Brane/Antibrane Configurations in Type IIA and M Theory

J Marsano
California Institute of Technology
Pasadena, CA

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1 Introduction

The study of supersymmetry breaking configurations in string theory has been a topic of significant interest during the past year. Many of these constructions have focused on stringy realizations of ISS type vacua in supersymmetric gauge theories. Another interesting class of examples has recently emerged, though, which admit no effective gauge theory description and therefore are, in a sense, inherently stringy[1, 2][1] . These involve wrapping D5 and $\overline{\text{D5}}$ branes on vanishing 2-cycles of local CY 3-folds in type IIB. If the vanishing cycles wrapped by branes and antibranes are homologous but grow to some finite nonzero size in between, classical brane/antibrane annihilation is prevented by a potential barrier and the resulting configurations are metastable. On the other hand, if the vanishing cycles are nonhomologous, one may be able to construct metastable or even stable nonsupersymmetric configurations.

What makes these noncompact type IIB constructions particularly interesting is that one has a great deal of computational control and can reliably study the scalar potential, and hence the vacua and phase structure, of the system. The reason for this is twofold. First, it has been conjectured that such brane/antibrane systems continue to exhibit the large N duality of [5, 6, 7], permitting one to describe them by deformed CY with flux. The only modification to the usual story in this nonsupersymmetric setting is the sign of the flux associated to the antibranes after geometric transition. The second crucial observation of [1] is that the $\mathcal{N}=2$ supersymmetry of type IIB on these noncompact CY 3-folds is only spontaneously broken by the fluxes, whose effect

[1]It should be noted, however, that some more recent work in this area focuses on brane/antibrane setups which do admit an effective gauge theory description in the IR [3, 4]

is simply to add electric and magnetic FI terms to the $\mathcal{N}=2$ abelian gauge theory that describes the IR physics. As such, the kinetic terms continue to be specified by special geometry and hence not only the superpotential, but in fact the full scalar potential can be determined reliably[2].

In the supersymmetric setting, the standard geometric transition story in type IIB has a well-understood T-dual counterpart in type IIA. The classical IIB setups with D5 branes wrapping vanishing cycles on local Calabi-Yau map to NS5/D4 brane configurations [9, 10, 11]. In type IIA, the tool that one utilizes to study the quantum system is M-theory, in terms of which our classical setup is described by nothing more than a curved M5 brane [12, 13]. This intuition instructs us to replace the NS5/D4 setup by a curved NS5 brane with flux in a manner quite analagous to the replacement of the classical configuration of D5's at CY singularities in IIB by a deformed CY geometry with flux. In the supersymmetric case, it is well-established that the vacuum configuration of the curved NS5 brane in type IIA is exactly T-dual to the deformed geometry on the type IIB side after geometric transition with moduli sitting at the supersymmetric vacuum [14, 15].

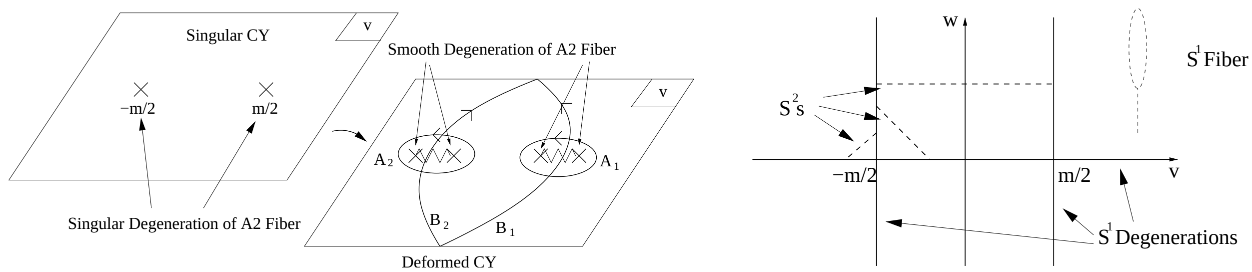
In this note, we shall describe how one can expand this story to the nonsupersymmetric setting, a task which is of interest for a variety of reasons. First, it will provide additional evidence for the two main conjectures of [1], namely the large N duality conjecture for brane/antibrane systems and the spontaneous breaking of $\mathcal{N}=2$ SUSY by fluxes and anti-fluxes on local CY[3]. Second, M-theory allows us to go beyond the $g_s \ll 1$ regime where one can reliably study the type IIB setup and peer into the world of strong string coupling. There, we will find new nonsupersymmetric features which do not have well-understood analogs in type IIB[4]

Before we study supersymmetry breaking configurations, though, we shall begin in section 2 by reviewing the supersymmetric story of the geometric transition in type IIB and its type IIA/M counterpart. Then, in section 3, we shall turn to the nonsupersymmetric story, describing non-holomorphic minimal area M5 curves and their properties. The general story we shall be telling here was first worked out in one example of a brane/antibrane system in [19]. Throughout this note, however, we shall make use of a simpler example, a study of which on the IIB side recently appeared in [3]. The type IIA/M description of this system is still part of work in progress and will appear soon [16].

[2]Note that this residual $\mathcal{N}=2$ structure is apparently more strongly broken and the system altered by warping effects after one couples to gravity [8].

[3]As far as we understand, the conjecture related to spontaneous breaking of SUSY is made at string tree level only. Indeed, we will find that, at least on the IIA side, it no longer holds in certain regimes at large g_s.

[4]Strictly speaking, these features are not directly related to anything in type IIB because the IIA and IIB descriptions are valid in different ranges of parameter space, depending on the size of the circle on which T-duality is applied. The new features in IIA are present when SUSY is not spontaneously broken so there is no structure to guarantee that they carry over to the IIB side. One might hope to find something qualitatively similar in IIB, though.



(a) Depiction of the singular CY (4) and its smooth deformation (6) as an A_2 fibration over the v-plane. The marked points indicate singular or smooth degenerations of one of the two S^2's in the A_2 fiber. The A and B cycles of the deformed CY are also labeled.

(b) Depiction of the singular CY (4) as a $\mathbb{C}^*$ fibration over $\mathbb{C}^2$. The solid lines denote smooth degenerations of the $S^1 \subset C^*$. The dotted are intervals over which one can fiber this S^1 to obtain a nontrivial S^2.

Figure 1: Two pictures of the CY geometry (4)

2 The Supersymmetric Story

2.1 The IIB Side

We start by considering type IIB on a simple class of local CY obtained by fibering deformed ALE singularities over the complex plane. To build a simple example of such a CY, consider for starters an A_2 singularity

$$xy + w^3 = 0 \tag{1}$$

We can view this as a $\mathbb{C}^*$ fibration over the w plane with a singular point at the origin. It is well-known that one can smooth out this singularity by turning on a pair of holomorphic deformations

$$xy + (w + t_1)(w + t_2)(w - t_1 - t_2) = 0 \tag{2}$$

which replace the multiple degeneration point of the $\mathbb{C}^*$ by three smooth degeneration points By fibering the $S^1 \subset \mathbb{C}^*$ over an interval connecting any pair of them, we find that holomorphic S^2's have grown in place of the singularity. There are in fact two homologously distinct such S^2's in the case of A_2.

We now build our local CY by fibering this deformed A_2 geometry over another copy of $\mathbb{C}$, parametrized by v. This is accomplished by replacing the t_i by holomorphic functions $t_i(v)$. For simplicity, we focus on the specific example[5]

$$xy + w\left(w - v - \frac{m}{2}\right)\left(w - v + \frac{m}{2}\right) = 0 \tag{3}$$

which can be put into an even simpler form by taking $v \to w - v$

$$xy + w\left(v^2 - \frac{m^2}{4}\right) = 0 \tag{4}$$

Two ways of visualizing this geometry will be useful for us. First, we have the usual picture, depicted in figure 1(a), of an A_2 fibration with one of the S^2's at the tip degenerating at $v = \frac{m}{2}$ and the other degenerating at $v = -\frac{m}{2}$. Alternatively, we can think of this geometry as a $\mathbb{C}^*$ fibration over the $\mathbb{C}^2$ spanned by w and v in which the fiber degenerates along the curves

$$w = 0, \quad v = \frac{m}{2}, \quad v = -\frac{m}{2} \tag{5}$$

As indicated in figure 1(b), the S^2's can be seen in this setup by fibering the $\mathbb{C}^*$ over any interval connecting a pair of these curves. Where two curves intersect, the corresponding S^2 degenerates. Note that our simple example is quite nongeneric because there is an S^2 in the geometry which never degenerates.

The local CY just constructed has two singular points but we can easily smooth them out by a suitable complex deformation

$$xy + w\left(v^2 - \frac{m^2}{4}\right) = a\left(v - \frac{m}{2}\right) + b\left(v + \frac{m}{2}\right) \tag{6}$$

As we see in figure 1(a), this deformation replaces the singular degeneration point of each S^2 with a pair of smooth degenerations. Fibering either S^2 over an interval connecting its smooth degeneration points yields an S^3 which has grown in place of the singularity.

Now that we have reviewed the relevant geometries, let us turn to the physics. We start with the singular geometry (4) and wrap N_1 D5 branes on the degenerate S^2 at $v = -\frac{m}{2}$ and N_2 D5's on the degenerate S^2 at $v = \frac{m}{2}$. We also turn on nontrivial B^{NS} and C_0 along these two cycles. This type of setup engineers $\mathcal{N} = 1$ $U(N_1) \times U(N_2)$ gauge theory with masses for the adjoint chiral superfields. The NS and RR 2-forms determine the bare gauge couplings and θ-angles, respectively.

To study this system at the quantum level, we perform a geometric transition whereby the singular geometry (4) with branes is replaced by the nonsingular geometry (6) with fluxes. The complex deformations of (6) are dynamical moduli whose values are fixed by the Gukov-Vafa-Witten superpotential, which is generated by the fluxes [17]

$$W_{GVW} = \int H \wedge \Omega \tag{7}$$

[5]To bring the CY into this form we have also shifted w by a constant.

Here, Ω is the holomorphic 3-form and H the 3-form flux. The periods of H along the A and B cycles in figure 1(a) are given by

$$\frac{1}{2\pi i}\oint_{A_j} H = N_j \qquad \frac{1}{2\pi i}\oint_{B_j} H = -\alpha_j \tag{8}$$

while, as usual, the A-periods of Ω parametrize the space of complex structure moduli

$$S_j = \frac{1}{2\pi i}\oint_{A_j}\Omega \tag{9}$$

It is often convenient to integrate H and Ω over the holomorphic S^2's in the A_2 fibers. In this case, each reduces to a 1-form defined on the Riemann surface

$$w\left(v^2-\frac{m^2}{4}\right) = a\left(v-\frac{m}{2}\right)+b\left(v+\frac{m}{2}\right) = 0 \tag{10}$$

More specifically, Ω reduces to

$$\int_{\text{fiber}}\Omega \equiv \omega = w\,dv \tag{11}$$

while H reduces to a 1-form h with periods given by the fluxes

$$\frac{1}{2\pi i}\oint_{A_j} h = N_j \qquad \frac{1}{2\pi i}\oint_{B_j} h = -\alpha_j \tag{12}$$

Supersymmetric vacua of this system now correspond to a Riemann surface whose period matrix $\hat{\tau}_{ij}$ satisfies the equation of motion that follows from (7)

$$\alpha_i + \hat{\tau}_{ij}N_j = 0 \tag{13}$$

The present example is particularly easy to solve because the holomorphic prepotential is known exactly [3, 16]

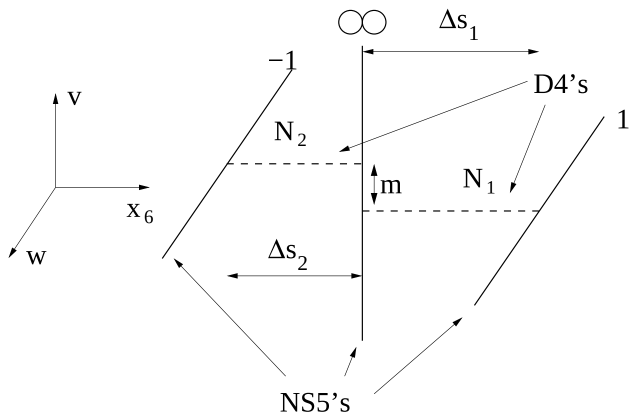
$$2\pi i\mathcal{F}(S) = \frac{1}{2}S_1^2\left(\ln\left(\frac{S_1}{m\Lambda_0^2}\right)-\frac{3}{2}\right)+\frac{1}{2}S_2^2\left(\ln\left(\frac{S_2}{m\Lambda_0^2}\right)-\frac{3}{2}\right) - S_1S_2\ln\left(\frac{m}{\Lambda_0}\right) \tag{14}$$

One immediately finds that the supersymmetric vacua occur for

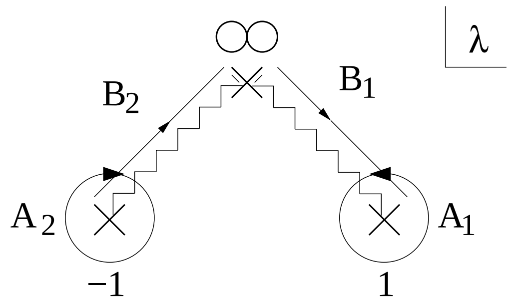
$$S_1^{N_1} = \Lambda_0^{2N_1-N_2}m^{N_1+N_2}e^{-2\pi i\alpha_1} \qquad S_2^{N_2} = \Lambda_0^{2N_2-N_1}m^{N_2+N_1}e^{-2\pi i\alpha_2} \tag{15}$$

2.2 The IIA Side

To study this system from the IIA point of view, we apply T-duality along the S^1 contained in the $\mathbb{C}^*$ fiber. The result of this is to replace the local CY geometry with a configuration of NS5



(a) NS5/D4 configuration related to D5's at singularities of the local CY (4). The marked point on the λ-plane (± 1 and ∞) to which each NS5 corresponds is also indicated.



(b) λ-plane used to parametrize the M-theory lift of the classical NS5/D4 configuration in figure 2(a). The A and B cycles T-dual to those of figure 1(a) are also indicated.

Figure 2: NS5/D4 setup in type IIA and the λ-plane used to parametrize its M-theory lift

branes in flat space. More specifically, one obtains three NS5 branes, each extended along the 0123 directions and one of the three degeneration curves of figure 1(b). Moreover, the three NS5's are further separated along the dual circle direction, which we denote by x_6, by an amount determined by $\int B$ along the two cycles connecting these curves. Finally, the D5 branes wrapping vanishing cycles dualize into D4 branes connecting adjacent NS5's. We study the limit in which the x_6 direction is essentially decompactified so that the resulting configuration is as in figure 2(a) and consists of three parallel NS5 branes with stacks of N_1 and N_2 D4 branes in between.

To study this system at the quantum level, we recall that NS5's and D4's are both different manifestations of a single object, namely the M5 brane. We therefore look for a smooth M5 brane extended along the 0123 directions and wrapping a minimal area Riemann surface in the remaining directions. We will parametrize this Riemann surface by the coordinates w and v as well as the holomorphic combination of the dual circle direction, x_6, and the M-circle, x_{10}

$$s = x_6 + ix_{10} \tag{16}$$

In the literature one typically gives an explicit representation of this embedding by specifying $w(v)$ and $s(v)$ but, because this is difficult to do in the nonsupersymmetric setting, we shall focus instead on a parametric description. For this, we note that the configuration at hand will lift to a genus 0 Riemann surface with three punctures, corresponding to the points at infinity on the three NS5's[6].

[6]We must have punctures because flux is allowed to come in from infinity along the NS5's.

We can therefore parametrize this surface by a single copy of $\mathbb{C}$ with three marked points, which can be taken to lie at -1, 1, and ∞.

To find the embedding relevant for the specific IIA configuration at hand, we must specify appropriate boundary conditions. First, along w and v we require that[7]

$$w(-1) = \frac{m}{2} \qquad w(\infty) = 0 \qquad w(1) = -\frac{m}{2} \tag{17}$$

and

$$v(-1) = \infty \qquad v(\infty) = \text{constant} \qquad v(1) = \infty \tag{18}$$

Now we must also specify the boundary conditions along s, which characterize the separation between NS5's and windings that originate from the D4's. These are most easily phrased in terms of periods of the 1-form ds about the A and B cycles depicted in figure 2(b)

$$\frac{1}{2\pi i}\oint_{A_j} ds = N_j \qquad \frac{1}{2\pi i}\oint_{B_j} ds = -\alpha_j \tag{19}$$

A holomorphic embedding satisfying these conditions is easy to write down

$$\begin{aligned} s(\lambda) &= N_1 \ln(\lambda - 1) - N_2 \ln(\lambda + 1) \\ w(\lambda) &= -\frac{2}{m}\left(\frac{A}{\lambda - 1} - \frac{B}{\lambda + 1}\right) \\ v(\lambda) &= -\frac{m\lambda}{2} \end{aligned} \tag{20}$$

where[8]

$$A^{N_1} = m^{N_1+N_2}\Lambda_0^{2N_1-N_2} e^{-2\pi i\alpha_1} \qquad B^{N_2} = m^{N_1+N_2}\Lambda_0^{2N_2-N_1} e^{-2\pi i\alpha_2} \tag{21}$$

From this parametric description, we can easily obtain the explicit one

$$\begin{aligned} 0 &= w\left(v^2 - \frac{m^2}{4}\right) - A\left(v - \frac{m}{2}\right) + B\left(v + \frac{m}{2}\right) \\ s &= \ln\left(\frac{m^{N_2-N_1}\left(v + \frac{m}{2}\right)^{N_1}}{\left(v - \frac{m}{2}\right)^{N_2}}\right) + \text{constant} \end{aligned} \tag{22}$$

which agrees with the M-theory lift for this setup originally found by Schmaltz and Sundrum [18]. If we interpret this M5 in IIA as a curved NS5 with flux, it is related by T-duality to a deformed CY with flux of precisely the form (6). Note, however, that the combination of holomorphy and our

[7] We also require that the embedding is locally 1-1 near the points where w and/or v diverge, which means that these correspond to single poles of the complex parameter λ.

[8] When imposing the B-period constraints on ds, which fix A and B in terms of the boundary data α_j, we must regularize, just as in the IIB case, by introducing a cutoff. In the following, we cutoff integrals over v at Λ_0 and integrals over w at $m\Lambda_0$. This is consistent with the fact that, before we did the rotation to the form (4), $w \sim \pm mv$ at infinity.

boundary conditions have completely fixed the moduli A and B of the resulting Riemann surface. To compare with the IIB side, we simply evaluate

$$S_1 = \oint_{A_1} w\, dv = A \qquad S_2 = \oint_{A_2} w\, dv = B \tag{23}$$

to see that A and B are nothing more than the moduli S_i. From (21), we finally see that the M5 curve is T-dual to the IIB deformed geometry with flux after geometric transition with moduli sitting precisely in the supersymmetric vacuum.

2.3 Some Further Remarks

Because the system considered thus far was supersymmetric, we were not particularly careful about enumerating when various analyses and interpretations were truly reliable. It is important to look at this issue a bit more carefully, though, if one wants to move to the nonsupersymmetric setting.

First of all, it must be noted that the IIB and IIA analyses presented here are never simultaneously valid. The reason for this is that on both the IIB and IIA sides, we had to assume the S^1 that gets T-dualized is large in string units. Consequently, each description is really probing a different regime of parameter space. For the SUSY configurations studied here, it is not surprising that the two sides nevertheless agree because they describe BPS vacua but, in more general settings, there is no *a priori* reason to expect any resemblance at all between the IIB and IIA descriptions. In a regime where $\mathcal{N} = 2$ SUSY is truly spontaneously broken, though, this residual structure provides enough "protection" to expect agreement. In this manner, a IIA description which is equivalent to the IIB one would provide some evidence for the existence of this extra structure and might provide a clue as to what parameter(s) controls it.

Secondly, we must be careful to enumerate precisely when minimal area M5 curves provide a reliable description of the vacuum and, in addition, when an interpretation at weak coupling in terms of a curved NS5 brane with flux can be trusted. Said more simply, we must ask when the probe approximation is truly reliable. In general, one requires that the curvatures and flux density are both small in the appropriate units, namely string units in IIA and 11-dimensional Planck units in M-theory. It is easy to demonstrate [19] that there are actually two quite separate regimes in which both of these conditions can be met, one at weak coupling $g_s \ll 1$ where we have a IIA description, and one at strong coupling $g_s \gg 1$ where we have an M-theory description. If we tune parameters to lie within one of these two regimes, we can use the probe approximation, and hence our minimal area curves, even to describe vacuum configurations that are not supersymmetric.

3 The Nonsupersymmetric Story

3.1 IIB Side

We now return to the local CY (4) and consider the possibility of replacing one set of D5's with $\overline{\text{D5}}$'s. An example of this sort was studied recently in [3]. According to the large N duality conjecture of [1], we can study this system using the same deformed CY (6) with flux provided we associate negative RR fluxes with the presence of antibranes before the geometric transition.

Once we arrive in the deformed CY with flux picture, we can use the fact that the fluxes spontanneously break $\mathcal{N}=2$ SUSY to study the vacuum structure. In particular, we still have the superpotential (7) and the Kahler potential is determined by special geometry according to $\mathcal{K}\sim\Im\hat{\tau}$. Our interest now, however, is in nonsupersymmetric solutions so instead of studying the condition (13) we look instead for minima of the full scalar potential

$$V=\overline{\left(\alpha_i+N^\ell\hat{\tau}_{\ell i}\right)}\left(\Im\tau^{-1}\right)^{ij}\left(\alpha_j+N^m\hat{\tau}_{mj}\right) \tag{24}$$

Using the prepotential (14), it is easy to see that there are in fact four critical points of this potential given by

$$S_1^{N_1}=\Lambda_0^{2N_1-N_2}m^{N_1+N_2}e^{-2\pi i\alpha_1} \qquad S_2^{N_2}=\Lambda_0^{2N_2-N_1}m^{N_2+N_1}e^{-2\pi i\alpha_2} \tag{25}$$

$$S_1^{N_1}=\Lambda_0^{2N_1-N_2}m^{N_1+N_2}e^{-2\pi i\bar{\alpha}_1} \qquad S_2^{N_2}=\Lambda_0^{2N_2-N_1}m^{N_2+N_1}e^{-2\pi i\bar{\alpha}_2} \tag{26}$$

$$S_1^{N_1}=\left(\frac{\Lambda_0^{2N_1}\bar{\Lambda}_0^{N_2}m^{N_1}}{\bar{m}^{N_2}}\right)e^{-2\pi i\alpha_1} \qquad S_2^{N_2}=\left(\frac{\Lambda_0^{2N_2}\bar{\Lambda}_0^{N_1}m^{N_2}}{\bar{m}^{N_1}}\right)e^{-2\pi i\bar{\alpha}_2} \tag{27}$$

$$S_1^{N_1}=\left(\frac{\Lambda_0^{2N_1}\bar{\Lambda}_0^{N_2}m^{N_1}}{\bar{m}^{N_2}}\right)e^{-2\pi i\bar{\alpha}_1} \qquad S_2^{N_2}=\left(\frac{\Lambda_0^{2N_2}\bar{\Lambda}_0^{N_1}m^{N_2}}{\bar{m}^{N_1}}\right)e^{-2\pi i\alpha_2} \tag{28}$$

For each of these critical points, however, we must determine when it is a minimum and when it lies within the physical regime $\Im\tau>0$. The importance of the second point cannot be understated as even the supersymmetric solutions can correspond to Riemann surfaces that become degenerate if the holomorphic volumes of the blown up three-cycles become sufficiently large compared to the cutoff scale, Λ_0. It is easy to demonstrate that, in fact, no two critical points can ever simultaneously exist in the physical regime and, moreover, that any critical point which lies within the physical regime is a minimum of the potential.

For simplicity, we now restrict our attention to the case $\Im\alpha_j<0$. In the supersymmetric scenario where $N_1,N_2>0$ it is easy to see that only the supersymmetric solution (25), which also satisfies (13), lies within the physical regime. Alternatively, if we take $N_1,N_2<0$ then the relevant solution becomes (26), which satisfies

$$\alpha_j+\bar{\hat{\tau}}_{jk}N^k=0 \tag{29}$$

and preserves the opposite half of the original $\mathcal{N} = 2$ supersymmetries as (25). This describes the vacuum configuration obtained by starting with two stacks of $\overline{\text{D5}}$'s before the geometric transition.

If we consider $N_2 > 0$ and $N_1 < 0$, though, neither (25) nor (26) lies within the physical regime[9]. Instead, the only minimum is a nonsupersymmetric one (28), which describes the vacuum configuration associated to N_2 D5's and N_1 $\overline{\text{D5}}$'s. Similarly, the solution (27) yields the nonsupersymmetric vacuum in the case $N_2 < 0$ and $N_1 > 0$ and describes the vacuum configuration associated to N_1 D5's and N_2 $\overline{\text{D5}}$'s.

3.2 The IIA Side

We now turn to the type IIA/M description of the nonsupersymmetric configurations just discussed. For definiteness, let us take $N_1 > 0$ and $N_2 < 0$. We also take α_j purely imaginary and m, Λ_0 purely real to keep things simple. The setup is as in figure 2(a) with the stack of N_2 D4's replaced by a stack of N_2 $\overline{\text{D4}}$'s. The holomorphic embedding (20) with N_2 taken to be negative has the right properties to describe a lift of this configuration but this simply corresponds to the supersymmetric solution (25) on the IIB side, which lies outside of the physical regime $\Im\tau > 0$. From the IIA/M point of view, this "breakdown" arises because the tubes into which the D4's and $\overline{\text{D4}}$'s blow up become large compared to the cutoff scale Λ_0[10]. What we would like to do here is find the configuration corresponding to the solution (27). This will take the form of a nonholomorphic minimal area embedding of the M5 into flat space.

The equations of motion governing such a minimal area embedding are familiar from bosonic string theory. As we recall from that context, the embedding coordinates must be harmonic

$$\partial\bar{\partial}s(\lambda,\bar{\lambda}) = \partial\bar{\partial}w(\lambda,\bar{\lambda}) = \partial\bar{\partial}v(\lambda,\bar{\lambda}) = 0 \tag{30}$$

and satisfy the "Virasoro" constraint[11]

$$g_s^2\partial s\partial\bar{s} + \partial w\partial\bar{w} + \partial v\partial\bar{v} = 0 \tag{31}$$

The search for nonholomorphic solutions to this system may seem hopeless but note that harmonic s, v, w can be written as sums of holomorphic and antiholomorphic functions

$$s(\lambda,\bar{\lambda}) = s_H(\lambda) + s_A(\bar{\lambda}) \qquad w(\lambda,\bar{\lambda}) = w_H(\lambda) + w_A(\bar{\lambda}) \qquad v(\lambda,\bar{\lambda}) = v_H(\lambda) + v_A(\bar{\lambda}) \tag{32}$$

[9]For this, we use the fact that one naturally chooses the mass m to be smaller than the cutoff scale, $m < \Lambda_0$

[10]It is not clear that this "breakdown" is a disaster in this IIA/M description. In type IIB, it causes the potential to become negative but the analagous quantity on the IIA/M side, namely the area of the curve, remains a manifestly positive definite quantity. What does break down when $\Im\tau$ ceases to be positive definite is the ability of the IIB potential to approximate the area of the M5 curve at weak coupling.

[11]Because we defined s without a factor of $R_{10} = g_s$, an explicit factor of g_s^2 appears here from the target space metric.

in terms of which (31) becomes

$$g_s^2 \partial s_H(\lambda)\partial s_A(\lambda) + \partial w_H(\lambda)\partial w_A(\lambda) + \partial v_H(\lambda)\partial v_A(\lambda) \tag{33}$$

Thus, even though (31) is a nonlinear condition, it is nevertheless a holomorphic one, significantly simplifying the task of finding solutions. In the case at hand, let us begin by looking for a nonholomorphic minimal area embedding satisfying the A period constraints

$$\frac{1}{2\pi i}\oint_{A_j} ds = N_j \tag{34}$$

and having first order poles in w (v) at $\lambda = \pm 1$ ($\lambda = \infty$). Such a solution is not difficult to find and takes the form

$$\begin{aligned} s(\lambda,\bar\lambda) &= g_s\left[\frac{N_1}{2}\ln\left(\frac{\lambda-1}{\bar\lambda-1}\right) - \frac{N_2}{2}\ln\left(\frac{\lambda+1}{\bar\lambda+1}\right)\right] \\ &\qquad + g_s N_1\cos\theta_1 \ln|\lambda-1| + g_s N_2 \cos\theta_2 \ln|\lambda+1| \\ w(\lambda,\bar\lambda) &= -\frac{2}{m}\left(\frac{A}{\lambda-1} - \frac{B}{\lambda+1}\right) \\ v(\lambda,\bar\lambda) &= -M\lambda + g_s N_1 \sin\theta_1 \ln|\lambda-1| - g_s N_2\sin\theta_2\ln|\lambda+1| \end{aligned} \tag{35}$$

where

$$\begin{aligned} 0 &= N_1\sin\theta_1 - N_2\sin\theta_2 \\ M &= \frac{g_s N_2\left[1+\cos(\theta_1+\theta_2)\right]}{2\sin\theta_1} \end{aligned} \tag{36}$$

and θ_1 is a real parameter. This is actually a three-parameter family of solutions which depends on our choices of A, B, and θ_1. To find the solution which yields a lift of the IIA configuration 2(a), we must further impose the B-period constraints

$$\frac{1}{2\pi i}\oint_{B_j} = -\alpha_j \tag{37}$$

and the condition

$$\oint_{B_1+B_2} dv = -m \tag{38}$$

which fixes the relative separation between the left- and right-most NS5's of figure 2(a) along v. These conditions can be studied in detail and indeed have some interesting structure[12] but for now let us focus on general features of the solution which do not require this level of scrutiny.

[12] By analogy with the brane/antibrane setup of [1, 2], there is a regime where we find two distinct solutions, of which only one is stable, and there is a regime where we find no solutions at all.

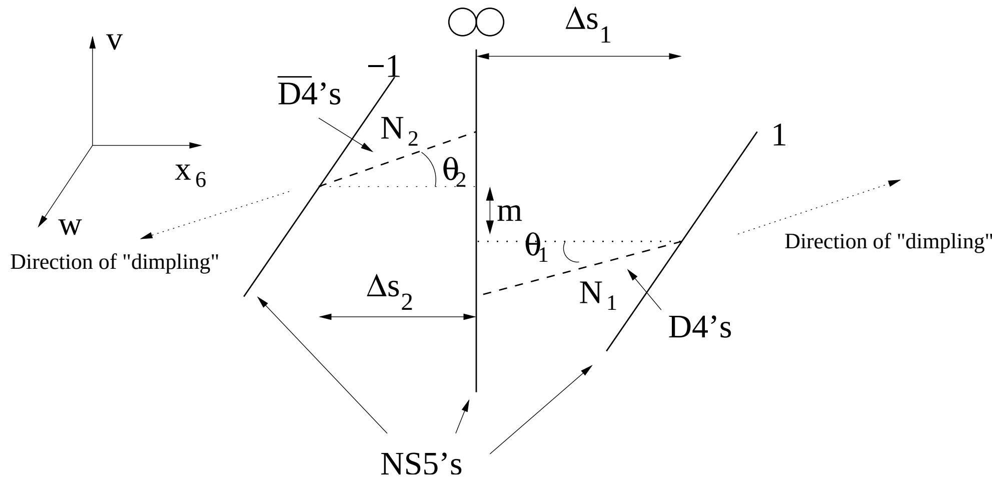


Figure 3: Tilting of D4 and $\overline{\text{D4}}$ stacks which gives rise to nonholomorphic "dimpling" of the NS5's along v

3.2.1 Understanding the Nonholomorphic Features

The first thing to note is that the embedding along w and v is now manifestly nonholomorphic. Consequently, when interpreted as describing the worldvolume of a curved NS5 brane, the configuration at hand does not appear to be T-dual to anything resembling a deformed CY. This is in stark contrast to what we might have expected from large N duality on the type IIB side and seems to indicate that the fragile relationship between the IIB and IIA stories is falling apart when supersymmetry is broken.

We might still hope, though, that these nonholomorphic features are negligible in some parameter regime. To see if this is true, let us try to understand their physical origin. From the form of (35), it appears that the logarithmic bending along x^6, which is a quantum effect that arises from D4's pulling on and "dimpling" the NS5's [12, 13], has been rotated into the $\Re v$ direction by an angle θ_1 near one NS5 and by an angle θ_2 near the other.

This rotation has a simple physical origin, though, in the repulsion between D4's and $\overline{\text{D4}}$'s ending on a common NS5 from opposite directions [20, 21]. This causes the D4's and $\overline{\text{D4}}$'s to rotate slightly which, in turn, means that they dimple the NS5's along a direction that is not exactly parallel to x^6. As a result, logarithmic bending leaks into the v-direction and the $w(v)$ embedding becomes manifestly nonholomorphic. This situation is depicted in figure 3.

To see what parameter controls the rotation of the D4's and $\overline{\text{D4}}$'s, let us follow a physical argument of the sort described in [21]. Momentarily setting $N_1 = N_2$ for simplicity, we note that the branes and antibranes behave like oppositely charged particles on the one transverse direction of the NS5 on which they both end. Denoting the distance that either the branes or antibranes move from the initial position (where they were separated by m) by x, the corresponding potential is

$$V_{\text{attractive}} \sim N^2 \ln\left(\frac{m-2x}{m}\right) \tag{39}$$

On the other hand, in order to move closer to one another, the D4's and $\overline{\text{D4}}$'s must stretch, paying an energy cost in tension. The corresponding contribution to the potential is

$$V_{\text{tension}} = \frac{2N}{g_s}\sqrt{\Delta s^2 + x^2} \tag{40}$$

At small x, the full potential $V_{\text{attractive}} + V_{\text{tension}}$ is minimized for a rotation angle

$$\theta \sim \tan\theta = \frac{x}{\Delta s} \sim \frac{g_s N}{m} + \mathcal{O}\left([g_s N/m]^2\right) \tag{41}$$

We can see this relation arise directly from our solution (35) as well, though it is not obvious as written because one must first impose the constraints (37) and (38) to derive it. Nevertheless, when $g_s N_j/m$ is small the rotation angle θ_1 scales like [13].

$$\sin\theta_1 \approx \frac{2g_s N_2}{m} + \mathcal{O}\left(\left[\frac{g_s N_j}{m}\right]^3\right) \tag{42}$$

and our full solution (35) reduces to

$$\begin{aligned} s(\lambda,\bar\lambda) &= g_s\left[\frac{N_1}{2}\ln\left(\frac{\lambda-1}{\bar\lambda-1}\right) - \frac{N_2}{2}\ln\left(\frac{\lambda+1}{\bar\lambda+1}\right)\right] \\ &\qquad + g_s N_1 \ln|\lambda-1| + g_s N_2 \ln|\lambda+1| \\ w(\lambda) &= -\frac{2}{m}\left(\frac{A}{\lambda-1} + \frac{B}{\lambda+1}\right) \\ v(\lambda) &= -\frac{m\lambda}{2} \end{aligned} \tag{43}$$

When interpreted as a curved NS5 with flux in type IIA, this solution is T-dual to the deformed CY (6) with flux. Moreover, if we follow the values of A and B forced on us by the constraints (37) and (38) in this limit, we find

$$A^{N_1} = \Lambda_0^{2N_1+N_2} m^{N_1-N_2} e^{-2\pi i\alpha_1} \qquad B^{N_2} = \Lambda_0^{2N_2+N_1} m^{N_2-N_1} e^{2\pi i\alpha_2} \tag{44}$$

Recalling that the moduli of the Riemann surface $w(v)$ are given by

$$S_1 = A \qquad S_2 = B \tag{45}$$

[13] Actually, there are two branches of solution for θ_j at small g_s. We focus here on the branch in which θ_j is proportional to g_s at small g_s as it seems to be the only stable one.

we see that our solution is exactly T-dual to the nonsupersymmetric minimum (27) [14].

What we have seen, then, is that when the parameters $g_s N_j/m$ are small, the IIA/M description exhibits complete agreement with the type IIB results obtained using large N duality and the assumption of spontaneous breaking of $\mathcal{N}=2$ SUSY.

3.2.2 Comments on more general setups

We can understand the importance of the parameter $g_s N_j/m$ for controlling the generation of nonholomorphic terms in $w(v)$ from a more general perspective by looking directly at the constraint (33). In order for the curve to take the approximate form of a holomorphic embedding $w(v)$ with harmonic s, the characteristic sizes of ∂w and ∂v must be small compared to that of ∂s, which is simply $g_s N_j$. In general, ∂w and ∂v will scale like the larger of S_j, m, which control holomorphic volumes in the curve, but in our example the actual contributions of w and v to (33) instead always scale like m. This is due to the extreme simplicity of the example under study and we expect that in general one needs all of $g_s N/S_j$ and $g_s N/m$ to be small in order for antiholomorphic terms in w and v to be negligible.

It is also important to note that, in order for the probe approximation in IIA to be reliable, one needs all of $g_s N/S_j$ and $g_s N/m$ to be small [19]. This means that, whenever the IIA description is valid, the resulting configuration is expected to take a form that is T-dual to a deformed CY with flux. Moreover, we can argue that, whenever the underlying Riemann surface is nondegenerate[15], the moduli resulting from the IIA/M analysis will always sit at a minimum of the IIB potential. To see this, note that when $\partial s \ll \partial w, \partial v$ the Nambu-Goto action becomes approximately

$$S_{NG} = \frac{1}{g_s}\int \sqrt{g(v,w,s)} = \frac{1}{g_s}\int \sqrt{\tilde{g}(v,w)} + \int |ds|^2 + \ldots \tag{46}$$

where $\tilde{g}$ is the induced metric associated to the v and w coordinates of the embedding only. The $\tilde{g}$ term implies that $w(v)$ must yield a minimal area embedding on its own which is necessarily holomorphic if we impose holomorphic boundary conditions along those directions. The complex structure moduli of $w(v)$ then yield flat directions of the $\tilde{g}$ term.

Turning now to the second term of (46), we find that ds must define a harmonic 1-form on the Riemann surface given by $w(v)$. The boundary conditions that we impose along s amount to fixing both the A- and B-periods of this harmonic 1-form and hence s is specified uniquely for a given choice of complex structure moduli. Because the complex structure moduli are not flat directions of $\int |ds|^2$, though, we must do a further minimization. A careful analysis reveals that,

[14] Recall that, for simplicity, we took α_j imaginary and Λ_0, m real.

[15] Recall that some minima of the IIB potential sit at regions in parameter space where holomorphic volumes of the Riemann surface are larger than the cutoff scale, rendering the regulated Riemann surface effectively degenerate.

for s satisfying the boundary conditions [19], $\int |ds|^2$ is nothing more than the scalar potential (24) that arises in type IIB [19][16] Consequently, this last minimization step is mathematically equivalent to the procedure that we follow in type IIB to find vacua. Agreement between the IIB and IIA/M pictures is thus guaranteed whenever a type IIA interpretation of our M5 curve is valid.

Does this mean that the nonholomorphic features of (35) are unphysical after all? Despite the fact that they cannot be reliably turned on at small g_s, it turns out [19] that they are indeed present over a wide range of parameter space at large g_s where the probe approximation is valid. In fact, for appropriate choices of other parameters in the problem, one can freely tune $g_s N_j/m$ from very small to quite large in the strongly coupled regime all the while maintaining reliability of the M5 probe approximation. This means that $g_s N_j/m$ seems to be relevant for controlling this nonholomorphic behavior, and hence spontaneous breaking of SUSY, at strong coupling as well.

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