{\dot{R}(t_s)} \gg \frac{\ln(\mu/\mu_*)}{\kappa} \tag{14}$$

and the idea is that we can always set up a matter system to meet this condition, tuning the velocity to be sufficiently small. Also, because there is no linear dilaton, this computation goes through in higher genera, I believe. It would also be interesting to perform a similar computation involving open strings, to see how the response of strings to D-brane sources behaves in this phase.

We have been working in conformal gauge for the strings. I would like to mention that if we do this in light-cone gauge, an interesting matching arises between the worldsheet and spacetime: a geometric crunch in one looks also like a crunch in the other.

**QFT with a mass growing exponentially in time**

A lot of the physics we are interested in, appears already at the level of quantum field theory of a particle with a mass growing in time, as Strominger and others emphasized in the case of open string tachyons. In a one dimensional worldline description this is

$$S_{worldline} = \int d\tau \left( -(\dot{X}^0)^2 + X^2 - (m_0^2 + \mu^2 e^{-2\kappa X^0}) \right) \tag{15}$$

The mass of the particle is $m_0$ in the far future $X^0 \to \infty$, and diverges in the far past $X^0 \to \infty$. The wave equation yields, in the Heisenberg picture,

$$u_k(x^0 \to -\infty) \quad \sim \quad \frac{1}{\sqrt{2w(x_0)}} e^{\pm i \int^{x^0} dt' w(t')} + WKB\ corrections \tag{16}$$

$$u_k(x^0 \to -\infty) \quad \sim \quad \frac{1}{\sqrt{2w_0}} e^{\pm i w_0 x^0} \tag{17}$$

where $w^2(x^0) = k^2 + m_0^2 + \mu^2 e^{-2\kappa X^0}$ and $w_0^2 = k^2 + m_0^2$ (here $k$ is the spatial momentum quantum number). The exact solution is given in terms of Bessel functions. As usual in these time-dependent cases, if we start with a vacuum state at the far past annihilated by half of the modes

$$a_{in}|in\rangle \tag{18}$$

with $a_{in}^{\dagger}$ denoting creation modes, then at the far future we get $a_{in}$ Bogoliubov-transformed into

$$\alpha a_{out} + \beta a_{out}^{\dagger}|in\rangle. \tag{19}$$

For bosons, say, we normalize these coeficients as

$$|\alpha|^2 - |\beta|^2 = 1 \tag{20}$$

and from the Bessel function of the exact solution we can read

$$\left|\frac{\beta}{\alpha}\right| = e^{-\frac{\pi w_0}{\kappa}} \tag{21}$$

so get that the number of created pairs goes like

$$|\beta|^2 = \frac{1}{e^{\frac{2\pi w_0}{k}} - 1} \tag{22}$$

**Coment (by W.Siegel)**: The equation of motion you are looking looks like that of a field in a deflationary universe. A scalar particle $\Phi$ of mass $m_0$ in a metric

$$ds^2 = a^2(t)\left(-dt^2 + d\vec{x}^2\right)$$

satisfies

$$\nabla^2\Phi + (d-2)\frac{\dot{a}}{a}\Phi = m_0^2 a^2 \Phi\,.$$

and then this is similar to a time-dependent mass! [up to the friction term].

In this mathematical audience, I might have been stopped by you complaining that our Hamiltonian,

$$H = -\frac{\delta^2}{\delta X^{0\,2}} - \mu^2 e^{-2\kappa X^0}\,, \tag{23}$$

is not Hermitian, since it grows quicker than $X^{0\,2}$. One way to express this is that eigenfunctions are no longer orthogonal,

$$\int dt\, \psi_E^*(t)\psi_{E'}(t) \neq \delta(E - E') \tag{24}$$

and in QFT, this makes the LSZ S-matrix break down. In the string context, this will be manifested in non-decoupling of BRST-trivial states from the S-matrix. In QFT a similar breakdown of the S matrix happens is one formulates the field theory on a semi-infinite time range (cutting it off in the past at some finite value). Interestingly, this is happening here by virtue of the time dependent mass, even though the field theory is a priori formulated on full Minkowski space. A standard way out is to impose a ”self adjoint extension”, as we are exploring and was studied by Schomerus in a nice paper [5]. This means that instead of allowing asymptotic solutions as in (16), we restrict to combinations of the form

$$u_k(x_0 \to -\infty) \quad \sim \quad \frac{1}{\sqrt{2w(x_0)}} cos\left(\int^{x^0} dt' w(t') + \frac{\pi}{4} - \pi\nu_0\right) \tag{25}$$

where $\nu_0$ is an arbitrary phase.

This corresponds to a certain set of squeezed states schematically of the form

$$|\nu >= (normalization\times) e^{e^{2\pi i\nu} a^\dagger a^\dagger} |in >, \tag{26}$$

cancelling the BRST anomaly. If we consider the linear combination of these solutions obtained by integrating over the phases, we get our "thermal" in vacuum $|in>$. In the string context, we are still in the process of determining the consequences of this BRST anomaly and its cancellation; it seems like a promising way to limit and characterize the allowed states of the system.

**Back to the strings: pair creation**

Finally let us complete our string computations. This section will be rather telescopic since the analysis is presented in the paper. In a realistic context, in particular to avoid bulk tachyons, we formulate our tachyon condensation process in a Type II or heterotic theory. In the latter case, for example, our winding tachyon condensate action is

$$\int d\sigma d\tau d\theta \left[\mathcal{L}_{kinetic+ghosts} - \mu\Psi e^{-\kappa\mathcal{X}^0}\cos(w\tilde{\Theta})\right], \tag{27}$$

where

$$\mathcal{X}^\mu = X^\mu + \theta^+\psi_+^\mu \qquad\qquad \Psi^a = \psi_-^a + \theta F^a\,. \tag{28}$$

The nice thing about the heterotic case is that the classical bosonic potential in the worldsheet is non-negative, given by $e^{-2\kappa X^0} cos^2(w\theta)$.

For computations , it is convenient to perform a Wick rotation by defining

$$\tau = e^{i\gamma}\tau_\gamma \qquad \mathcal{X}^\mu = e^{i\gamma}\mathcal{X}^\mu_\gamma \qquad \mu = e^{i\gamma}\mu_\gamma \qquad k = e^{-i\gamma}k_\gamma \tag{29}$$

and then taking $\gamma = \frac{\pi}{2}$. We can include additionally a shift to obtain a linear dilaton in the rotated theory, and hence conformal invariance of the semiclassical field theory path integral there, though this does not affect the results (see v2 of the paper for comments on this).

Previously we computed the vacuum amplitude without such a rotation. Doing the rotation corresponds to working in a different vacuum. When performing this Wick rotation in our vacuum amplitude (11) we get

$$Z_1^E = \hat{Z}_{free} \left( -\frac{\ln(\mu/\mu_*)}{k} - \frac{i\pi}{2\kappa} \right) \tag{30}$$

from which we see that we have a thermal vacuum. The rate of pair creation is obtained from the two point function of vertex operators, and corresponds to $|\beta/\alpha|$ in the field theory (see (21)). In the rotated theory, this corresponds to the reflection coefficient divided by the incoming flux on the Liouville wall, which has magnitude 1. An integral over the zero modes similar to the one we did before yields a result proportional to $\mu^{i\sum\omega/\kappa}$ so that rotating the magnitude 1 result back yields

$$\langle V_w V_w \rangle \sim e^{-\frac{\pi w}{k}} \tag{31}$$

which notably matches our previous field theory result. Hence we obtain a thermal spectrum at the linearized level, with amplitudes dying before strong coupling effects kick in.

Let me finish by saying I find it fascinating that there is this simple regime where perturbative string effects smooth the singularity. For reasons I mentioned above, this is reminiscent of Euclidean quantum gravity computations of old but it is a perturbative effect rather than a (perhaps less clear) tunneling effect. This is very much in line with many physical arguments and now also with some more precise string theoretic computations. We are presently exploring the consequences of all this for black-hole physics, related also to the recent result [6].

## References

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