Pseudospin Quantum Computation in Semiconductor Nanostuctures

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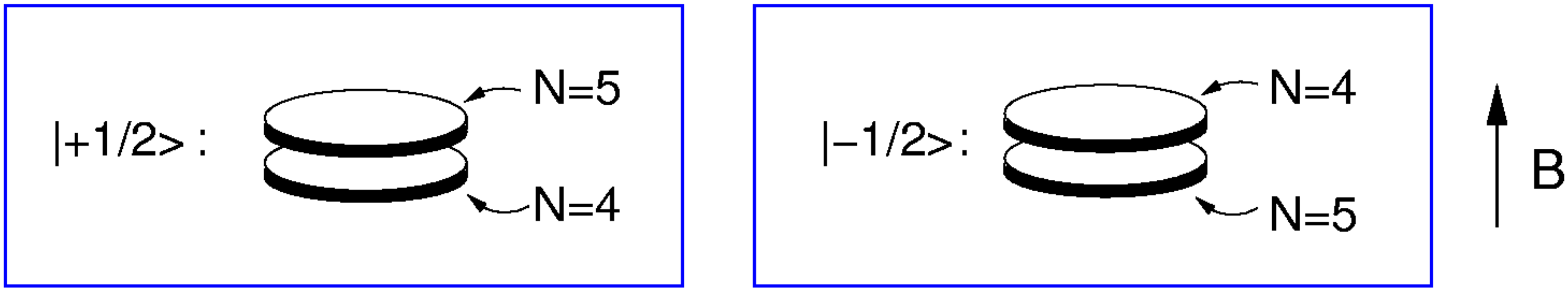
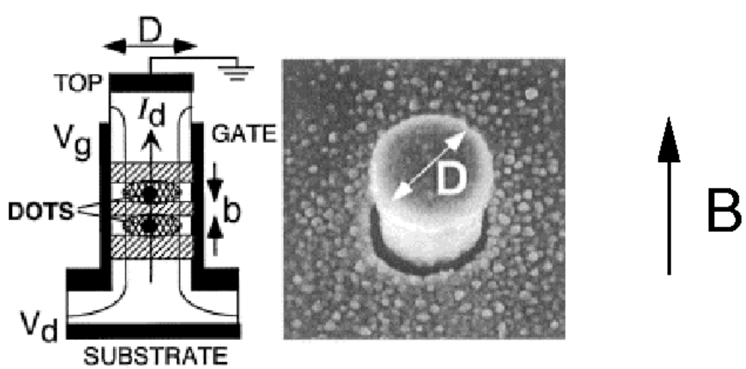
Bilayer quantum Hall droplets

- Schematic diagram: for example, take a system with $N_{tot} = 9$.



- Bilayer quantum Hall droplets are already being made.



(Austing et al.,Physica E,2001)

Analogy

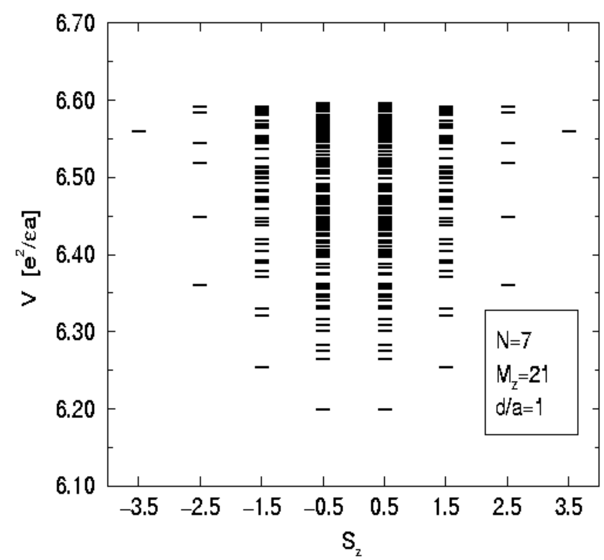
	superconductors	bilayer quantum Hall systems at $\nu=1$
origin of gap	e–e pairing	e–h pairing
fluctuation	number of Cooper pairs	relative number difference between the two layers: $2S_z$
coherence (Josephson effect)	coherent superposition between systems of N and N+1 Cooper pairs	coherent superposition between systems of S_z and S_z+1
mesoscopic version	superconducting grains (Cooper pair box)	vertically coupled quantum dots under magnetic field (quantum Hall droplets)

Energy spectrum and excitations

- Method: Exact diagonaliztion in Fock–Darwin basis

energy spectrum as a function of relative number difference (S_z)

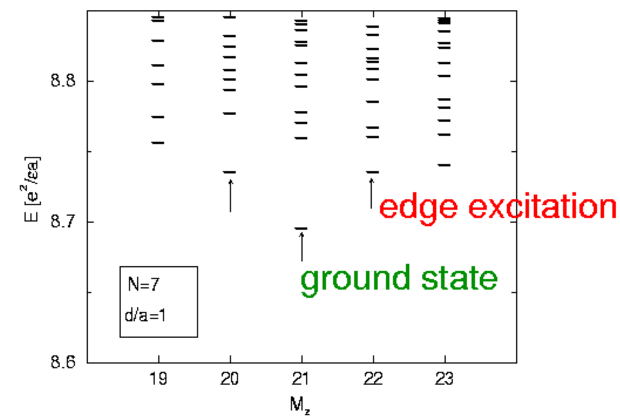
The ground state is obtained in the Hilbert space of the lowest value of $|S_z|$ to minimize the (relative) charging energy.



Energy spectrum and excitations

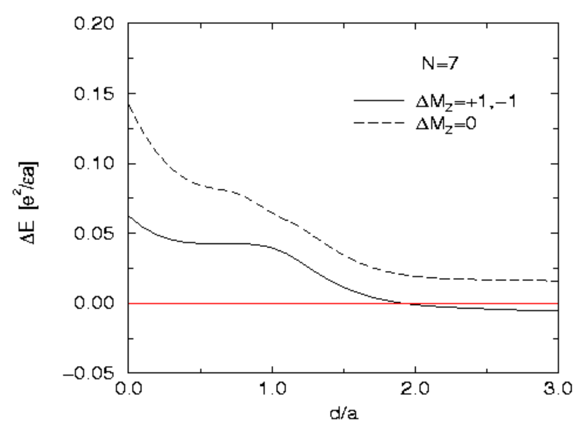
energy spectrum as a function of total angular momentum:

A stable ground state is formed in a range of magnetic field due to the cusp in the Coulomb interaction energy.



lowest excitation gaps as a function of interlayer distance:

The lowest excitation gap persists roughly up to d/a=1.5.



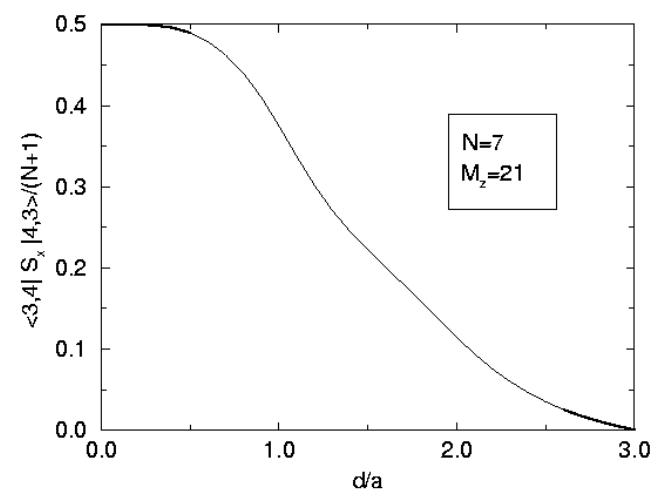
Spontaneous interlayer coherence

There is an interlayer coherence even in the limit of zero tunneling:

$$\lim_{t \to 0} \frac{\langle +1/2 | H_t | -1/2 \rangle}{t} \equiv \langle +1/2 | S_x | -1/2 \rangle \neq 0$$

pseudo–spin magnetization

In fact, it can be shown that
$\langle +1/2 | S_x | -1/2 \rangle = (N+1)/2$
exactly at d=0, which is a consequence
of formation of pseudo–spin ferromagnet
(Halperin's (1,1,1) state).



The reduced Hamiltonian for two–level system

$$H_{red} = -\Delta_x \sigma_x + \Delta_z \sigma_z$$

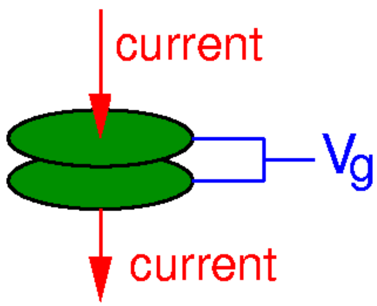
generic Hamiltonian for two–level system (pseudo–spin S=1/2 system)

$$\Delta_x = t \langle 1/2 | S_x | -1/2 \rangle$$

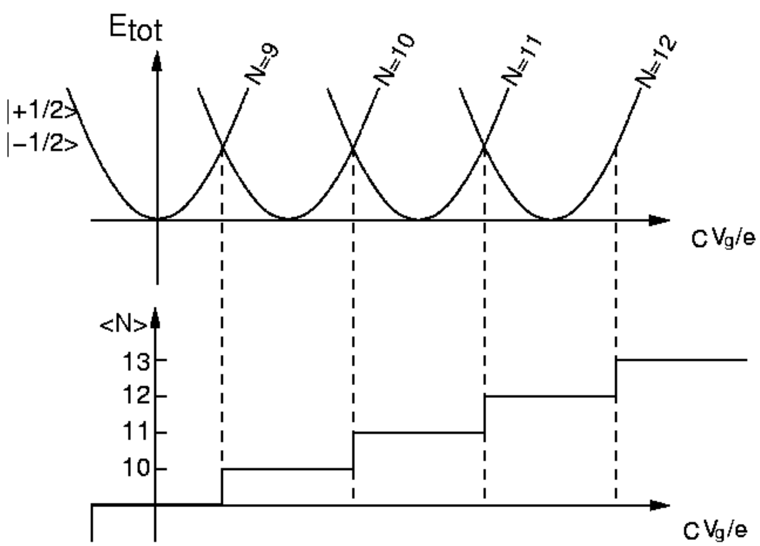
renormalized tunneling gap (Remember that Δ_x is proportional to N.)

Δ_z external relative bias voltage

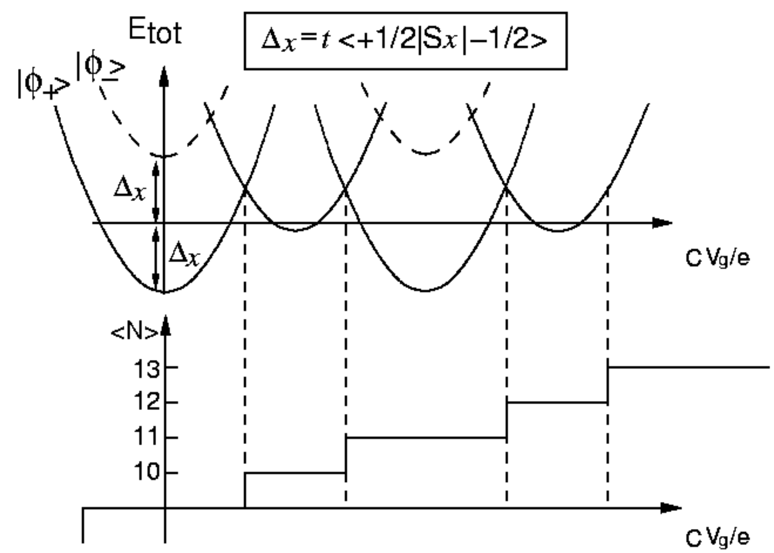
Even–odd effect in the Coulomb–blockade peaks



$$H_{charging} = \frac{e^2}{2C_\Sigma}\left(N - \frac{CV_g}{e}\right)^2$$



Without interlayer coherence:
Conductance peaks are evenly spaced.



With interlayer coherence:
Spacing of conductance peaks oscillates, depending on whether N is even or odd.

Decoherence due to voltage fluctuation

spin–boson model (Leggett et al. 1987)

$$H = H_{spin} + H_{boson} + H_{coupling}$$

$$\begin{aligned} H_{spin} &= -\Delta_x \sigma_x + \Delta_z \sigma_z \\ &= \Delta E(\cos\eta\sigma_z + \sin\eta\sigma_x) \end{aligned}$$

$$H_{boson} = \sum_a \left(\frac{p_a^2}{2m_a} + \frac{1}{2} m_a \omega_a^2 x_a^2 \right)$$ environment

$$H_{coupling} = \sigma_z \sum_a \lambda_a x_a = X\sigma_z$$

$$\langle X_\omega^2 \rangle = \hbar J(\omega) \coth\frac{\omega}{2k_B T}$$

$$J(\omega) = \frac{\pi}{2} \sum_a \frac{\lambda_a^2}{m_a \omega_a} \delta(\omega - \omega_a)$$ bath spectral density

In Ohmic dissipation

$$J(\omega) = \frac{\pi}{2} \alpha \hbar \omega$$

$$\frac{1}{\tau_{relax}} = \pi\alpha \sin^2\eta \frac{\Delta E}{\hbar} \coth\frac{\Delta E}{2k_B T}$$ relaxation time

$$\frac{1}{\tau_\phi} = \frac{1}{2\tau_{relax}} + \pi\alpha\cos^2\eta \frac{2k_B T}{\hbar}$$ dephasing time

Decoherence due to voltage fluctuation

$$H_{coupling} = X\sigma_z = \gamma e \delta V \sigma_z$$

$$\gamma = \frac{C_g}{C_\Sigma} \sim 10^{-2}$$

$$\langle \delta V \delta V \rangle_\omega = Re[Z(\omega)]\hbar\omega \coth \frac{\hbar\omega}{2k_B T}$$

Johnson–Nyquist power spectrum

$$\frac{1/\tau}{\Delta E} = \gamma^2 \frac{4R_v}{R_K} \sim 10^{-5}$$

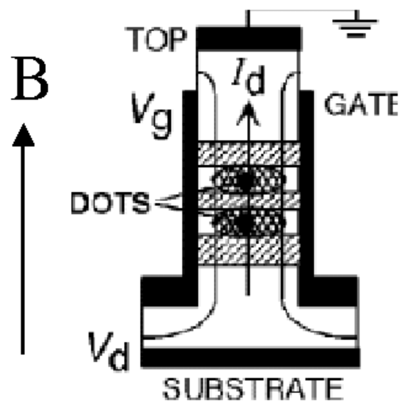
Ratio of dephasing rate to elementary logic operation rate

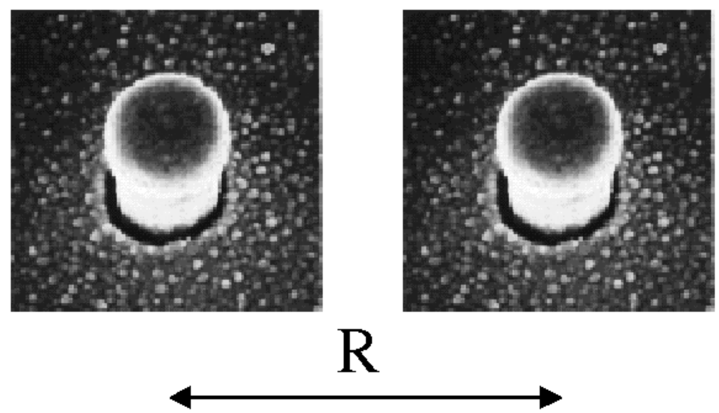
$$R_v \sim 50\Omega \quad for \ typical \ voltage \ circuits$$

$$R_K = h/e^2$$

Can we entangle bilayer quantum Hall droplets ?

- Use the Coulomb interaction to couple adjacent droplets in the MDD phase





- Find the interaction matrix within the two level subspace
- Simple form for the interaction

Single BQHD model

- Odd number of electrons
- Begin with no interlayer tunneling ($t \rightarrow 0$)
- Real spins polarized
- Large magnetic field (MDD phase)

$$H = H_0 + \hat{P} V_{Coul} \hat{P}$$

$$H_0 = \frac{1}{2}\left(\sqrt{\omega_c^2 + 4\omega_0^2} - \omega_c\right)\hat{L}_z$$

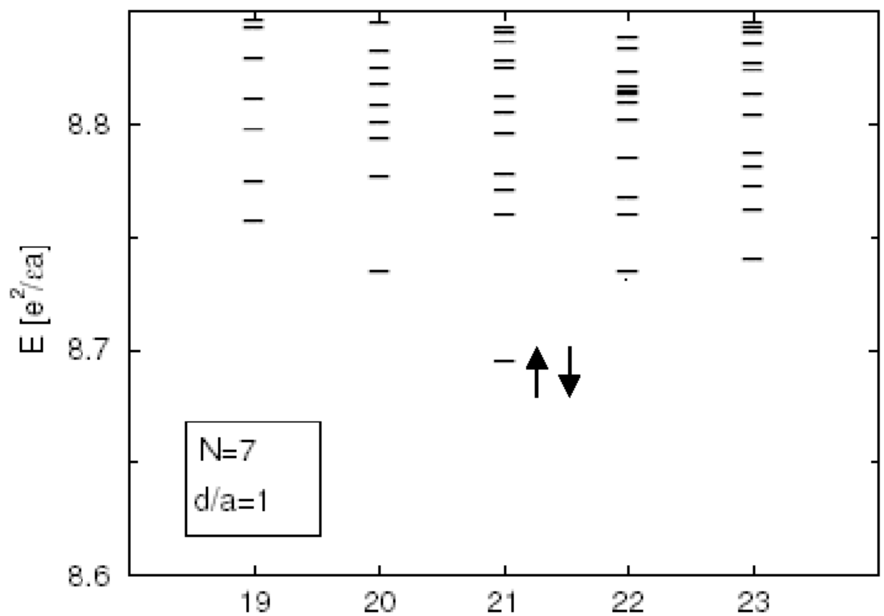
↑ angular momentum

$$\frac{V_{Coul}}{e^2/\epsilon a} = \sum_{i<j\in\uparrow} \frac{1}{r_{ij}} + \sum_{k<l\in\downarrow} \frac{1}{r_{kl}} + \sum_{i\in\uparrow, k\in\downarrow} \frac{1}{\sqrt{r_{ik}^2 + (\frac{d}{a})^2}}$$

↑ layer separation

- Diagonalize in lowest Landau level Fock-Darwin basis

Energy spectrum in a single BQHD



Angular Momentum

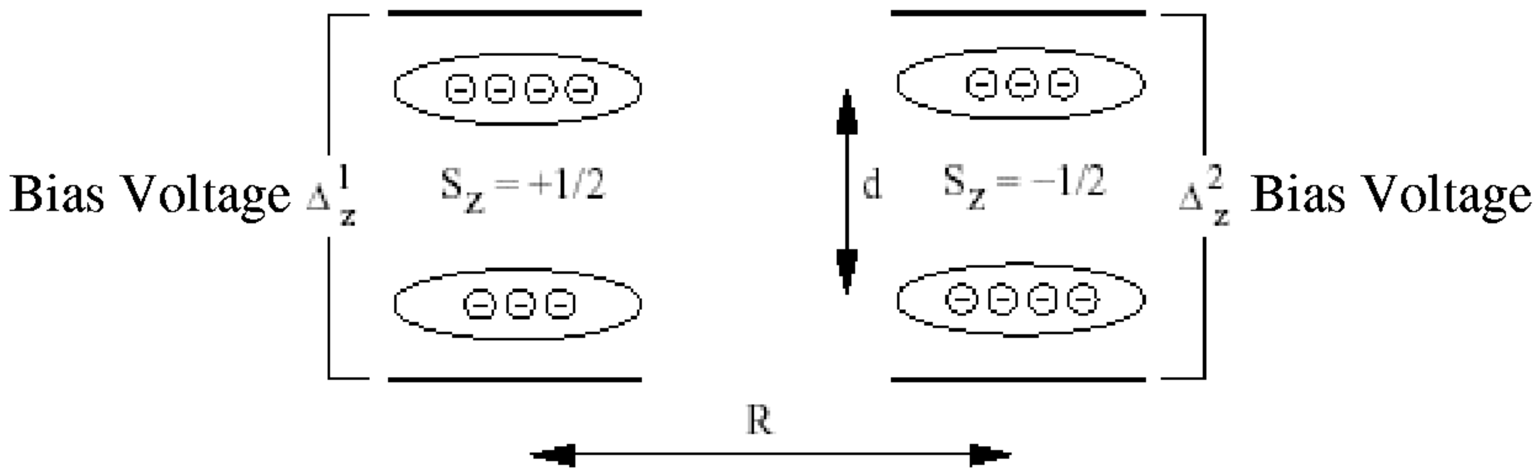
Two lowest levels are pseudospin up and down

$$\mathbf{S} \equiv \frac{1}{2}\sum_{m} c_a^{\dagger}(m)\vec{\sigma}_{ab}c_b(m)$$

S_Z measures half the number difference between layers

S_X measures interlayer coherence

Coulomb Coupled BQHDs



- Four degenerate basis states without inter-BQHD Coulomb interaction

$$|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$$

- Coulomb interaction does not flip pseudospin

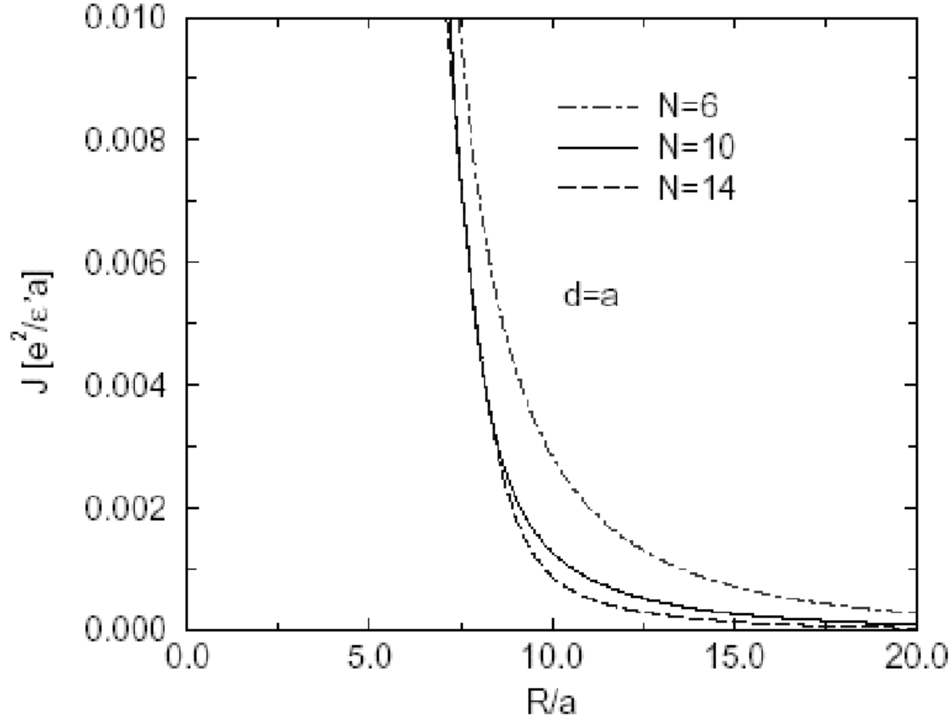
- No off-diagonal terms in interaction matrix

Ising Interaction Between BQHDs

$$H_I = \frac{J}{2}\sigma_z^1\sigma_z^2$$

$$J = \langle\uparrow\uparrow|\sum_{i,j} V(R,d;\mathbf{r}_i,\mathbf{r}'_j)|\uparrow\uparrow\rangle - \langle\uparrow\downarrow|\sum_{i,j} V(R,d;\mathbf{r}_i,\mathbf{r}'_j)|\uparrow\downarrow\rangle$$

- Coulomb interaction favors antiferromagnetic coupling $J>0$



- Dipole-Dipole interaction at large R
- $J \sim R^{-3}$
- J ~ 0.02 meV at R=9a and B=9T

Include Vertical Tunneling

$$H_t = -t\hat{S}_x$$

t=single particle tunneling gap

In the reduced Hilbert space, $|\uparrow> \& |\downarrow>$, of the ith BQHD:

$$H^i_{red} = -\Delta^i_x \sigma^i_x + \Delta^i_z \sigma^i_z$$

↑ renormalized tunneling

↑ bias voltage

$$\Delta_x = t\langle \uparrow |\hat{S}_x| \downarrow \rangle$$

Multiple BQHD Hamiltonian

$$H_{total} = \sum_i [-\Delta_x^i \sigma_x^i + \Delta_z^i \sigma_z^i] + \sum_{ij} \frac{J^{ij}}{2} \sigma_z^i \sigma_z^j$$

- Quantum Ising Model

R=9a J ~ 0.02 meV

Δ_x ~ 0.03 meV

- Δ_x and J originate from Coulomb interaction

Bilayer Quantum Hall Droplets as Qubits

- Ising interaction allows for simple implementation of CNOT gate

- Tuning Parameters

  t tunable via gates or in plane magnetic field

  J tunable with interstitial screening

  example: intermediate BQHD or a metal

- Dephasing due to fluctuations in Δ_z

Conclusions

- Bilayer quantum Hall droplets are studied in the context of quantum computing.

- Spontaneous interlayer phase coherence is quantified by directly computing the expectation value of pseudo–spin magnetization.

- We predict the even–odd effect in the tunneling conductance peaks which is the precise quantum Hall analog of the Josephson effect in the Cooper pair box problem.

- The even–odd effect can serve as a convenient tool to measure the formation of interlayer coherence and to determine whether the two–level system (pseudo–spin S=1/2 system) is obtained.

Conclusion

- Odd number of electrons in a Bilayer Quantum Hall droplet in MDD phase forms a two-level system

- In the low energy sector the Coulomb interaction between BQHDs maps onto a Quantum Ising model

$$H_{total} = \sum_i [-\Delta_x^i \sigma_x^i + \Delta_z^i \sigma_z^i] + \sum_{ij} \frac{J^{ij}}{2} \sigma_z^i \sigma_z^j$$