Puff Field Theory

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Puff Field Theory (PFT) is a conjectured nonlocal, Lorentz violating, but rotationally invariant, quantum field theory, decoupled from gravity. It was constructed from string theory in references [1]-[2], on which this short summary is based.

We start with type-IIA compactified on T^3 in the form of $S^1 \times S^1 \times S^1$ with compactification radii R'_i ($i = 1, 2, 3$) and string coupling constant g'_{st}. We add a geometric twist as follows. Let x_1, x_2, x_3 be the compact coordinates on T^3 with periodicities $2\pi R_1, 2\pi R_2, 2\pi R_3$ and let $\vec{y}$ denote the vector of coordinates in the six transverse directions. We then replace the original periodicity conditions of T^3 with

$$(x_1, x_2, x_3, \vec{y}) \sim (x_1 + 2\pi R_1, x_2, x_3, \Omega\vec{y}),$$

where Ω is some element of the rotation group SO(6), which can be extended to act on fermions by taking $\Omega \in \text{Spin}(6)$. Next, we add N Kaluza-Klein particles in the 1^{st} direction and look for the low-energy description of this configuration in the limit

$$\alpha'^{-1/2} R'_1 \longrightarrow 0, \quad \alpha'^{-1/2} R'_2 \rightarrow \text{finite}, \quad \alpha'^{-1/2} R'_3 \rightarrow \text{finite}, \quad g'_{\text{st}} \rightarrow \text{finite}, \tag{1}$$

combined with

$$\Omega = \exp\left(2\pi\alpha'^{-\frac{13}{4}} {g'_{\text{st}}}^{-\frac{1}{2}} {R'_1}^{\frac{3}{2}} R'_2 R'_3 \zeta\right) \longrightarrow I, \qquad \zeta \rightarrow \text{finite}. \tag{2}$$

Here, ζ is an element of the Lie algebra $so(6) \simeq su(4)$, and the cumbersome factor in front of it will be discussed later. Note that if ζ is in an appropriate $su(3)$ ($su(2)$) subgroup of $so(6)$ then $\mathcal{N} = 1$ ($\mathcal{N} = 2$) SUSY is preserved, respectively.

When $\zeta = 0$, it is not hard to see that the low-energy description is simply 3+1D $U(N)$ $\mathcal{N} = 4$ SYM. The boundary conditions are periodic, and T-duality in the 1^{st} direction, followed by S-duality, followed by T-dualities in the 2^{nd} and 3^{rd} directions, convert the system to N D3-branes in type-IIB compactified on T^3 with compactification radii

$$R_1 = \alpha'^{\frac{3}{4}} g_{\text{st}}'^{-\frac{1}{2}} R_1'^{-\frac{1}{2}} , \qquad R_k = \alpha'^{\frac{5}{4}} g_{\text{st}}'^{\frac{1}{2}} R_1'^{-\frac{1}{2}} R_k'^{-1} , \quad (k = 2, 3), \tag{3}$$

and string coupling constant $g_{\text{st}} = \alpha' R_3'^{-1} R_2'^{-1}$. The limit (1) was chosen so that the ratios R_2/R_1 and R_3/R_1 and the coupling constant g_{st} remain finite, while $\alpha'^{-1/2} R_k \longrightarrow \infty$ $(k = 1, 2, 3)$. For nonzero ζ, U-duality is less useful in the limit (1). However, we will argue below that the low-energy limit still describes a decoupled QFT on $R^{3,1}$, but a nonlocal one. We will define R_1, R_2, R_3 as in (3), even for $\zeta \neq 0$. The cumbersome factor in the exponent of (2) then simplifies to $(2\pi/R_1 R_2 R_3)\zeta$.

The string-theory construction of PFT described above was inspired by the Douglas-Hull construction [3] of NCSYM (super Yang-Mills theory on a noncommutative space), and by Witten's construction of the duals of (p, q) 5-branes [4].

We now proceed to study some simple aspects of PFT. If ζ is chosen appropriatedly, we get $\mathcal{N} = 2$ supersymmetric PFT, and we can calculate the energies of BPS states that include electric and magnetic fluxes, as well as momentum. If we express ζ as an $su(2) \subset su(4)$ matrix, it has eigenvalues $\pm\beta$. Suppose we have ℓ units of R-charge, k_i units of momentum, e_i units of electric flux, and m_i units of magnetic flux in the i^{th} direction, for $i = 1, 2, 3$. (All of these are integers and are associated with Kaluza-Klein, D-brane, or string charges in the string theory construction.) With the notation $V \equiv R_1 R_2 R_3$, and

$$\mathbf{P} \equiv \sum_{i=1}^{3} \frac{k_i}{R_i} \hat{\mathbf{n}}_i, \qquad \mathbf{E} \equiv \sum_{i=1}^{3} \frac{e_i R_i}{2\pi V} \hat{\mathbf{n}}_i, \quad \mathbf{B} \equiv \sum_{i=1}^{3} \frac{m_i R_i}{2\pi V} \hat{\mathbf{n}}_i,$$

we get the BPS energy

$$E \;=\; \frac{2\ell\beta}{g_{\text{st}}\alpha'^2} + \frac{2\pi^2 V^2}{|NV + 2\ell\beta|} \left(\frac{g_{\text{YM}}^2}{2\pi} \mathbf{E}^2 + \frac{2\pi}{g_{\text{YM}}^2} \mathbf{B}^2 \right) + |\mathbf{P} - \frac{4\pi^2 V^2}{|NV + 2\ell\beta|} \mathbf{E} \times \mathbf{B}|.$$

Analogous formulas for NCSYM have been derived, for instance, in [5]-[7]. We see from the last term that the dispersion relation of massless Kaluza-Klein particles remains relativistic. The combination $NV + 2\ell\beta$, which appears in the denominator of various terms in the BPS formula, suggests that ℓ units of R-charge formally carry an intrinsic volume of $2\ell\beta$. For more motivation for this interpretation, see [1].

The supergravity dual of PFT, found using the string theory construction in a way similar to [8]-[9], is given by

$$\begin{aligned} ds^2 &= \frac{R^2}{r^2} K^{-\frac{1}{2}} \left[dx^2 + dy^2 + dz^2 - \left(dt - \frac{4\pi N}{r^2} \vec{n}^T \zeta d\vec{n} \right)^2 \right] + \frac{R^2}{r^2} K^{\frac{1}{2}} dr^2 + R^2 K^{\frac{1}{2}} d\Omega_5^2 \, , \\ C_4' &= \frac{\pi N}{r^4} K^{-1} dt \wedge dx \wedge dy \wedge dz - \frac{\pi N}{g_{\text{st}} \alpha'^2 r^6} K^{-1} \vec{n}^T \zeta d\vec{n} \wedge dx \wedge dy \wedge dz, \end{aligned} \tag{4}$$

where C_4' is the RR flux, and

$$K \equiv 1 + \frac{16\pi^2 N^2}{r^6}\vec{n}^T\zeta^T\zeta\vec{n}, \qquad \vec{n}\in S^5, \qquad d\Omega_5^2 = \sum_{I=1}^{6} dn_I^2, \qquad R^4 \equiv 4\pi g_{\rm st} N {\alpha'}^2 \ .$$

The IR fixed-point, which can be read-off from the supergravity dual, is $\mathcal{N}=4$ SYM. At higher energy scales, the deviation of PFT from $\mathcal{N}=4$ SYM is parameterized by ζ, which has dimensions of volume. To lowest order in ζ, PFT is therefore a deformation of $\mathcal{N}=4$ SYM by an IR-irrelevant operator of dimension 7. The form of this operator can be found in [2]. It involves a coupling of the energy-momentum tensor to the R-symmetry current.

In the UV limit the supergravity dual becomes singular. However, at a somewhat relaxed level of rigor, it is possible to dualize the extreme UV region to a weakly coupled M-theory background, at least for a special choice of ζ which preserves a $u(3)\subset so(6)$ subgroup of the R-symmetry. The advantange of this choice of ζ is that it is compatible with the Hopf-fibration of S^5. The duality that converts (4) to a weakly coupled background can be described as T-duality along the fiber of the Hopf-fibration and a lift to M-theory (cf. [10]), yielding

$$\begin{aligned}
{\ell_P}^{-2} ds_M^2 &= (4\pi N)^{-1/3} g_{\rm st}^{-1}\rho^2\left[\Delta^{-2/3}(dx_1^2+dx_2^2+dx_3^2) - \Delta^{1/3}dx_0^2\right] \\
&\quad + (4\pi N)^{2/3}\Delta^{1/3}\rho^{-2}d\rho^2 + (4\pi N)^{2/3}\Delta^{1/3} ds_B^2 \\
&\quad + (4\pi N)^{-1/3}\Delta^{1/3}(g_{\rm st}^{-1}d\xi^2 + g_{\rm st} d\eta^2), \\
{\ell_P}^{-3} G_4 &= 2\pi N\omega\wedge\omega + 2\omega\wedge d\eta\wedge d\xi \\
&\quad + \frac{2}{3}(4\pi N)^{-2} g_{\rm st}^{-3}\beta d\Big(\frac{\rho^6}{\Delta}\Big)\wedge dx_1\wedge dx_2\wedge dx_3,
\end{aligned} \tag{5}$$

where

$$\Delta \equiv 1 + (4\pi N)^{-1} g_{\rm st}^{-3}\beta^2\rho^6 \ ,$$

ℓ_P is the Planck length, x_0, x_1, x_2, x_3 are coordinates on $R^{3,1}$, η and ξ are periodic coordinates with period 2π (parameterizing T^2), ds_B^2 is an appropriately normalizaed metric on $\mathbf{CP^2}$, ω is a harmonic 2-form whose cohomology class generates $H^2(\mathbf{CP^2},\mathbf{Z})$, and G_4 is the 4-form flux. We also replaced the coordinate r with $\rho\propto 1/r$, so that $\rho\to\infty$ is the UV limit. The background (5) becomes weakly coupled as $\rho\to\infty$.

This background has interesting properties, such as a timelike boundary corresponding to the extreme UV similarly to AdS, an infinite redshift in frequency near the boundary, and an infinite blueshift in wavelength. This combination of infinite frequency redshift and infinite wavelength blueshift can be used to argue for a discrete spectrum. The argument is as follows. Consider particle trajectories in (5) with fixed energy E and spatial momentum $\vec{p}$ (defined with respect to the Killing vectors $\partial/\partial x_0$ and $\partial/\partial x_1,\dots\partial/\partial x_3$, respectively). Then, for timelike or lightlike trajectories we get

$$\rho \le \rho_{\max} \equiv (4\pi N)^{1/6} g_{\rm st}^{1/2}\beta^{-1/3}\left(\frac{E}{|\vec{p}|}\right)^{1/3} \ . \tag{6}$$

To test the discreteness of the PFT spectrum, let's compactify on a fixed T^3 and assume that zero modes of bosonic fields are removed by appropriate twisted boundary conditions. Then $|\vec{p}|$ is bounded from below (by a value of the order of the inverse of the longest side of the T^3). Then (6) implies that ρ is bounded from above by an expression that is proportional to the cube root of the energy, so that only a finite portion of the background (5) is accessible, for any finite energy. This suggests a discrete spectrum.

The fundamental formulation of PFT is at this point unknown. The arguments presented above, howoever, and especially the existence of a supergravity dual with a geodesically complete metric [as can be checked for (5) in the UV, and as is obvious for (4) in the IR] suggest that it is a decoupled theory. The existence of simple constructions of Lorentz violating models in string theory could provide a further incentive to look for such violations experimentally, as even a minute violation of Lorentz invariance is experimentally testable [11].

References

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