Pseudospin Quantum Computation in Semiconductor Nanostuctures

Sankar Das Sarma
Kwon Park
Vito Scarola

Condensed Matter Theory Center
University of Maryland

Supported by: ARDA and LPS

Preprints: www.physics.umd.edu/cmtc
Bilayer quantum Hall droplets

- Schematic diagram: for example, take a system with $N_{\text{tot}}=9$.

- Bilayer quantum Hall droplets are already being made.

(Austing et al., Physica E, 2001)
<table>
<thead>
<tr>
<th></th>
<th>superconductors</th>
<th>bilayer quantum Hall systems at $v=1$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>origin of gap</strong></td>
<td>e–e pairing</td>
<td>e–h pairing</td>
</tr>
<tr>
<td><strong>fluctuation</strong></td>
<td>number of Cooper pairs</td>
<td>relative number difference between the two layers: $2S_z$</td>
</tr>
<tr>
<td><strong>coherence</strong></td>
<td>coherent superposition between systems of N and N+1 Cooper pairs</td>
<td>coherent superposition between systems of $S_z$ and $S_z +1$</td>
</tr>
<tr>
<td><strong>(Josephson effect)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>mesoscopic version</strong></td>
<td>superconducting grains (Cooper pair box)</td>
<td>vertically coupled quantum dots under magnetic field (quantum Hall droplets)</td>
</tr>
</tbody>
</table>
Energy spectrum and excitations

Method: Exact diagonalization in Fock–Darwin basis

energy spectrum as a function of relative number difference ($S_z$)

The ground state is obtained in the Hilbert space of the lowest value of $|S_z|$ to minimize the (relative) charging energy.
energy spectrum as a function of total angular momentum:

A stable ground state is formed in a range of magnetic field due to the cusp in the Coulomb interaction energy.

lowest excitation gaps as a function of interlayer distance:

The lowest excitation gap persists roughly up to $d/a=1.5$. 
Spontaneous interlayer coherence

There is an interlayer coherence even in the limit of zero tunneling:

\[ \lim_{t \to 0} \frac{<+1/2|H_t|-1/2>}{t} \equiv <+1/2|S_x|-1/2> \neq 0 \]

In fact, it can be shown that
\[ <+1/2|S_x|-1/2> = (N+1)/2 \]

exactly at \( d=0 \), which is a consequence of formation of pseudo-spin ferromagnet (Halperin’s (1,1,1) state).
The reduced Hamiltonian for two–level system

$$H_{\text{red}} = -\Delta_x \sigma_x + \Delta_z \sigma_z$$

generic Hamiltonian for two–level system
(pseudo–spin $S=1/2$ system)

$$\Delta_x = t <1/2|S_x|-1/2>$$
renormalized tunneling gap
(Remember that $\Delta_x$ is proportional to $N$.)

$\Delta_z$ external relative bias voltage
Even–odd effect in the Coulomb–blockade peaks

\[ H_{\text{charging}} = \frac{e^2}{2C_\Sigma} \left( N - \frac{CV_g}{e} \right)^2 \]

Without interlayer coherence: Conductance peaks are evenly spaced.

With interlayer coherence: Spacing of conductance peaks oscillates, depending on whether \( N \) is even or odd.
Decoherence due to voltage fluctuation

spin–boson model  (Leggett et al. 1987)

\[
H = H_{\text{spin}} + H_{\text{boson}} + H_{\text{coupling}}
\]

\[
H_{\text{spin}} = -\Delta x \sigma_x + \Delta z \sigma_z = \Delta E(\cos \eta \sigma_z + \sin \eta \sigma_x)
\]

\[
H_{\text{boson}} = \sum_a \left( \frac{p_a^2}{2m_a} + \frac{1}{2}m_a\omega_a^2 x_a^2 \right)
\]

\[
H_{\text{coupling}} = \sigma_z \sum_a \lambda_a x_a = X \sigma_z
\]

\[
\left\langle X_{\omega}^2 \right\rangle = \hbar J(\omega) \coth \frac{\omega}{2k_B T}
\]

\[
J(\omega) = \frac{\pi}{2} \sum_a \frac{\lambda_a^2}{m_a \omega_a} \delta(\omega - \omega_a)
\]

**In Ohmic dissipation**

\[
J(\omega) = \frac{\pi}{2} \alpha \hbar \omega
\]

\[
\frac{1}{\tau_{\text{relax}}} = \pi \alpha \sin^2 \eta \frac{\Delta E}{\hbar} \coth \frac{\Delta E}{2k_B T}
\]

\[
\frac{1}{\tau_\phi} = \frac{1}{2\tau_{\text{relax}}} + \pi \alpha \cos^2 \eta \frac{2k_B T}{\hbar}
\]

relaxation time

dephasing time
Decoherence due to voltage fluctuation

\[ H_{\text{coupling}} = X\sigma_z = \gamma e\delta V \sigma_z \]

\[ \gamma = \frac{C_g}{C_\Sigma} \sim 10^{-2} \]

\[ \langle \delta V \delta V \rangle_\omega = \text{Re}[Z(\omega)] \frac{\hbar \omega}{2k_B T} \text{ coth} \frac{\hbar \omega}{2k_B T} \]

Johnson–Nyquist power spectrum

\[ \frac{1/\tau}{\Delta E} = \gamma^2 \frac{4R_v}{R_K} \sim 10^{-5} \]

Ratio of dephasing rate to elementary logic operation rate

\[ R_v \sim 50\Omega \text{ for typical voltage circuits} \]

\[ R_K = \frac{\hbar}{e^2} \]
Can we entangle bilayer quantum Hall droplets?

- Use the Coulomb interaction to couple adjacent droplets in the MDD phase

- Find the interaction matrix within the two level subspace
- Simple form for the interaction
Single BQHD model

- Odd number of electrons
- Begin with no interlayer tunneling (t→ 0)
- Real spins polarized
- Large magnetic field (MDD phase)

\[ H = H_0 + \hat{P} V_{Coul} \hat{P} \]

\[ H_0 = \frac{1}{2} \left( \sqrt{\omega_c^2 + 4\omega_0^2} - \omega_c \right) \hat{L}_z \]

\[ \frac{V_{Coul}}{e^2/\epsilon a} = \sum_{i<j \in \uparrow} \frac{1}{r_{ij}} + \sum_{k<l \in \downarrow} \frac{1}{r_{kl}} + \sum_{i \in \uparrow, k \in \downarrow} \frac{1}{\sqrt{r_{ik}^2 + (\frac{d}{a})^2}} \]

- Diagonalize in lowest Landau level Fock-Darwin basis
Energy spectrum in a single BQHD

Angular Momentum

Two lowest levels are pseudospin up and down

\[ S = \frac{1}{2} \sum_{m} c_{a}^{\dagger}(m) \vec{\sigma}_{ab} c_{b}(m) \]

\( S_z \) measures half the number difference between layers

\( S_x \) measures interlayer coherence
Coulomb Coupled BQHDs

\[
\begin{array}{ccc}
\text{Bias Voltage } & S_z = +1/2 & \text{Bias Voltage} \\
\Delta_z^1 & \text{d} & \Delta_z^2 \\
\text{R} & & \\
\end{array}
\]

- Four degenerate basis states without inter-BQHD Coulomb interaction

\[|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\]

- Coulomb interaction does not flip pseudospin

- No off-diagonal terms in interaction matrix
Ising Interaction Between BQHDs

\[ H_I = \frac{J}{2} \sigma_z^1 \sigma_z^2 \]

\[ J = \langle \uparrow \uparrow | \sum_{i,j} V(R, d; r_i, r_j') | \uparrow \uparrow \rangle - \langle \uparrow \downarrow | \sum_{i,j} V(R, d; r_i, r_j') | \uparrow \downarrow \rangle \]

- Coulomb interaction favors antiferromagnetic coupling \( J > 0 \)
• Dipole-Dipole interaction at large $R$

• $J \sim R^{-3}$

• $J \sim 0.02 \text{ meV at } R=9a$ and $B=9T$
Include Vertical Tunneling

\[ H_t = -t \hat{S}_x \]

\( t = \) single particle tunneling gap

In the reduced Hilbert space, \(|\uparrow\rangle \& |\downarrow\rangle\), of the \(i\)th BQHD:

\[ H_{\text{red}}^i = -\Delta_x^i \sigma_x^i + \Delta_z^i \sigma_z^i \]

\[ \Delta_x = t(\uparrow |\hat{S}_x| \downarrow) \]

renormalized tunneling \hspace{1cm} \text{bias voltage}
Multiple BQHD Hamiltonian

\[ H_{\text{total}} = \sum_i \left[ -\Delta_x^{i} \sigma_x^{i} + \Delta_z^{i} \sigma_z^{i} \right] + \sum_{ij} \frac{J_{ij}}{2} \sigma_x^{i} \sigma_x^{j} \]

- Quantum Ising Model

\[ R=9a \quad J \sim 0.02 \text{ meV} \]
\[ \Delta_x \sim 0.03 \text{ meV} \]

- \( \Delta_x \) and \( J \) originate from Coulomb interaction
Bilayer Quantum Hall Droplets as Qubits

• Ising interaction allows for simple implementation of CNOT gate

• Tuning Parameters
  t tunable via gates or in plane magnetic field
  J tunable with interstitial screening
  example: intermediate BQHD or a metal

• Dephasing due to fluctuations in $\Delta_z$
Conclusions

- Bilayer quantum Hall droplets are studied in the context of quantum computing.

- Spontaneous interlayer phase coherence is quantified by directly computing the expectation value of pseudo-spin magnetization.

- We predict the even–odd effect in the tunneling conductance peaks which is the precise quantum Hall analog of the Josephson effect in the Cooper pair box problem.

- The even–odd effect can serve as a convenient tool to measure the formation of interlayer coherence and to determine whether the two–level system (pseudo–spin S=1/2 system) is obtained.
Conclusion

- Odd number of electrons in a Bilayer Quantum Hall droplet in MDD phase forms a two-level system

- In the low energy sector the Coulomb interaction between BQHDs maps onto a Quantum Ising model

\[ H_{total} = \sum_i \left[ -\Delta_x^i \sigma_x^i + \Delta_z^i \sigma_z^i \right] + \sum_{ij} \frac{J_{ij}}{2} \sigma_x^i \sigma_x^j \]