Sally Dawson, BNL Collider Physics for String Theorists Stony Brook, Summer, 2007

- Introduction to the Standard Model
 - **\Box** Review of the SU(2) x U(1) Electroweak theory
 - Experimental status of the EW theory
 - Constraints from Precision Measurements
- Searching for the Higgs Boson
- The Importance of the TeV Scale
- Why the fuss over the MSSM?

Collider Physics Timeline



First collisions in May, 2008





Planned shut-down in 2009

Large Hadron Collider (LHC)

- proton-proton collider at CERN (2008)
- 14 TeV energy
 - 7 mph slower than the speed of light
 - cf. 2 TeV @ Fermilab
 - (307 mph slower than the speed of light)
- Typical energy of quarks and gluons 1-2 TeV



Requires Detectors of Unprecedented Scale



- Two large multipurpose detectors
- CMS is 12,000 tons (2 x's ATLAS)
- ATLAS has 8 times the volume of CMS

Detectors

ATLAS and CMS will be ready for pilot physics run in May, 2008



ATLAS, 9/06

14 TeV physics run in 2008

- Initially run at low luminosity $(2 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1})$
- Ramp to full luminosity in 2010 (10^{34} cm⁻² s⁻¹)

Standard Model Synopsis



Gauge bosons:
 SU(3): G_μⁱ, i=1...8
 SU(2): W_μⁱ, i=1,2,3
 U(1): B_μ

- Gauge couplings: g_s, g, g'
- SU(2) Higgs doublet: Φ

SM Higgs Mechanism

 Standard Model includes complex Higgs SU(2) doublet

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$

- With SU(2) x U(1) invariant scalar potential $V = \mu^2 \Phi^+ \Phi + \lambda (\Phi^+ \Phi)^2$
- If $\mu^2 < 0$, then spontaneous symmetry breaking
- Minimum of potential at: $\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$
 - Choice of minimum breaks gauge symmetry

• Why is
$$\mu^2 < 0?$$

More on SM Higgs Mechanism

 Couple Φ to SU(2) x U(1) gauge bosons (W_i^μ, i=1,2,3; B^μ)

$$L_{S} = (D^{\mu}\Phi)^{+}(D^{\mu}\Phi) - V(\Phi)$$
$$D_{\mu} = \partial_{\mu} - i\frac{g}{2}\sigma^{i}W^{i}{}_{\mu} - i\frac{g}{2}Y_{\Phi}B_{\mu}$$

• Gauge boson mass terms from:

$$(D_{\mu}\Phi)^{+}D^{\mu}\Phi \to \dots + \frac{1}{8}(0,v)(gW_{\mu}^{a}\sigma^{a} + g'B_{\mu})(gW^{b\mu}\sigma^{b} + g'B^{\mu})\begin{pmatrix}0\\v\end{pmatrix} + \dots + \frac{v^{2}}{8}(g^{2}(W_{\mu}^{1})^{2} + g^{2}(W_{\mu}^{2})^{2} + (-gW_{\mu}^{3} + g'B_{\mu})^{2}) + \dots$$

More on SM Higgs Mechanism

• With massive gauge bosons:

$$W_{\mu}^{\pm} = (W_{\mu}^{1} \mp W_{\mu}^{2}) / \sqrt{2}$$

$$Z_{\mu}^{0} = (g W_{\mu}^{3} - g' B_{\mu}) / \sqrt{(g^{2} + g'^{2})}$$

$$M_W = g_V/2$$

 $M_Z = \sqrt{(g^2 + g'^2)_V/2}$

• Orthogonal combination to Z is massless photon

$$A_{\mu}^{0} = (g' W_{\mu}^{3} + gB_{\mu}) / \sqrt{(g^{2} + g'^{2})}$$

More on SM Higgs Mechanism

Weak mixing angle defined
$$\cos \theta_{W} = \frac{g}{\sqrt{g^{2} + {g'}^{2}}} \quad \sin \theta_{W} = \frac{g'}{\sqrt{g^{2} + {g'}^{2}}}$$

$$Z = -\sin \theta_{W}B + \cos \theta_{W}W^{3}$$

$$A = \cos \theta_{W}B + \sin \theta_{W}W^{3}$$

$$M_{W} = M_{Z} \cos \theta_{W}$$

Natural relationship in SM—Provides stringent restriction on Beyond the SM models

Fermi Model

Current-current interaction of 4 fermions

 $L_{FERMI} = -2\sqrt{2}G_F J_\rho^+ J^\rho$

Consider just leptonic current

$$J_{\rho}^{lept} = \overline{v}_{e} \gamma_{\rho} \left(\frac{1 - \gamma_{5}}{2}\right) e + \overline{v}_{\mu} \gamma_{\rho} \left(\frac{1 - \gamma_{5}}{2}\right) \mu + hc$$

- Only left-handed fermions feel charged current weak interactions (maximal P violation)
- This induces muon decay



Muon decay

- Consider $\nu_{\mu} e \rightarrow \mu \nu_{e}$
- Fermi Theory:







$$-i2\sqrt{2}G_F g_{\mu\nu}\overline{u}_{\mu}\gamma^{\mu}\left(\frac{1-\gamma_5}{2}\right)u_{\nu_{\mu}}\overline{u}_{\nu_{e}}\gamma^{\nu}\left(\frac{1-\gamma_5}{2}\right)u_{e}$$

For
$$|\mathbf{k}| << M_W, 2\sqrt{2G_F} = g^2/2M_W^2$$

$$\frac{ig^2}{2}\frac{1}{k^2-M_W^2}g_{\mu\nu}\overline{u}_{\mu}\gamma^{\mu}\left(\frac{1-\gamma_5}{2}\right)u_{\nu_{\mu}}\overline{u}_{\nu_{e}}\gamma^{\nu}\left(\frac{1-\gamma_5}{2}\right)u_{e}$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2}$$

For
$$|k| >> M_W, \sigma \sim 1/E^2$$

Parameters of $SU(2) \ge U(1)$ Sector

- g, g', μ , $\lambda \Rightarrow$ Trade for:
 - $\square \alpha = 1/137.03599911(46)$ from (g-2)_e and quantum Hall effect
 - \Box G_F=1.16637(1) x 10⁻⁵ GeV⁻² from muon lifetime
 - □ M_Z=91.1875±0.0021 GeV
 - Plus Higgs and fermion masses

SM is VERY PREDICTIVE THEORY!!!

Inadequacy of Tree Level Calculations

Mixing angle is predicted quantity

- On-shell definition $\cos^2\theta_W = M_W^2/M_Z^2$
- \Box Predict M_W

$$M_{W}^{2} = \pi \sqrt{2} \frac{\alpha}{G_{F}} \left(1 - \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_{F}M_{Z}^{2}}} \right)^{-1}$$

$$s_W^2 c_W^2 = \frac{\pi \alpha}{G_F M_Z^2}$$

- Plug in numbers:
 - M_W predicted =80.939 GeV
 - $M_W(exp) = 80.398 \pm 0.025$ GeV
- Need to calculate beyond tree level

Modification of tree level relations

$$G_F = \frac{\pi\alpha}{\sqrt{2}M_W^2 \sin^2 \theta_W} \frac{1}{(1-\Delta r)}$$

 $\Box \Delta r$ is a physical quantity which incorporates 1-loop corrections

 \Box Contributions to Δr from top quark and Higgs loops

$$\Delta r^{t} = -\frac{3G_{F}m_{t}^{2}}{8\sqrt{2}\pi^{2}} \left(\frac{\cos^{2}\theta_{W}}{\sin^{2}\theta_{W}}\right)$$

Extreme sensitivity of

precision measurements to m_t

$$\Delta r^{h} = \frac{11G_{F}M_{W}^{2}}{24\sqrt{2}\pi^{2}} \left(\ln \frac{M_{h}^{2}}{M_{W}^{2}} - \frac{5}{6} \right)$$

World Average for W mass

- Direct measurements (Tevatron/LEP2) and indirect measurements (LEP1/SLD) in excellent agreement
- Indirect measurements
 assume a Higgs mass



W Mass Measurement



Location of peak gives M_W

Shape of distribution sensitive to $\Gamma_{\rm W}$

Statistics enough to best LEP 2

Why doesn't the top quark decouple?

- In QED, running of α at scale μ not affected by heavy quarks with $m_q >> \mu$
- Decoupling theorem: diagrams with heavy virtual particles don't contribute at scales $\mu \ll m_q$ if
 - \Box Couplings don't grow with m_q
 - Gauge theory with heavy quark removed is still renormalizable
- Spontaneously broken SU(2) x U(1) theory violates both conditions
 - Longitudinal modes of gauge bosons grow with mass
 - **Theory without top quark is not renormalizable**

Latest Value for Top Quark Mass



Quantum Corrections are sensitive to the Higgs Mass

- Direct observation
 of W boson and
 top quark (blue)
- Inferred values from precision measurements (pink)



Higgs Searches at LEP2

- LEP2 searched for $e^+e^- \rightarrow Zh$
- Rate turns on rapidly after threshold, peaks just above threshold, σ~β³/s

■ LEP2 limit, M_h > 114.1 GeV



e⁺e⁻ -> Z h



Data prefer light Higgs

- Low Q² data not included
 - Doesn't include atomic parity violation in cesium, parity violation in Moller scattering, & neutrinonucleon scattering (NuTeV)
- M_h< 182 GeV
 - 1-sided 95% c.l. upper limit, including direct search limit
 - Best fit is in excluded region



Understanding Higgs Limit

$$M_{W} = 80.364 - 0.0579 \ln\left(\frac{M_{h}}{100 \ GeV}\right) - 0.008 \ln^{2}\left(\frac{M_{h}}{100 \ GeV}\right)$$
$$-0.5098\left(\frac{\Delta \alpha_{had}^{(5)}(M_{Z})}{0.02761} - 1\right) + 0.525\left[\left(\frac{M_{t}}{172 \ GeV}\right)^{2} - 1\right]$$
$$-0.085\left(\frac{\alpha_{s}(M_{Z})}{0.118} - 1\right)$$

 M_W (experiment)=80.398 ± 0.025 GeV

This assumes the Standard Model



$$\sigma_{Z-peak}\left(e^+e^- \to f\bar{f}\right) \approx \frac{N_c G_F^2 M_Z^4}{24\pi\Gamma_Z^2} (R_e^2 + L_e^2)(R_f^2 + L_f^2)$$

Where are we with Z's?

• At the Z pole:

- \square 2 x 10⁷ unpolarized Z's at LEP
- □ 5 x 10⁵ Z's at SLD with $P_e \sim 75\%$
- What did we measure at the Z?
 - $\Box \ Z \text{ lineshape} \Rightarrow \sigma, \Gamma_Z, M_Z$
 - **Z** branching ratios
 - Asymmetries
- W⁺W⁻ production at 200 GeV
 - Searches for Zh

Z cross section



Requires precise calibration of energy of machine

Number of light neutrinos: $N_0 = 2.9840 \pm 0.0082$

Electroweak Theory is Precision Theory

We have a model.... And it works to the 1% level

Gives us confidence to predict the future!



2 3

0

The Moral:

- Experimental measurements of M_W, M_t and electroweak observables at LEP/SLC are sufficiently precise that they limit not only M_h, but possible extensions of the Standard Model
- Only missing element of the Standard Model is the Higgs Boson, which must be lighter than a few 100 GeV if the Standard Model is the whole story

Does SM work at Low Energy?

 Moller scattering, e⁻e⁻→e⁻e⁻ $\sin^2 \theta_{W}^{eff}(Q)$ 0.242 E158 v-nucleon scattering NuTeV 0.24 Atomic parity 0.238 violation in Cesium 0.236 0.234 Qw(Cs) 0.232 We believe we know how to PDG2004 evolve coupling constants: 10-2 10⁻¹ Q (GeV) 10 102 this understanding necessary for grand unified theories

Limits from Precision Measurements in Models beyond the SM

- How to incorporate physics beyond the Standard Model in limits from precision measurements?
- S,T,U approach assumes new physics is dominantly in gauge boson
 2-point functions at scale M >> M_Z
- For example, parameterize: $M_W^2 = (...)S + (...)T + (...)U$

Neglects box and vertex contributions

$$\frac{\alpha}{4s_{\theta}^{2}c_{\theta}^{2}}S = \frac{\Pi_{ZZ}(M_{Z}^{2})}{M_{Z}^{2}} - \frac{\Pi_{ZZ}(0)}{M_{Z}^{2}}$$

$$\alpha T = \frac{\Pi_{WW}(0)}{M_{W}^{2}} - \frac{\Pi_{ZZ}(0)}{M_{Z}^{2}}$$

Easy to calculate in model of the week: often a good approximation

S,T,U

- As Higgs gets heavy, predictions get further and further from data
 - Compensate with large $\rho = \alpha T$

Heavy degenerate 4^{th} generation: $\Delta S=2/(3\pi), \ \Delta T=0$

Non-degenerate 4^{th} generation: $\Delta T = N_c G_F \Delta m^2 / (8 \sqrt{2} \pi^2) > 0$





Higgs can be heavy in models with new physics

- Specific examples of heavy Higgs bosons exist in Little Higgs Models and Triplet Models
- $M_H \approx 450-500$ GeV allowed with large isospin violation $(\alpha \Delta T = \rho)$ and higher dimension operators



We don't know what the model is which produces the operators which generate large ΔT

Review of Higgs Couplings

- Higgs couples to fermion mass
 - Largest coupling is to heaviest fermion

$$L = -\frac{m_f}{v} \bar{f}fh = -\frac{m_f}{v} \left(\bar{f}_L f_R + \bar{f}_R f_L\right)h$$

- Top-Higgs coupling plays special role?No Higgs coupling to neutrinos
- Higgs couples to gauge boson masses

$$L = gM_{W}W^{+\mu}W_{\mu}^{-}h + \frac{gM_{Z}}{\cos\theta}Z^{\mu}Z_{\mu}h + \dots$$

- Only free parameter is Higgs mass!
- Everything is calculable....*testable theory*

Higgs Decays



- h \rightarrow ff proportional to m_f^2 $\frac{BR(h \rightarrow b\overline{b})}{BR(h \rightarrow \tau^+ \tau^-)} = N_c \left(\frac{m_b^2}{m_\tau^2}\right) \left(\frac{\beta_b}{\beta_\tau}\right)^3$
- β³ typical of scalar
 (pseudo-scalar decay ≈β)



to bb most important

Higgs decays to gauge bosons

- $h \rightarrow gg$ sensitive to top loops
 - Remember no coupling at tree level
- h → γγ sensitive to W loops, only small contribution from top loops
- h→W⁺W⁻→ffff has sharp threshold at 2 M_W, but large branching ratio even for M_h=130 GeV

For any given M_h , not all decay modes accessible
Higgs Branching Ratios



Bands show theory errors

>Largest source of uncertainty is b quark mass

Data points are $e^+e^{-1}LC$ at $\sqrt{s}=350$ GeV with L=500 fb⁻¹

Total Higgs Width

- al width sensitive funct. In Small Mh, Higgs is narrower than rector resolution romes large, width alsc
- As M_h becomes large, width alsc

 - \Box For M_h ~1.4 TeV, Γ_{tot} ~M_h

$$\Gamma(h \to W^+ W^-) \approx \frac{\alpha}{16 \sin^2 \theta_W} \frac{M_h^3}{M_W^2}$$
$$\approx 330 GeV \left(\frac{M_h}{1TeV}\right)^3$$





Higgs production at Hadron Colliders

- Many possible production mechanisms; Importance depends on:
 - Size of production cross section
 - Size of branching ratios to observable channels
 - Size of background
- Importance varies with Higgs mass
- Need to see more than one channel to establish Higgs properties and verify that it is a Higgs boson

Production in Hadron Colliders

- Gluon fusion
 - □ Largest rate for all M_h at LHC
 - Gluon-gluon initial state
 - Sensitive to top quark Yukawa λ_t
- Lowest order cross section:

$$\hat{\sigma}_{0}(gg \to h) = \frac{\alpha_{s}(\mu_{R})^{2}}{1024\pi v^{2}} \left| \sum_{q} F_{1/2}(\tau_{q}) \right|^{2} \delta(M_{h}^{2} - \hat{s})$$

- $\Box \quad \tau_q = 4M_q^2/M_h^2$
- □ Light Quarks: $F_{1/2} \rightarrow (M_b/M_h)^2 log(M_b/M_h)$
- Heavy Quarks: $F_{1/2} \rightarrow -4/3$

In SM, b-quark loops unimportant





Rapid approach to heavy quark limit

Gluon fusion, continued

Integrate parton level cross section with gluon parton distribution functions

$$\sigma_0(pp \to h) = \hat{\sigma}_0 z_j^1 \frac{dx}{x} g(x, \mu_F) g(\frac{z}{x}, \mu_F)$$

□ $z=M_h^2/S$, S is hadronic center of mass energy

- Rate depends on μ_R , μ_F
- Rate for gluon fusion independent of M_t for $M_t >> M_h$
 - Counts number of heavy fermions

Vector Boson Fusion

- $W^+W^- \rightarrow X$ is a real process:
- Rate increases at large s: $\sigma \approx (1/M_W^2) \log(s/M_W^2)$
- Integral of cross section over final state phase space has contribution from W boson propagator:

$$\int \frac{d\theta}{\left(k^2 - M_W^2\right)^2} \approx \int \frac{d\theta}{\left(2EE'\left(1 - \cos\theta\right) + M_W^2\right)^2}$$

Peaks at small θ

Outgoing jets are mostly forward and can be tagged



Idea: Look for h decaying to several different channels

Ratio of decay rates will have smaller systematic errors

W(Z)-strahlung

- W(Z)-strahlung ($qq \rightarrow Wh$, Zh) important at Tevatron
 - Same couplings as vector boson fusion
 - □ Rate proportional to *weak* coupling
 - □ Below 130-140 GeV, look for $q\bar{q} \rightarrow Vh, h \rightarrow bb$
 - □ For $M_h > 140$ GeV, look for $h \rightarrow W^+W^-$
- Theoretically very clean channel



Comparison of rates at Tevatron



≻Luminosity goals for Tevatron: 4-8 fb⁻¹

≻Higgs very, very hard at Tevatron

Tevatron Run 2



Collider Run II Integrated Luminosity

~3 fb⁻¹ recorded 4-8 fb⁻¹ by 2009

Higgs at the Tevatron

■ Largest rate, gg→h, h →bb, is overwhelmed by background



 $\sigma(gg \rightarrow h) \sim 1 \text{ pb} \ll \sigma(bb)$

Fermilab looks for the Higgs in Many Channels



Can the Tevatron discover the Higgs?



This relies on statistical combination of multiple weak channels

Comparison of production rates at LHC



Search Channels at the LHC

 $gg \rightarrow h \rightarrow bb$ has huge QCD bckd: Must use rare decay modes of h

- $gg \rightarrow h \rightarrow \gamma \gamma$
 - **Small BR** $(10^{-3} 10^{-4})$
 - Only measurable for M_h < 140 GeV
- Largest Background: QCD continuum production of γγ
- Also from γ -jet production, with jet faking γ , or fragmenting to π^0
- Fit background from sidebands of data

M_h=120 GeV; L=100 fb⁻¹



•Gives 1% mass measurement

Vector Boson Fusion

- Outgoing jets are mostly forward and can be tagged
- Vector boson fusion and QCD background look different





Vector Boson Fusion for light Higgs

> For $M_h = 115$ GeV combined significance ~ 5σ

Vector boson fusion effective for measuring Higgs couplings

> Proportional to g_{WWh} and g_{ZZh}

>Often assume they are in SU(2) ratio: $g_{WWh//}g_{ZZh}=\cos^2\theta_W$



Vector boson fusion for Heavy Higgs

$200 \; GeV < M_h < 600 \; GeV:$

discovery in h → ZZ → l+l-l+lBackground smaller than signal
Higgs width larger than experimental resolution (M_h > 300 GeV)

- confirmation in $h \rightarrow ZZ \rightarrow l^+l^-$ jj channel

 $M_h > 600 \text{ GeV}$: 4 lepton channel statistically limited $h \rightarrow ZZ \rightarrow l^+l^- \nu\nu$ $h \rightarrow ZZ \rightarrow l^+l^- jj$, $h \rightarrow WW \rightarrow l \nu jj$

-150 times larger BR than 4l channel



 $h \rightarrow ZZ \rightarrow l^+l^-l^+l^-$



tth at the LHC

- $gg \rightarrow t\bar{t}h \rightarrow t\bar{t}b\bar{b}$
- Spectacular signal
 - $\Box t \rightarrow Wb$
 - Look for 4 b jets, 2 jets, 1 lepton

Unique way to measure top quark Yukawa coupling



Early studies looked promising

BUT...Large QCD background to tth

Current $t\bar{t}H, H \rightarrow b\bar{b}$ outlook: (30 fb⁻¹)



ATLAS Sensitivity for a light SM Higgs



If there is a light SM Higgs, we'll find it at the LHC



What if we find a "Higgs-like" object?

• We need to:

- Measure Higgs couplings to fermions & gauge bosons
- Measure Higgs spin/parity
- Reconstruct Higgs potential
- □ Is it the SM Higgs?
- Reminder: Many models have other signatures:
 - New gauge bosons (little Higgs)
 - Other new resonances (Extra D)
 - □ Scalar triplets (little Higgs, NMSSM)
 - Colored scalars (MSSM)
 - etc

Is it a Higgs?

- How do we know what we've found?
- Measure couplings to fermions & gauge bosons

$$\frac{\Gamma(h \to b\overline{b})}{\Gamma(h \to \tau^+ \tau^-)} \approx 3 \frac{m_b^2}{m_\tau^2}$$

Measure spin/parity

$$J^{PC} = 0^{++}$$

Measure self interactions

$$V = \frac{M_h^2}{2}h^2 + \frac{M_h^2}{2v}h^3 + \frac{M_h^2}{8v^2}h^4$$

Very hard at hadron collider

Absolute measurements of Higgs couplings @ LHC

► Ratios of couplings more precisely measured than absolute couplings



≻10-40% measurements of most couplings

Can we reconstruct the Higgs potential?

$$V = \frac{M_h^2}{2}h^2 + \lambda_3 vh^3 + \frac{\lambda_4}{4}h^4$$
$$SM : \lambda_3 = \lambda_4 = \frac{M_h^2}{2v^2}$$

≻Fundamental test of model!



 \succ We have no idea how to measure λ_4

Reconstructing the Higgs potential



λ₃ requires 2 Higgs production
 M_h<140 GeV, h→bbbb
 Overwhelming QCD background

Can determine whether $\lambda_3=0$ at 95% cl with 300 fb⁻¹ for 150<M_h<200 GeV

Initial Physics Program at the LHC



• Large numbers of events even at low LHC luminosity

$\sqrt{s}=14$ TeV-- the first 10 pb⁻¹



- Small statistical errors in precision measurements
- Search for rare processes
- Large samples for studies of systematic effects



Masses at one-loop in the SM

• First consider a fermion coupled to a massive complex Higgs scalar

$$L = \overline{\Psi}(i\partial)\Psi + \left|\partial_{\mu}\phi\right|^{2} - m_{s}\left|\phi\right|^{2} - \left(\lambda_{F}\overline{\Psi}_{L}\Psi_{R}\phi + h.c.\right)$$

Assume symmetry breaking as in SM:

$$\phi = \frac{(h+v)}{\sqrt{2}} \qquad \qquad m_F = \frac{\lambda_F v}{\sqrt{2}}$$

Masses at one-loop, #2

• Calculate mass renormalization for Ψ



Compute using a high scale momentum cutoff, Λ

Symmetry and the fermion mass

- $\delta m_F \approx m_F$
 - \square m_F=0, then quantum corrections vanish
 - When $m_F = 0$, Lagrangian is invariant under
 - $\Psi_L \rightarrow e^{i\theta_L} \Psi_L$
 - $\Psi_R \rightarrow e^{i\theta_R} \Psi_R$
 - □ $m_F \rightarrow 0$ increases the symmetry of the threey
 - Yukawa coupling (proportional to mass) breaks symmetry and so corrections ≈ m_F

Scalars are very different





M_h depends quadratically on high mass scales

Light Scalars are Unnatural

- Higgs mass grows with scale of new physics, Λ
- No additional symmetry for $M_h=0$, no protection from large corrections





$$\delta M_h^2 = \frac{G_F}{4\sqrt{2}\pi^2} \Lambda^2 \left(6M_W^2 + 3M_Z^2 + M_h^2 - 12M_t^2 \right)$$
$$= -\left(\frac{\Lambda}{0.7 \,\text{TeV}} \, 200 \,\text{GeV} \right)^2$$

 $M_h \leq 200 \ GeV \ requires \ large \ cancellations$

Try to cancel quadratic contributions by adding new particles

- SUSY models add scalars with same quantum numbers as fermions, but different spin
- Little Higgs models cancel quadratic dependences with new particles with same spin

Arguments like this are basis for believing that "something new" happens at the TeV energy scale

Landau Pole

- M_h is a free parameter in the Standard Model
- Can we derive limits on the basis of consistency?
- Consider a scalar potential:

$$V = \frac{M_h^2}{2}h^2 + \frac{\lambda}{4}h^4$$

- This is potential at electroweak scale
- Parameters evolve with energy in a calculable way
High Energy Behavior of λ

Renormalization group scaling

$$\frac{1}{\lambda(Q)} = \frac{1}{\lambda(\mu)} + (\dots) \log\left(\frac{Q}{\mu}\right)$$

$$16\pi^2 \frac{d\lambda}{dt} = 12\lambda^2 + 12\lambda g_t^2 - 12g_t^4 + (gauge)$$

$$t \equiv \log\left(\frac{Q^2}{\mu^2}\right) \qquad \qquad g_t = \frac{M_t}{v}$$

- Large λ (Heavy Higgs): self coupling causes λ to grow with scale
- Small λ (Light Higgs): coupling to top quark causes λ to become negative

Theoretical bounds on SM Higgs Boson

 If SM valid up to Planck scale, only a small range of theoretically allowed Higgs Masses



Unitarity Limits

• Consider $2 \rightarrow 2$ elastic scattering

 $\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \left| A \right|^2$

Partial wave decomposition of amplitude

$$A = 16\pi \sum_{l=0}^{\infty} (2l+1)P_l(\cos\theta)a_l$$

- a_l are the spin *l* partial waves
- Unitarity requirement:

 $\left|\operatorname{Re}(a_{l})\right| \leq \frac{1}{2}$

More on Unitarity

Idea: Use unitarity to limit parameters of theory

Cross sections which grow with energy always violate unitarity at some energy scale

$\omega^+\omega^- \rightarrow \omega^+\omega^-$

Two interesting limits: s, t >> M_h²

$$A(\omega^+\omega^- \to \omega^+\omega^-) \to -2\frac{M_h^2}{v^2}$$

$$a_0^0 \rightarrow -\frac{M_h^2}{8\pi v^2}$$

$$\square$$
 s, t << M_h^2

$$A(\omega^+\omega^- \to \omega^+\omega^-) \to -\frac{u}{v^2}$$

$$a_0^0 \rightarrow -\frac{s}{32\pi v^2}$$

Use Unitarity to Bound Higgs $|\operatorname{Re}(a_l)| \le \frac{1}{2}$

High energy limit:

$$a_0^0 \rightarrow -\frac{M_h^2}{8\pi v^2}$$

$$M_h < 800 \text{ GeV}$$

Heavy Higgs limit

$$a_0^0 \rightarrow -\frac{s}{32\pi v^2}$$

$$E_c \sim 1.7 \text{ TeV}$$

 \rightarrow New physics at the TeV scale

Can get more stringent bound from coupled channel analysis

Conclusion

- Data from the Tevatron, SLC, and LEP support (with exquisite precision) the SM picture with a single Higgs boson
- If a SM-like Higgs boson exists, we should find it at the LHC
- BUT....the SM is not completely satisfactory theoretically