Controlled coupling of charge qubits

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Quantum dynamics of Josephson junctions

- Superconductor can be thought of as a BEC of Cooper pairs: one single-particle state

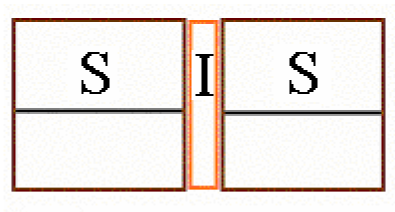
$$\Psi = \sqrt{n}e^{ij}$$

occupied with macroscopic number of particles. The phase f and the number of particles n are conjugate quantum variables (Anderson, 64; Ivanchenko, Zil’berman, 65):

$$[n, j\,] = \mathrm{i}.$$

This relation describes dynamics of addition or removal of particles to/from the condensate.

• This dynamics manifests itself most directly in Josephson tunnel junctions, and was studied as an example of *macroscopic* quantum dynamics (Leggett, 80).



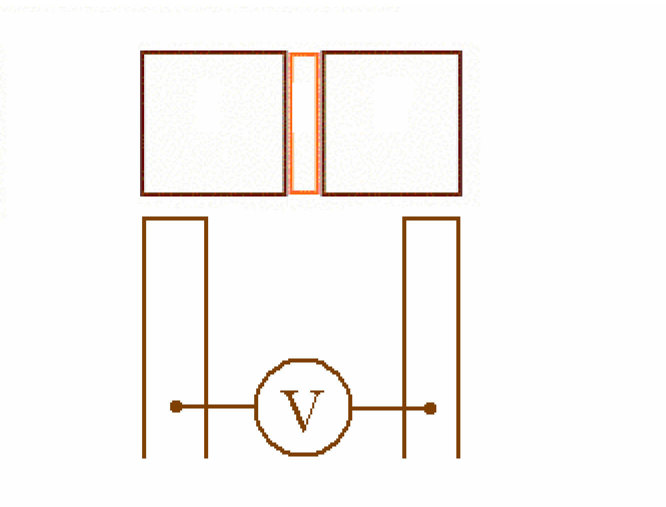
$$H = -E_C\, \partial^2 / \partial \varphi^2 - E_J \cos \varphi + U_{ext}(\varphi), \qquad E_C \equiv (2e)^2 / 2C\,.$$

• If quantum fluctuations of phase φ become large, junction behavior can be described as a semiclassical dynamics of charge that leads to controlled transfer of individual Cooper pairs (Averin, Zorin, Likharev, 1985).

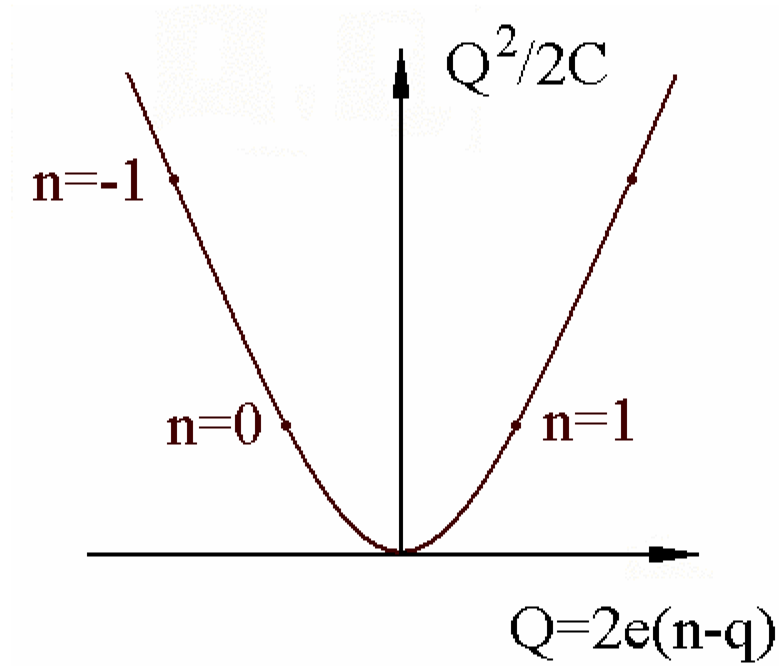
$$H = E_C (n-q)^2 - E_J/2 \left(|n\rangle\langle n \pm 1| + |n \pm 1\rangle\langle n| \right).$$

Charge qubits

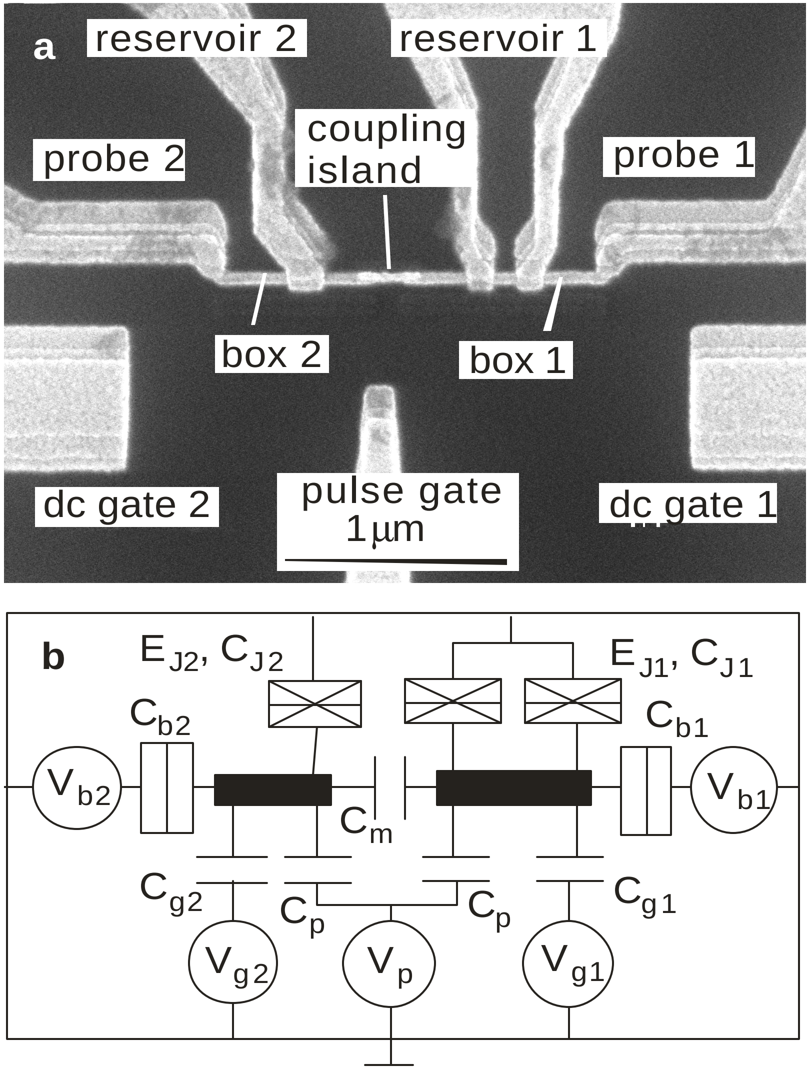
For $E_J << E_C$ and $q˜ 1/2$, the charge tunneling dynamics in an isolated individual junction is directly reduced to the two-state form.



$$H = -E_C (q - 1/2)\, s_z - (E_J/2)\, s_x$$



Two coupled charge qubits

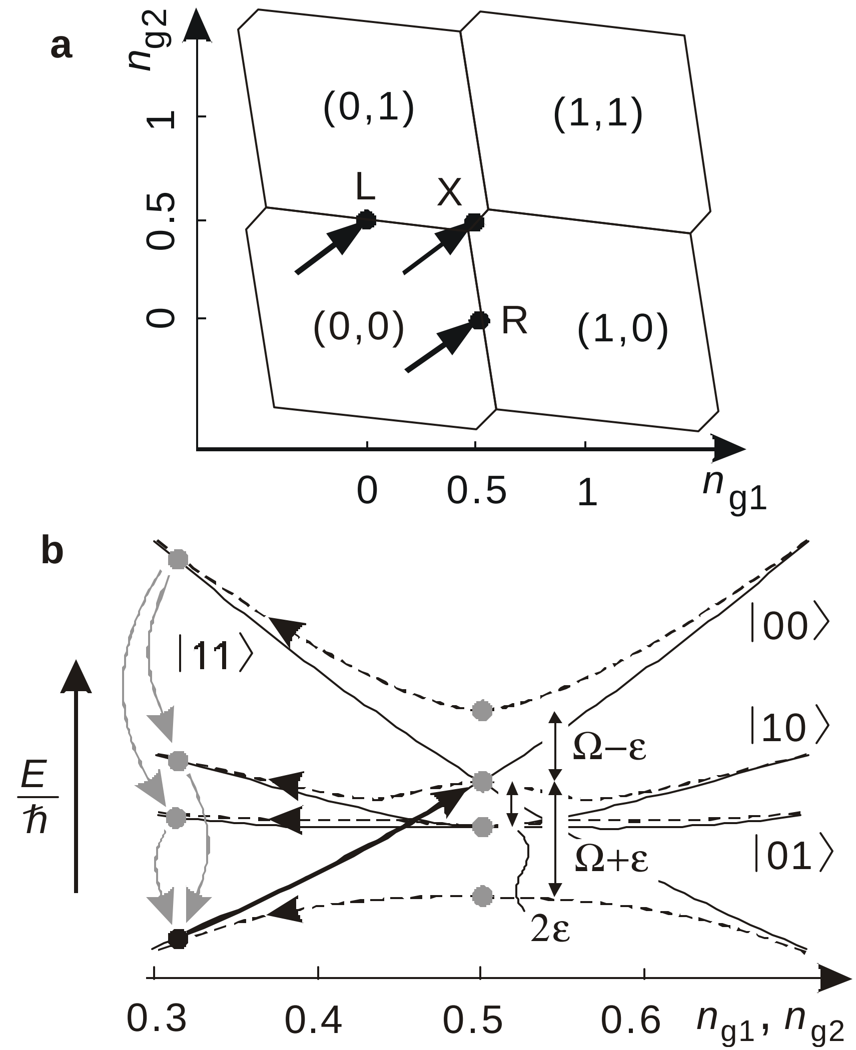


$$H = \begin{bmatrix} E_{00} & -\frac{1}{2}E_{J1} & -\frac{1}{2}E_{J2} & 0 \\ -\frac{1}{2}E_{J1} & E_{10} & 0 & -\frac{1}{2}E_{J2} \\ -\frac{1}{2}E_{J2} & 0 & E_{01} & -\frac{1}{2}E_{J1} \\ 0 & -\frac{1}{2}E_{J2} & -\frac{1}{2}E_{J1} & E_{11} \end{bmatrix}$$

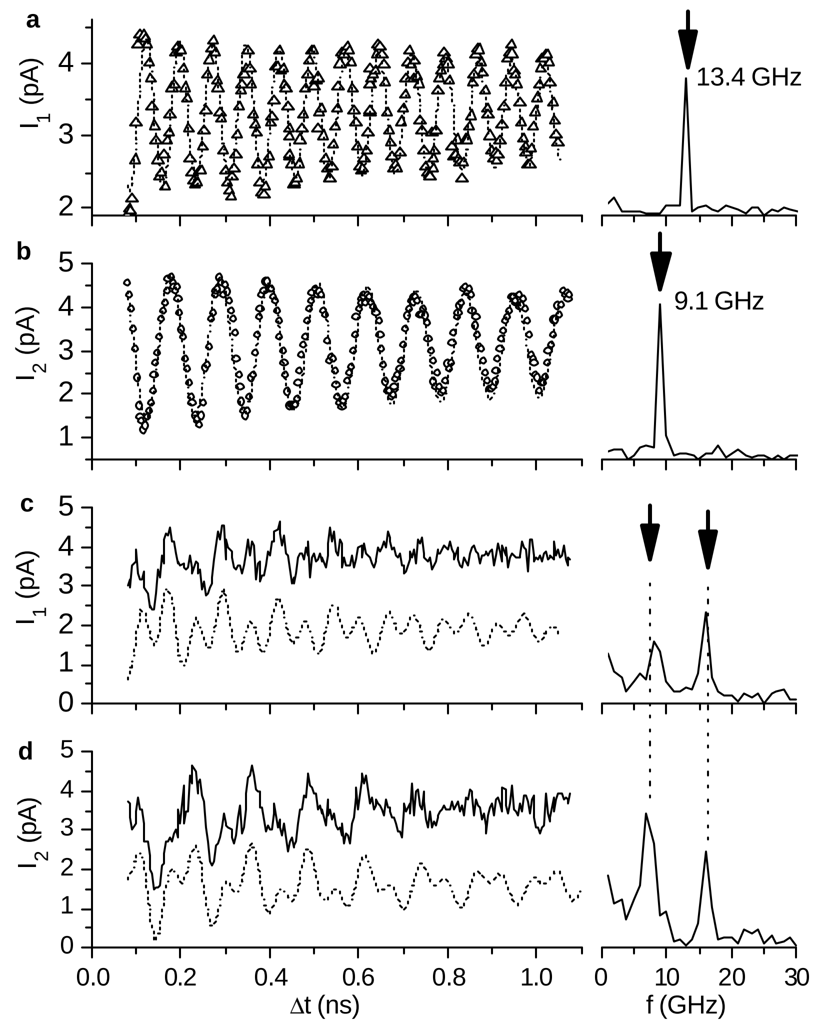
$$E_{n1n2} = E_{c1}(n_{g1}-n_1)^2 + E_{c2}(n_{g2}-n_2)^2 + \\ +E_m(n_{g1}-n_1)(n_{g2}-n_2),$$

$$E_m = e^2C_m/(C_{S\,1}C_{S\,2} - C_m^{\ 2})$$

Yu. A. Pashkin *et al*.,
Nature **421**, 823 (2003).



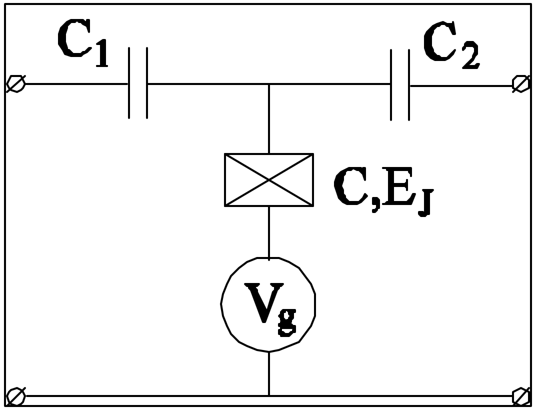
a
n_{g2}
1
0.5
0
(0,1)
(1,1)
L
X
R
(0,0)
(1,0)
0
0.5
1
n_{g1}
b
$|11\rangle$
$|00\rangle$
$|10\rangle$
$|01\rangle$
$\Omega-\varepsilon$
$\Omega+\varepsilon$
2ε
$\frac{E}{\hbar}$
0.3
0.4
0.5
0.6
n_{g1}, n_{g2}



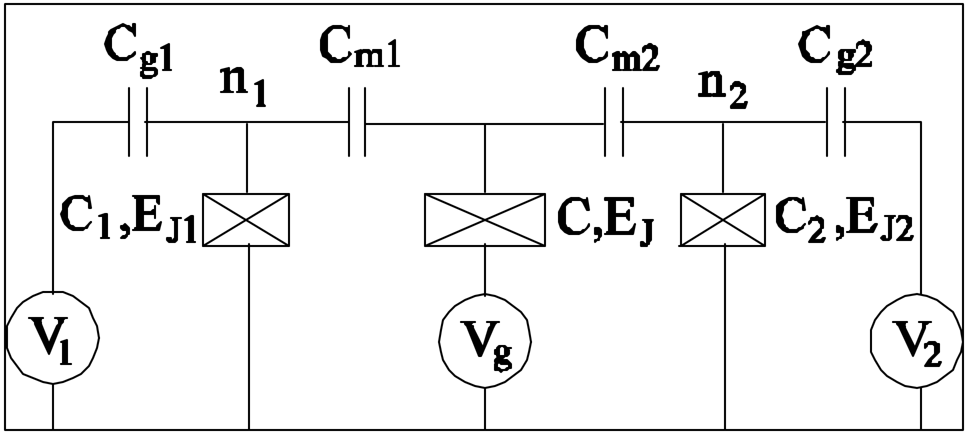
a
I_1 (pA)
13.4 GHz
b
I_2 (pA)
9.1 GHz
c
I_1 (pA)
d
I_2 (pA)
0.0
0.2
0.4
0.6
0.8
1.0
Δt (ns)
0
10
20
30
f (GHz)

Variable electrostatic transformer: controlled coupling of charge qubits

Equivalent circuit of the variable electrostatic transformer:



Gate-controlled qubit coupling:



coupling capacitance:

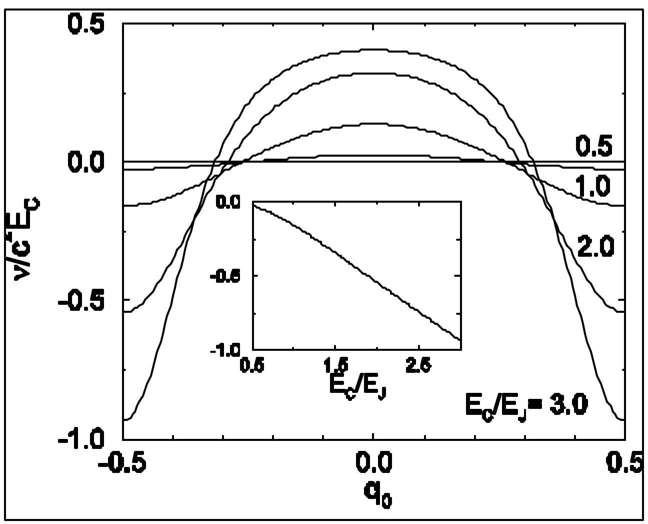
$$C \equiv \partial V_{out} / \partial q = \partial^2 e_0 (q_g + q) \big/ \partial q^2$$

Coupling strength:

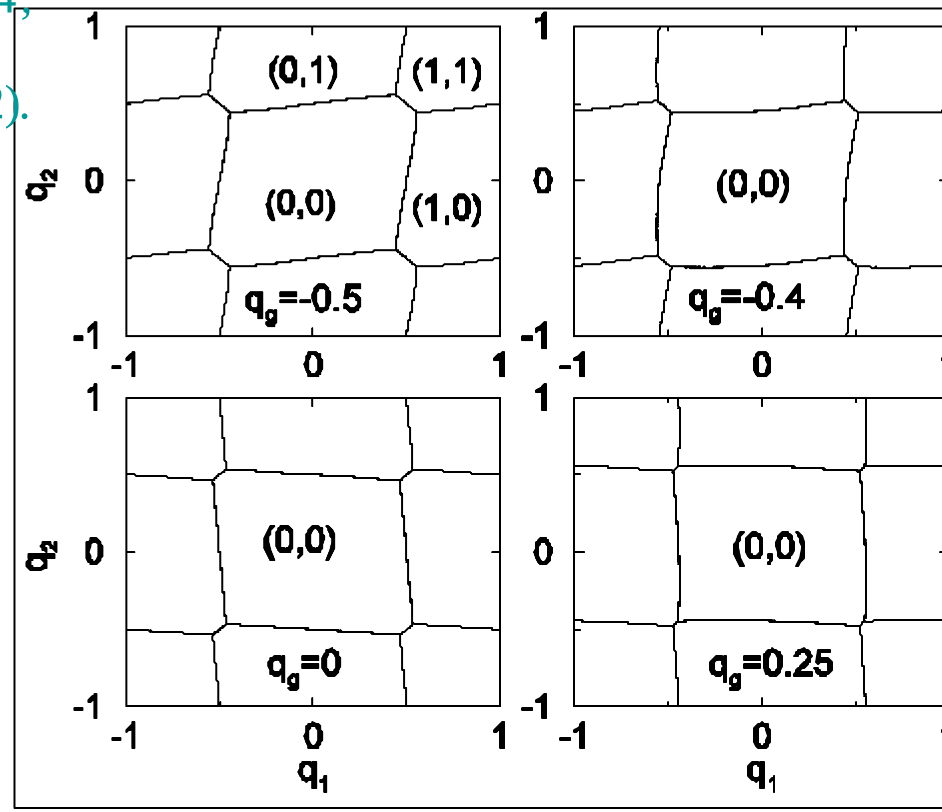
$$H = n\, s_{z1} s_{z2},$$

$$n = [e_0(q_0 + c) + e_0(q_0 - c) - 2e_0(q_0 + c)]/4,$$

$$c = C_m / C_\Sigma, \qquad q_0 = q_g + c\sum_{i=1,2}(q_i - 1/2).$$

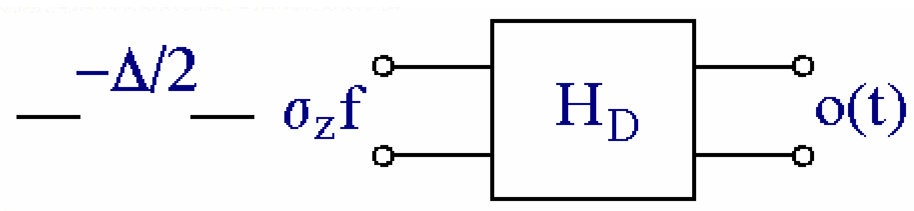


Charging diagram demonstrates transition from positive to negative coupling



Continuous monitoring of the MQC oscillations

The trade-off between the information acquisition by the detector and back-action dephasing manifests itself in the directly measurable quantity in the case of measurement of coherent quantum oscillations in a qubit.



$$H = -\frac{\Delta}{2}\sigma_x + \sigma_z f + H_D$$

Spectral density $S_o(\omega)$ of the detector output reflects coherent quantum oscillations of the measured qubit:

$$S_o(\omega) = S_q + \frac{\Gamma \lambda^2}{4\pi} \frac{\Delta^2}{(\omega^2 - \Delta^2)^2 + \Gamma^2 \omega^2}.$$

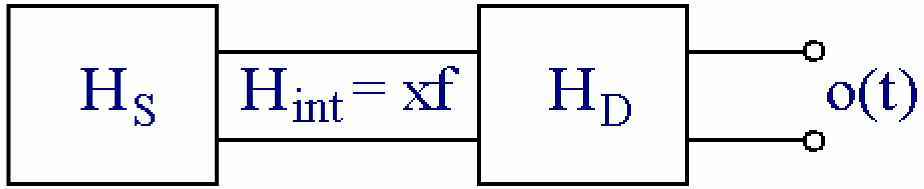
The height of the oscillation peak in the output spectrum is limited by the link between the information and dephasing:

$$S_{\max} / S_q \leq 4.$$

Linear quantum measurements

Linear-response theory enables one to develop quantitative description of the quantum measurement process with an arbitrary detector provided that it satisfies some general conditions:

- the detector/system coupling is weak so that the detector's response is linear;
- the detector is in the stationary state;
- the response is instantaneous.



$$H = H_S + H_D + xf$$

D.V.A., cond-mat/00044364, cond-mat/0010052, and to be published.
S.Pilgram and M. Büttiker, PRL 89, 200401 (2002).
A.A. Clerk, S.M. Girvin, and A.D.Stone, cond-mat/0211001.

FDT analog for quantum measurements

$$\eta|l| \leq 4p\,[S_f S_q - (\mathrm{Re}\,S_{fq})^2]^{1/2},$$

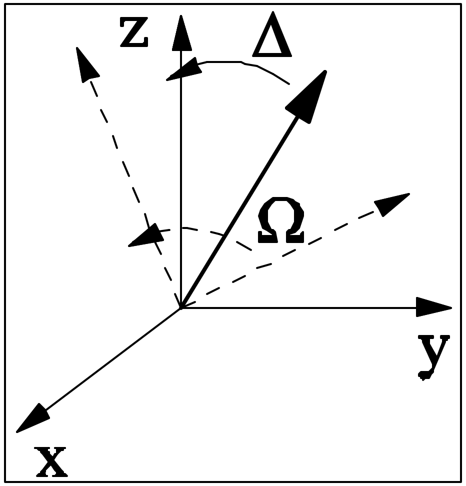
where ? is the linear response coefficient of the detector, S_f and S_q are the low-frequency spectral densities of the, respectively, back-action and output noise, $\mathrm{Re}S_{fq}$ is the classical part of their cross-correlator.

This inequality shows that finite response coefficient implies that that noise generated by the detector is non-vanishing. Although it was obtained from the linear-response theory, it has broader meaning in that it characterizes the efficiency of the trade-off between the information acquisition by the detector and back-action dephasing of the measured system. The detector that satisfies this inequality as equality is called ``ideal'' or ``quantum-limited''.

Quantum non-demolition measurements of a qubit

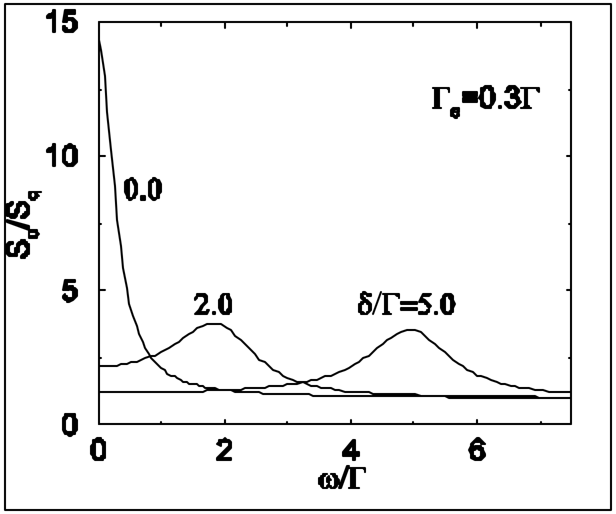
QND measurement avoids the detector backaction by employing specially designed detector-qubit coupling which effectively measures qubit in the rotating frame that follows the qubit oscillations:

$$H = -\frac{\Delta}{2}s_x - \frac{1}{2}(s_z \cos\Omega t + s_y \sin\Omega t) f + H_D$$



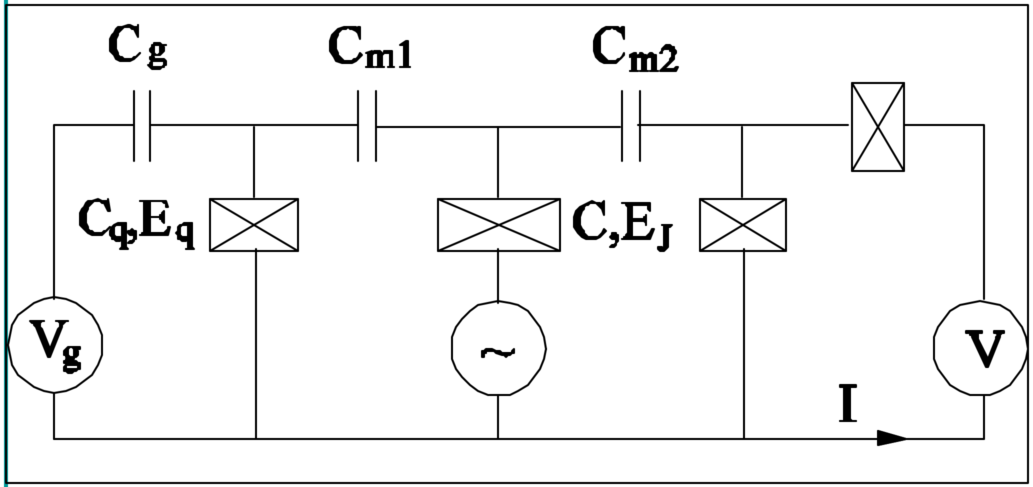
D.V.A., PRL **88**, 207901 (2002).

Suppression of backaction should manifest itself as more pronounced oscillation line in the output spectrum of detector S_0 when the detuning d=? -O is small in comparison to the backaction dephasing rate G:



For flux qubits, the QND coupling can be implemented with SFQ circuits, either directly or as a periodic sequence of the ``single-shot'' measurements.

Semi-QND qubit measurements



$$H = -\frac{\Delta}{2} s_x - \frac{1}{2}\cos(\Omega t) s_z f + H_D$$

Spectral density of the detector output:

$$|\Delta - \Omega| >> \Gamma$$

$$S_o(w) = S_q + \frac{l^2}{32p} \frac{\Gamma/4}{(w - |\Delta - \Omega|)^2 + (\Gamma/4)^2},$$

$$|\Delta - \Omega| << \Gamma$$

$$S_o(w) = S_q + \frac{l^2}{16p}\left[\frac{\Gamma/8}{w^2 + (\Gamma/8)^2} + \frac{1}{2}\frac{3\Gamma/8}{w^2 + (3\Gamma/8)^2}\right].$$