ON QUANTUM COMPUTER AS AN IDEAL QUANTUM RANDOM GENERATOR AND USELESSNESS OF THE REVERSIBILITY IN QUANTUM COMPUTIONS

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ABSTRACT. We prove nonexistence of the reversible classical-quantum interaction and conclude the fundamental impossibility of the reversible interface between classical and quantum computers. We show how the reversible unitary transformations on an infinite-dimensional Hilbert space can implement irreversible endomorphic computations without taking a projection or any partial trace, and how they can induce randomness on the classical part of a hybrid semi-quantum systems from a pure state. This can be used to explain the dynamical emergence of the decoherence in quantum unitary computations, and to reconcile the unitarity of these computations aa well as the Schr̈dinger dynamics with any reduction postulate of quantum measurements.

'I think I can safely say that nobody understands quantum mechanics' – RICHARD FEYNMAN.

0.0.1. *Contents of the extended paper:*

1. Nonexistence of Quantum-Classical Reversible Interactions
2. Quantum Stochastic Dynamics and the Eventum Mechanics
3. Reconciliation of Unitarity, Decoherence and Measurement
4. Quantum Hidden Generators of the Classical Visible Chaos

1. INTRODUCTION

The advantages of the reversible quantum computations, if they ever be realized, should be checked in conjunction with the quantum-classical interface which will allow our interaction with quantum computers through the semi-quantum devices realizing the quantum measurements. Usually such devices are modeled by irreversible phenomenological operations such as orthoprojections which are in clear contradiction with the unitarity of the quantum reversible computations. On the other hand, the reversible classical computations, which can also be described in terms of the special unitary operations, may in principle remain reversible even if they are interfaced with the output devices such as printers, displays, etc. Ideally they all can be thought as the interacting parts of the larger classical reversible system which can still be represented by the special unitary transformations on an extended Hilbert space. Since every pure state of the classical system is an eigen state of every possible observable of this system on the Hilbert space, no reduction is occurred when one of the possible results of the computation is read.

Key words and phrases. Quantum Computations, Classical-Quantum Interface, Quantum Information Processing, Decoherence and Measurement.

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Recent advances in quantum measurement theory (see the cited in the end review paper) made possible to reconcile the irreversible quantum operations with the unitary transformations as in classical case. It has been proved that every phenomenological quantum measurement operation (instantaneous, spontantaneous, or even time-continuous) can be realized on the classical part of an extended semi-quantum system unitary evolving from a pure initial state. No quantum-mechanical reduction is needed as the observables of the classical part (Bell's beables) are comparable with any potential observable of the extended hybrid system. The irreversibility and decoherence emerge from the unitary dynamics not as the result of the reduction (orthoprojection) or ignorance (partial tracing) but due to irreversibility of classical-quantum interaction which is still represented by a selfadjoint Hamiltonian in thus extended, necessarily infinitely-dimensional Hilbert space.

If both, classical and quantum reversible computations,\and even irreversible measurement operations can be represented by the unitary transformations, it should also be possible thus represent hybrid classical-quantum computers, and even the quantum-classical interface should be described in this unified way as a part of the hybrid semi-quantum computational system. This would allow a fair comparison of quantum computers as the parts of semi-quantum, including the interface, systems with purely classical computational systems.

Our analysis shows that the reversible interaction of the classical and quantum computers is fundamentally impossible, and the irreversible interaction can be represented by a unitary operator only if they are parts of a larger, infinite-rank hybrid system. By interaction we mean not one side, classical-quantum action which can be realized as the classical control of the parameters of the Hamiltonian quantum-mechanical system. If quantum computers will ever be useful, they should interact with the classical also in the opposite way, allowing the father use of the quantum computation in the ordinary computers and eventually comprehended by us, the classical computers.

The usual way to avoid this important question is to leave the measurement problem to the later, final stage, on which some sort of maximal likelihood argument is used to ensure the plausibility of the measured computational result. On the intermediate, computational stage the only such decoherence, if any, is admitted that can be error corrected by finding the decoherence-free subspace. Such approach does not address this fundamental problem due to the following reasons. First, the interaction with heat bath even under zero temperature will produce such system-environment entangled states that effective computational operations will be strongly mixing, and will not have decoherence-free spaces. And second, even more important, it excludes the quantum multi-stage computations when the problem is divided into a sequence of computational tasks and outputs are allowed on the intermediate stages. This might be in particular useful for constructing ε-error correction of the irreversible computation by optimal quantum feedback control based on the quantum filtering method which does not require decoherence free subspaces. The fair comparison of the single stage quantum computations with the measurement at the final stage when already no feedback can be arranged with the multistage feedback computations can be done only in the framework of the irreversible computations in the classical-quantum hybrid systems. One should expect as in the classical case that irreversible multistage, ideally time-continuous feedback computations will be advantageous to the single-stage, seemingly reversible quantum computations, but in fact irreversible as they should be combined with the final, fundamentally irreversible measurement stage.

The semi-quantum endomorphic dynamics of the classical-quantum hybrid systems leads to a new, event-enhanced quantum mechanics, called the eventum mechanics, with the unitary propagators given by the solutions of a Schrödinger self-adjoint boundary value problem in an extended Hilbert space.

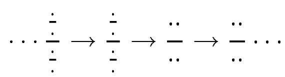
2. On unitary representation of classical computing

The primary purpose of this paper is to show that the first of the following two questions has always negative answer, while the second one has positive answer only in the infinite dimensional Hilbert space:

Problem 1. *Is it possible to have a reversible interaction between classical and quantum systems?*

Problem 2. *Is it possible to induce the classical chaos by a unitary interaction from a pure state?*

First we should remind that classical reversible evolutions (computations) $g : X \to X$ over a finite set $X = \{1, \dots, d\}$ of pure point states $x \in X$ can be described in terms of the unitary transformations $s : \langle\varphi| \mapsto \langle\varphi|s$ of the probability amplitudes $\langle\varphi| = (c_1, \dots, c_d)$ (unit row-vectors) in a Hilbert space $\mathcal{F} = \mathbb{C}^d$ as $\langle\varphi|s|x\rangle = c_{g(x)}$. The map $\kappa : \mathcal{A} \ni a \to \kappa(a) = sas^\dagger$ on the algebra $\mathcal{A} = \mathbb{A}(X)$ of diagonal matrices is automorphism. Converse: if an Abelian algebra $\mathcal{A}$ on a Hilbert space $\mathcal{F}$ is invariant under a unitary implemented κ, then $\langle\varphi|\kappa(a)|x\rangle = a(g(x))\, c_{g(x)}$ for a bijection g. However κ can be irreversible $\dim \mathcal{A} < \infty$ as it is illustrated on the following diagram:



Here the unitary operator on the Hilbert space of the two-side infinite chain of classical bit pairs is simply given by the right shift onto itself, however it induces irreversible dynamics if the states of the right bits are pairwise identified, being seen on the right as single bits. Note that the dynamical map of on the state space of such infinite chain is still surjective and not mixing, which corresponds to the injectivity of the irreversible Heisenberg endomorphism on the commutative algebra of (bounded) functions on such state space, and nonmixing property of the Abelian endomorphisms.

This is not true in the quantum case. One can easily show an irreversible injective endomorphism of the algebra of all bounded operators on the (infinite dimensional) Hilbert space that due to entanglement can mix pure states.

3. Nonexistence of q-c Reversible Interactions

In order to formulate this negative result we shall use the following

3.1. Vocabulary and notations of quantum stochastics:

- Macro system = a diagonal algebra $\mathcal{A}$ of events and beables.
- Extended system = a product operator algebra $\mathsf{B} = \mathcal{A} \otimes \mathcal{B}$.
- Pure state = an extreme expectation $\epsilon : \mathsf{B} \to \mathbb{C}$, $\epsilon = \varrho_0 \otimes \varsigma$.

- Invertible dynamics = an endomorphism $\kappa : \mathsf{b} \mapsto \mathsf{ubu}^\dagger \in \mathsf{B}$.
- Induced chaos = a mixed state $\varrho(a) = \epsilon\left(\mathsf{u}(a \otimes I)\mathsf{u}^\dagger\right)$, $a \in \mathcal{A}$

Theorem 1. *Let* u *be a unitary operator in* $\mathcal{H} = \mathcal{F} \otimes \mathcal{G}$ *such that* $\kappa(\mathsf{b}) = \mathsf{ubu}^\dagger \in \mathcal{A} \otimes \mathcal{B}$ *for any operator* $\mathsf{b} = a \otimes b$ *with the diagonal* $a \in \mathcal{A}$ *and arbitrary* $b \in \mathcal{B}$ *from the algebra* $\mathcal{B}$ *of bounded operators on a Hilbert space* $\mathcal{G}$*. If* $\dim \mathcal{F} < \infty$*, then* κ *is revertible,* $\kappa(\mathsf{B}) = \mathsf{B}$*. If* κ *is reversible, then* $\mathcal{A}$ *is autonomous of* $\mathcal{B}$ *in the sense that there exists a unitary operator* s *on* $\mathcal{F}$ *such that* $\mathsf{u}(a \otimes I)\mathsf{u}^\dagger = sas^\dagger \otimes I$*.*

Proof. The decomposable algebra $\mathsf{B} = \mathcal{A} \otimes \mathcal{B}$ is uniquely defined as the set A' of all operators on $\mathcal{H}$ commuting with $\mathsf{A} = \mathcal{A} \otimes I$. If the injective dynamical map $\kappa(\mathsf{b}) = \mathsf{ubu}^\dagger$ is authomorphic on B, then $\mathsf{A}' = \kappa(\mathsf{B}) \subseteq \kappa(\mathsf{A})'$, i.e. $\kappa(\mathsf{A}) \subseteq \kappa(\mathsf{B})' = \mathsf{A}$. Due to the same argument $\kappa^{-1}(\mathsf{A}) = \mathsf{u}^\dagger \mathsf{A} \mathsf{u} \subseteq \mathsf{A}$ and thus $\kappa(\mathsf{A}) = \mathsf{A}$. Hence κ induces an authomorphism on the diagonal algebra $\mathcal{A} = \mathbb{A}(X)$, which is implemented by a unitary operator s on $\mathcal{F} = \mathbb{C}^{|X|}$. ∎

Corollary 1. *The only option left to make use of quantum computations is to allow the irreversibility* $\kappa(\mathsf{B}) \subset \mathsf{B}$ *of the endomorphisms* $\kappa(\mathsf{b}) = \mathsf{ubu}^\dagger$ *which is possible iff* $|X| = \dim \mathcal{F} = \infty$ *as is illustrated below on the diagram*

$$\cdots \to 0 \to 0 \to \div \to \div \to \cdots$$

Here 0 denote the qubit state spaces (Bloch spheres) forming the left-infinite chain which interacts at the boundary with the right-infinite chain of the classical bits having the two-point state spaces, and the irreversible dynamics is implemented by the unitary right shift in the two-side infinite tensor product of the two dimensional Hilbert spaces. This model implements the solution of the following decoherence problem in the single bit case $X = \{0, 1\}$.

4. Reconstruction of the Decoherence

Problem 3. *Given a probability law* P *on the algebra* $A = A(X)$ *of measurable functions* a *on the data space* X *with* $\varrho(a) = \sum \alpha(x) P(x) \equiv (a, P)$ *find an auxiliary system* $\mathcal{B}$ *with a pure state* ς *and endomorphism* κ *of the algebra* $A \otimes \mathcal{B}$ *such that* $\epsilon(\kappa(a \otimes I)) = (a, P)$*, where* $\epsilon = \varrho_0 \otimes \varsigma$*, and* ϱ_0 *is pure, given by a* δ*-distribution* $P_0 = \delta$ *at an initial point-state* $x_0 \in X$*.*

One can embed A into the matrix algebra $\mathbb{B} = \mathbb{B}(X)$ as the diagonal subalgebra $\mathbb{A} = \{\widehat{a} : a \in A\} \equiv \mathbb{A}(X)$ and easily find $\varkappa(b) = sbs^\dagger$, $s^\dagger = s^{-1}$ such that $\langle 0| s = P^{1/2}$, $\varepsilon(b) = \langle 0|b|0\rangle$. However $\mathbb{B} \neq \mathbb{A} \otimes \mathcal{B}$. There is no solution with an Abelian or finite-dimensional $\mathcal{B}$ unless ϱ is a pure state, $P = \delta_x$, $\mathcal{B} = \mathbb{C}$. Thus we are bound to explore the only left possibility: to construct a non-Abelian infinite-dimensional auxiliary algebra $\mathcal{B}$.

4.0.1. *The decoherence as a result of the irreversibility of the classical-quantum computation.*

: If $X = \{0, 1\}$, $\varrho(a) = \langle 0| s\widehat{a}s^\dagger |0\rangle$, but $s\mathbb{A}s^\dagger \nsubseteq \mathbb{A}$ if $p_0 p_1 \neq 0$ as one can see from the explicit construction

$$\widehat{a} = \begin{matrix} a_0 & 0 \\ 0 & a_1 \end{matrix}, \quad s = \begin{matrix} p_0^{1/2} & p_1^{1/2} \\ p_1^{1/2} & -p_0^{1/2} \end{matrix}, \quad |0\rangle = \begin{matrix} 1 \\ 0 \end{matrix}.$$

: Let us take $\mathcal{B} = \mathcal{M} \otimes \mathcal{N}$, $\varsigma(B) = \langle 0,0|B|0,0\rangle$, where $\mathcal{M} = \mathbb{A}(\mathbb{Z}_+)$ is the algebra of diagonal bounded matrices indexed by $\mathbb{Z}_+ = \{0,1,2,\dots\}$, $\mathcal{N} = \mathbb{B}(\mathbb{Z}_+)$ is the algebra of all bounded matrices, and $|m,n\rangle = |m\rangle \otimes |n\rangle$.

: On the H-span $\ell^2 \otimes \mathbb{C}^2 \otimes \ell^2$ of $|m,x,n\rangle = |m\rangle \otimes |x\rangle \otimes |n\rangle$, $x \in \{0,1\}$ we define the operator $\mathsf{u} = p_0^{1/2}\mathsf{u}_0 + p_1^{1/2}\mathsf{u}_1$ by

$$\langle m,x,n|\mathsf{u}_0 = (-1)^n \left\langle 2m+x, x(n), \frac{1}{2}(n - x(n))\right|,\ m \in \mathbb{Z}_+,$$

$$\langle m,x,n|\mathsf{u}_1 = \left\langle 2m+x, x(n)+(-1)^n, \frac{1}{2}(n - x(n))\right|, n \in \mathbb{Z}_+$$

where $x(n) = (1-(-1)^n)/2 \in \{0,1\}$ is the evenness of n.

Theorem 2. *Thus defined operator* u *is unitary and* $\mathsf{uBu}^\dagger \subset \mathsf{B}$*, where* $\mathsf{B} = \mathcal{M} \otimes \mathbb{A} \otimes \mathcal{N}$*. Moreover, if* ϵ *is the pure state* $\varphi_0 \otimes \varphi_0 \otimes \varphi_0$ *defined by* $\varphi_0(\cdot) = \langle 0|\cdot|0\rangle$ *on* $\mathcal{M}$*,* $\mathcal{N}$ *and* $\mathbb{A}$ *respectively by the vacuum vector* $|0\rangle$ *in* ℓ^2 *and in* $\mathbb{C}^2$*, then* $\epsilon\left(\mathsf{u}(I \otimes \widehat{a} \otimes I)\mathsf{u}^\dagger\right) = a_0 p_0 + a_1 p_1$*.*

Proof. Use the isomorphism of $\ell^2 \otimes \mathbb{C}^2 \otimes \ell^2$ and two-sided infinite vacuum-product $\dots\mathfrak{h}\otimes\mathfrak{h}\otimes\mathfrak{h}\dots$ of $\mathfrak{h} = \mathbb{C}^2$ by $|m,x,n\rangle \simeq |\dots,0,x_k^-,\dots,x_2^-,x_1^-,x,x_1^+,x_2^+\dots,x_l^+,0,\dots\rangle$ given by the binary bit representations $\left(x_k^-,\dots,x_2^-,x_1^-\right)$ of $m = 2^{k-1}x_k^- + \dots + 2x_2^- + x_1^-$ and $\left(x_1^+,x_2^+,\dots,x_l^+\right)$ of $n = x_1^+ + 2x_2^+ + \dots + 2^{l-1}x_l^+$. Then u is the composition ts of two unitarities: a scattering $\mathsf{s} = I^- \otimes s \otimes I^+$ and left shift $\langle\dots,x_1^-|\langle x|\langle x_1^+,x_2^+,\dots|\mathsf{t} = \langle\dots,x_1^-,x|\langle x_1^+|\langle x_2^+,\dots|$ with $\langle 0^-,0,0^+|\mathsf{t} = \langle 0^-,0,0^+|$ and $\epsilon\left(\mathsf{u}\widehat{a}\mathsf{u}^\dagger\right) = \langle 0|s\widehat{a}s^\dagger|0\rangle$. The stability of B such that $\mathsf{u}^\dagger\mathsf{Bu} \not\subseteq \mathsf{B}$ follows from $\mathsf{t}(\mathcal{M} \otimes \mathbb{B} \otimes \mathcal{N})\mathsf{t}^\dagger = \mathsf{B}$ and $\mathsf{sBs}^\dagger \subset \mathcal{M} \otimes \mathbb{B} \otimes \mathcal{N}$. ∎

Corollary 2. *The decoherence reconstruction theorem proves that any classical random number can be generated from a pure initial state by a quantum computation as the result of a Hamiltonian boundary interaction of an infinite number of the classical and quantum bits. However this convertible computation is necessarily irreversible: there is no time symmetric Hamiltonian interaction of the classical and quantum bits!*

Corollary 3. *The von Neumann projection wave function collapse and any other reduction postulate can also be derived from a purely Hamiltonian time asymmetric interaction of the quantum system with an infinite semi-classical apparatus in a pure initial state. There is no need in a nonlinear non Hamiltonian quantum mechanics to explain quantum measurement, however it should be causal, enhanced by events.*

Conclusion 1. *The event enhanced causal quantum mechanics, or Eventum Mechanics, is a time asymmetric quantum mechanics of hidden variables. However it doesn't explain the microworld by classical hidden variables. Instead, it simply recognizes that all quantum variables of the microworld are hidden and the only visible are classical variables of macro events and beables. This is the only possible extension of the Hamiltonian quantum mechanics in which there is no EPR paradox that makes it fully consistent with the Copenhagen interpretation. It was envisaged by Schrödinger when he tried to explain EPR by the decoherence of an atom (as invisible microworld) interacting with his cat (as a visible macroworld). The cat, initially hidden in the box (quantum), simply becomes visible (classical) due to a free evolution (shift) in an infinite semiclassical string of the cats!*

> *'To those who do not understand mathematics it is difficult to get across a real filling as to the beauty, the deepest beauty, of the nature'* – RICHARD FEYNMAN.

4.0.2. *All references are form the review paper.* Belavkin V P: Quantum Causality, Stochastics, Trajectories and Information, Rep. Prog. Phys. 65 (2002) (quant-ph/0208087)

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