Quantum Information Processing with Ultra-Cold Atomic Qubits

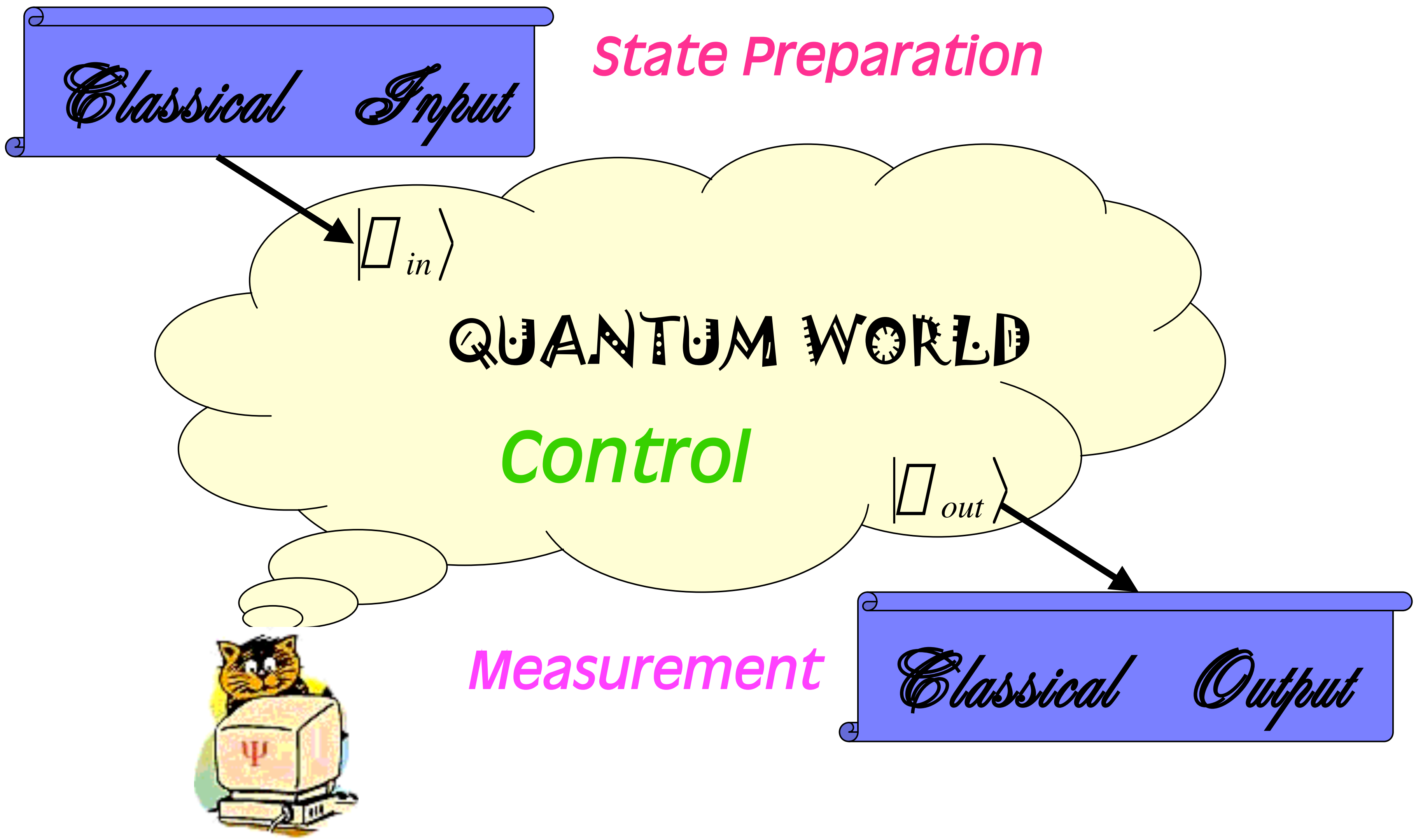
Ivan H. Deutsch

University of New Mexico

Information Physics Group

http://info.phys.unm.edu

Quantum Information Processing



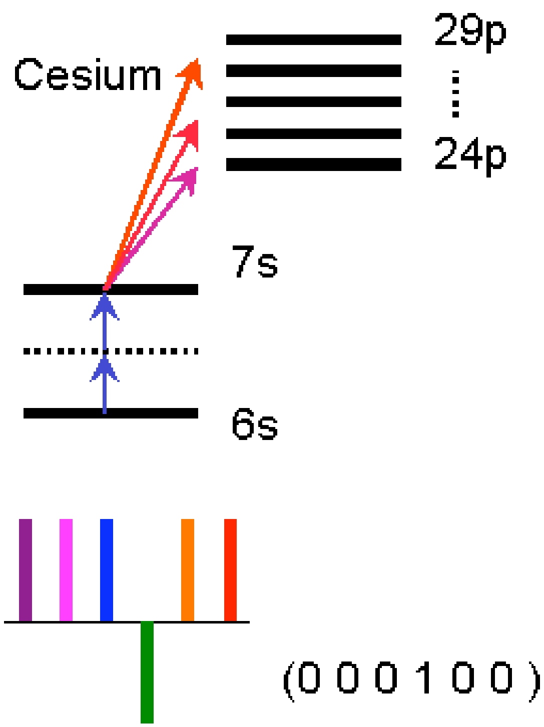
Example: Rydberg atom

http://gomez.physics.lsa.umich.edu/~phil/qcomp.html

Grover's database search algorithm

Data register: Rydberg wave packet

Read-in: phase information

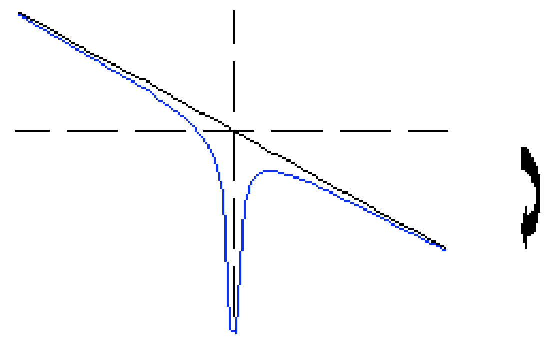


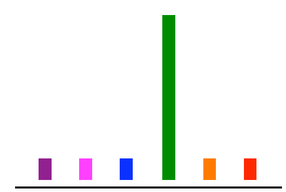
Optical Short Pulse (A)

Decoding pulse

Optical pulse or terahertz Half-cycle Pulse (B)

Read-out: amplitude information





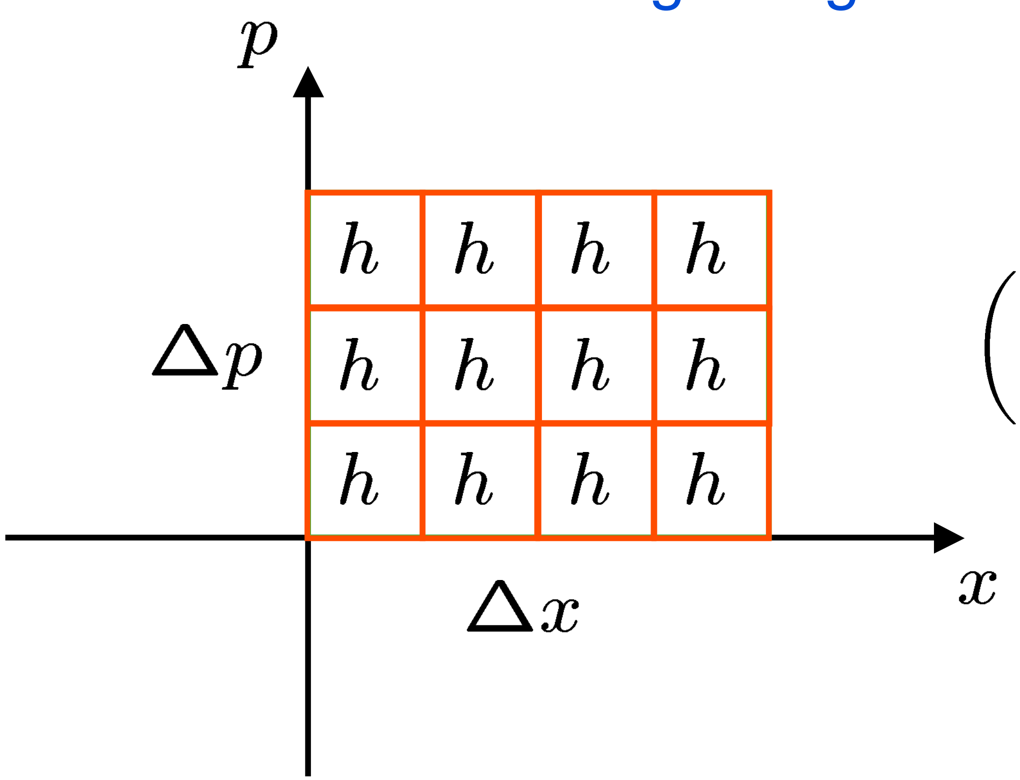
State-selective field ionization signal

Hilbert space and physical resources

The primary resource for quantum computation is Hilbert-space dimension.

Hilbert spaces of the same dimension are fungible, but the available Hilbert-space dimension is a physical quantity that costs physical resources.

Single degree of freedom



Action

$$\begin{pmatrix}\text{Available} \\ \text{Hilbert-space} \\ \text{dimension}\end{pmatrix} \simeq \frac{\Delta x \Delta p}{h} = \frac{A}{h}$$

Hilbert space and physical resources

Many degrees of freedom

$$2^N = \begin{pmatrix}\text{Available}\\ \text{Hilbert-space}\\ \text{dimension}\end{pmatrix} \simeq \frac{A_1 \cdots A_T}{h^T} = \left(\frac{A}{h}\right)^T$$

Number of degrees of freedom (T)

Identical degrees of freedom

Hilbert-space dimension measured in qubit units. (2^N)

$$\frac{A}{h} \sim 2^{N/T}$$

Scalable resource requirement

$$\frac{A}{h} \sim \text{poly}(N) \qquad T \sim \frac{N}{\log(\text{poly}(N))}$$

Strictly scalable resource requirement

$$T \sim \frac{N}{\log D} \qquad \frac{A}{h} \sim D$$

qudits

Quantum computing in a single atom

Characteristic scales are set by "atomic units"

Length $r_c = \frac{\hbar^2}{me^2} = a_0$

Momentum $p_c = \frac{me^2}{\hbar} = \frac{\hbar}{a_0}$

Action $L_c = r_c p_c = \hbar$

Energy $E_c = \frac{e^2}{a_0} = \frac{p_c^2}{m}$

Bohr

$$L_n = n\hbar \qquad r_n = n^2 a_0 \qquad p_n = \frac{1}{n}\frac{\hbar}{a_0} \qquad E_n = -\frac{1}{2n^2}\frac{e^2}{a_0}$$

Hilbert-space dimension up to *n*

$$2^N = \sum_{k=1}^{n}\sum_{l=0}^{k-1}(2l+1) \sim \frac{1}{3}n^3 \sim \left(\frac{L_n}{\hbar}\right)^3 = \left(\frac{r_n p_n}{\hbar}\right)^{③}$$

③ ← 3 degrees of freedom

Quantum computing in a single atom

Characteristic scales are set by "atomic units"

Length	Momentum	Action	Energy
$r_c = \frac{\hbar^2}{me^2} = a_0$	$p_c = \frac{me^2}{\hbar} = \frac{\hbar}{a_0}$	$L_c = r_c p_c = \hbar$	$E_c = \frac{e^2}{a_0} = \frac{p_c^2}{m}$

Bohr

$$L_n = n\hbar \qquad r_n = n^2 a_0 \qquad p_n = \frac{1}{n}\frac{\hbar}{a_0} \qquad E_n = -\frac{1}{2n^2}\frac{e^2}{a_0}$$

Poor scaling in this *physically unary* quantum computer

$$r_n \sim 2^{2N/3} a_0$$

$$N = 100 \text{ qubits} \Longrightarrow r_n \sim 10^{20} a_0 = 6 \times 10^6 \text{ km}$$

5 times the diameter of the Sun

First Conclusions

- All Hilbert Spaces of the same dimension are mathematically isomorphic and *fungible*.
- The dimension of Hilbert is a *resource*.
- *Physics* determines the structure of Hilbert space.
- Systems with *multiple physical degrees of freedom* give Hilbert space a *tensor-product structure*.
- Arbitrary superpositions lead to *entangled states*.
- Control of a many-body system is a *necessary condition* to have an exponentially large Hilbert space with out using an exponential physical resource.

R. Blume-Kohout, C. M. Caves, and I. H. Deutsch, *Found. Phys*. **32**, 1641(2002).

QIP = Many-body Control

n-body Hilbert Space

$$\mathcal{H} = h_1 \quad h_2 \quad \cdots \quad h_n \quad \text{(dimension } d^n\text{)}$$

"subsystem" = "body" (dimension d)

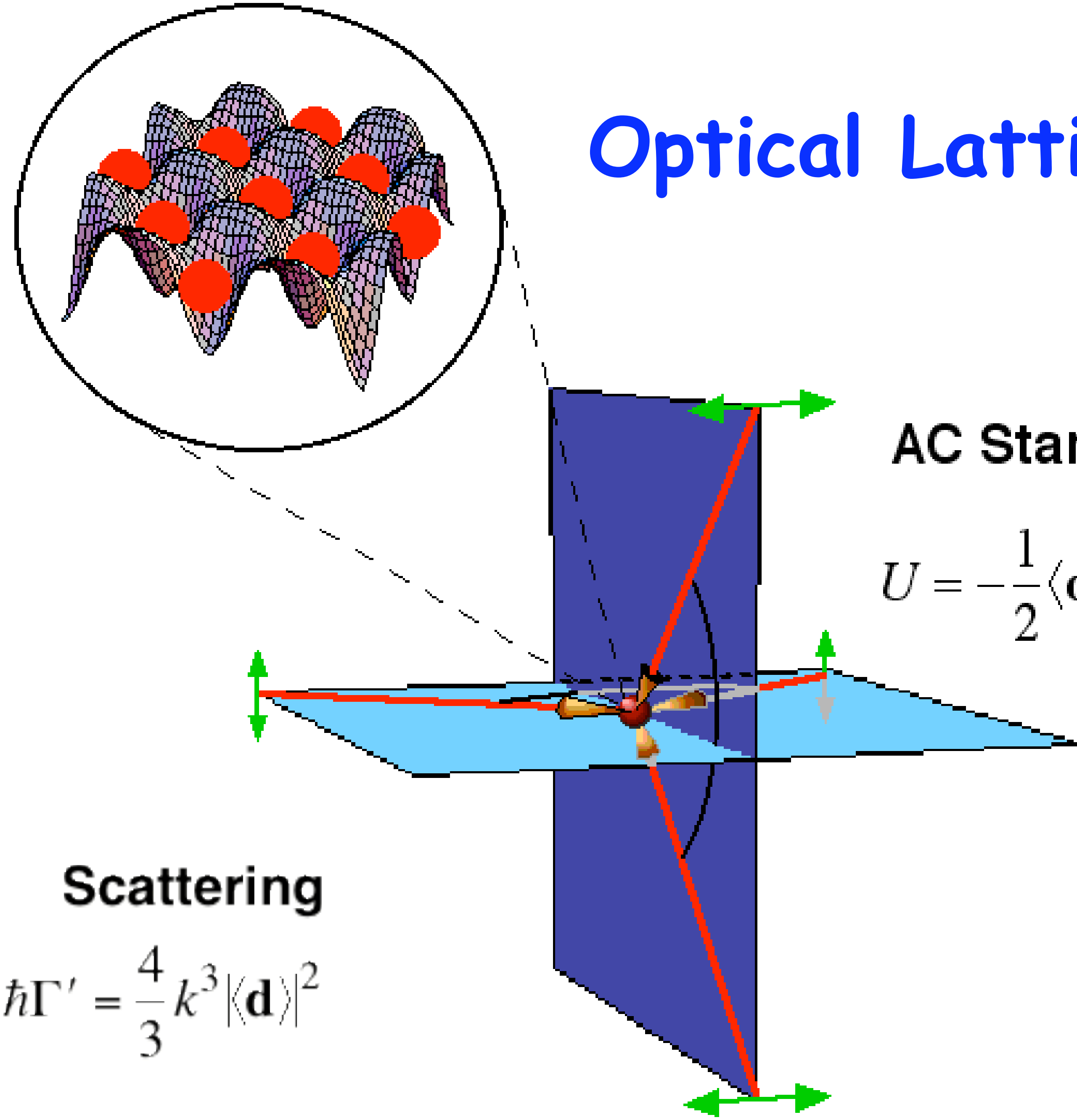
Fundamental Theorem of QIP

An ***arbitrary*** unitary map on $\mathcal{H}$ can be constructed from a tensor product of:

- A finite set of single-body unitaries . $\{u_i^{(1)}\}$
- Any chosen *entangling* two-body unitary.

$$u_{ij}^{(2)} \neq u_i^{(1)} \quad u_j^{(1)}$$

Optical Lattices



AC Stark Shift

$$U = -\frac{1}{2}\langle \mathbf{d} \rangle \cdot \mathbf{E}$$

Scattering

$$\hbar\Gamma' = \frac{4}{3}k^3 |\langle \mathbf{d} \rangle|^2$$

Why Optical Lattices?

State Preparation

- Initialization
- Entropy Dump

Laser cooling

State Manipulation

- Potentials/Traps
- Control Fields
- Particle Interactions

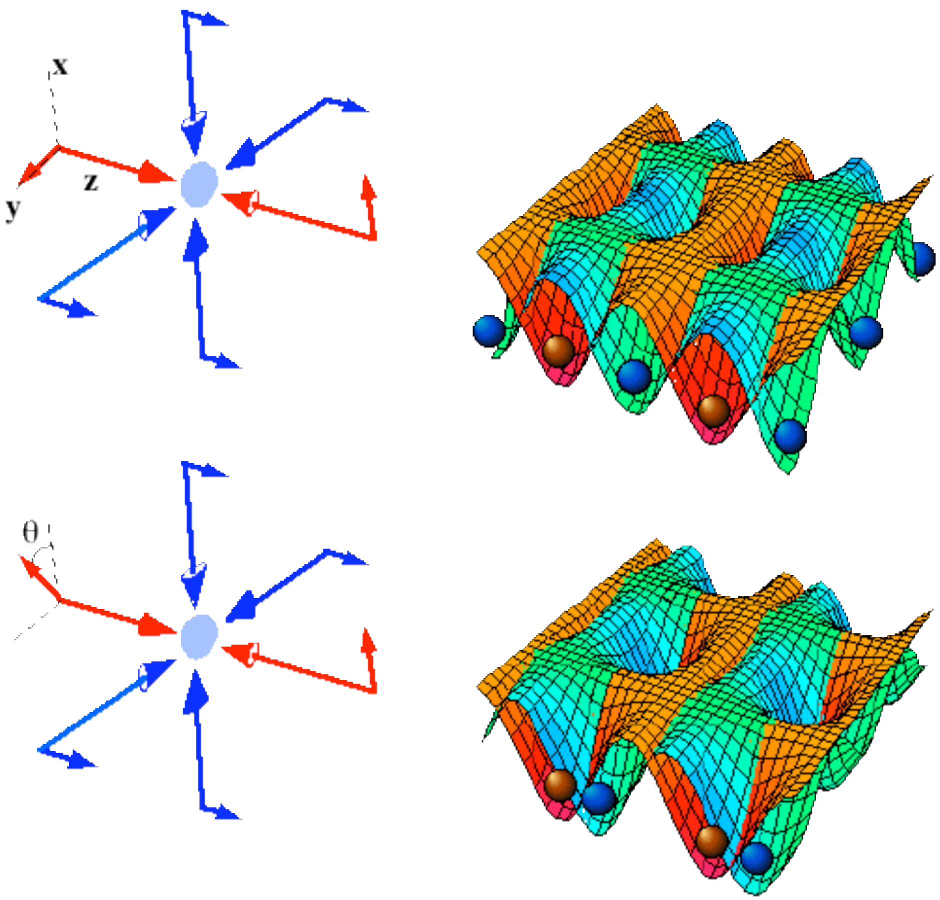
Quantum Optics
NMR

State Readout

- POVM
- State Tomography
- Process Tomography

Fluorescence

Two Qubit Interaction:
Three dimensional picture



Planes of atoms interact pairwise
(parallel operations)

Qubit-Qubit Interactions

- Electric dipole-dipole interaction.
  - Optical AC Stark $d \sim (s/2)ea_0$ G. K. Brennen *et al.* PRL (1999)
  - DC Stark (Rydberg) $d \sim n^2 e a_0$ D. Jaksch *et al.* PRL (2000)

- Ground electronic collision. D. Jaksch *et al.* PRL (1999)

$$V = \frac{1}{4}V_S(r) + \frac{3}{4}V_T(r) + (V_T(r) - V_S(r))\vec{s}_1 \cdot \vec{s}_2$$

- Magnetic dipole-dipole interaction.

$$V = \frac{\vec{\mu}_1 \cdot \vec{\mu}_2 - 3(\vec{\mu}_1 \cdot \vec{\mathbf{e}}_r)(\vec{\mu}_2 \cdot \vec{\mathbf{e}}_r)}{r^3}$$

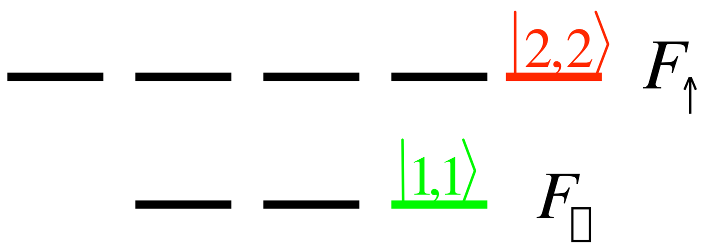
L. You, M. Chapman PRA (2001)

- Real photon exchange (cavity QED)

T. Pellizzari *et al.* PRL (1995)

Example: S-wave collisions

D. Jaksch *et al.* PRL (1999)



$$|\uparrow\rangle|\uparrow\rangle \to |\uparrow\rangle|\uparrow\rangle$$

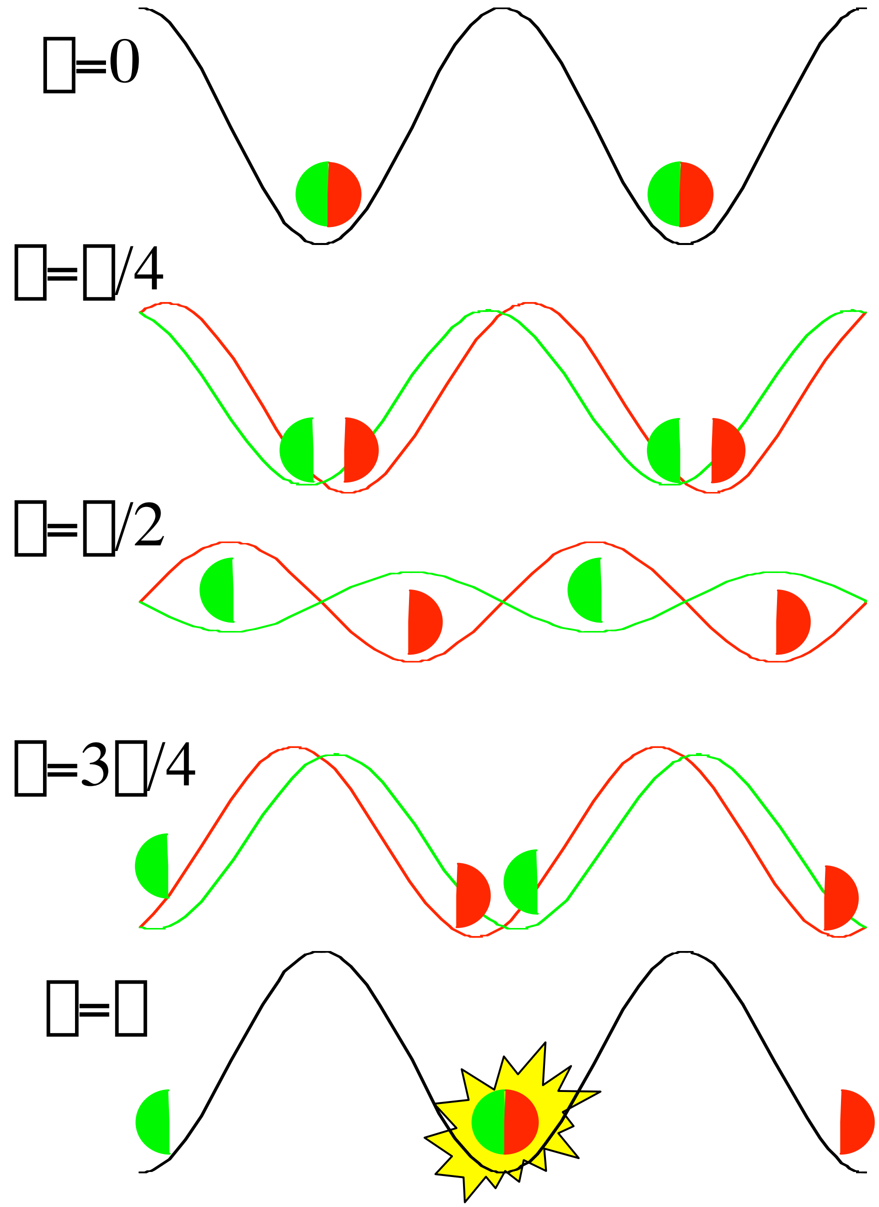
$$|\uparrow\rangle|\downarrow\rangle \to e^{i\phi}|\uparrow\rangle|\downarrow\rangle$$

$$|\downarrow\rangle|\uparrow\rangle \to |\downarrow\rangle|\uparrow\rangle$$

$$|\downarrow\rangle|\downarrow\rangle \to |\downarrow\rangle|\downarrow\rangle$$

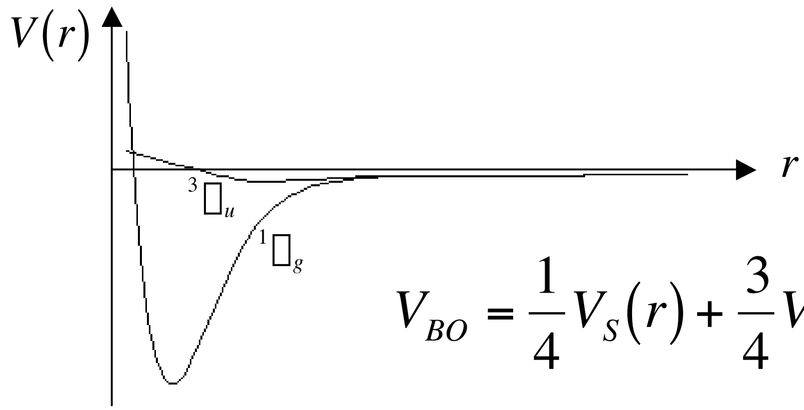
$$\phi \sim (k_L a)\omega_{osc}\tau$$

O. Mandel et al quant-ph 0301169



General Ground-Electronic Collisions

- Exchange Interactions:



$$V_{BO} = \frac{1}{4} V_S(r) + \frac{3}{4} V_T(r) + \left(V_T(r) + V_S(r)\right) \vec{s}_1 \cdot \vec{s}_2$$

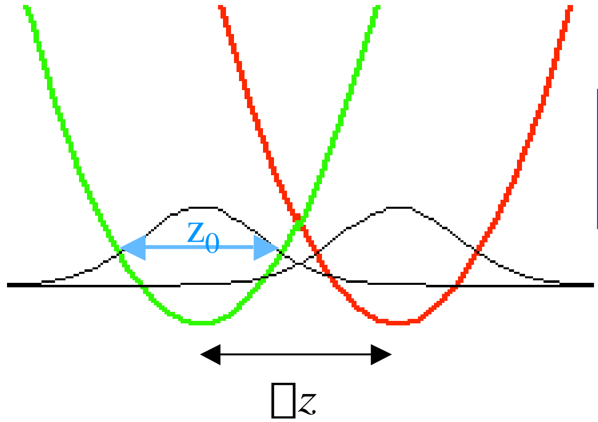
- Problem:

Interaction does not conserve atomic quantum numbers

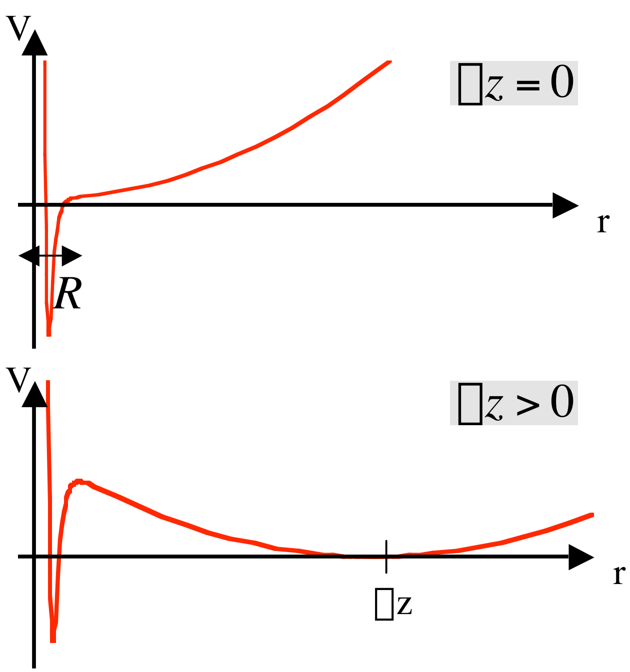
- Solution:

Collisions between trapped but *separated* atoms

Atoms in Separated Traps



$$\hat{H} = \frac{\hat{p}_1^2}{2m} + \frac{\hat{p}_2^2}{2m} + V_{trap}\left(\vec{r}_1 - \frac{\Delta\vec{z}}{2}\right) + V_{trap}\left(\vec{r}_2 + \frac{\Delta\vec{z}}{2}\right) + \hat{V}_{\mathrm{int}}(r)$$



Separation in harmonic case:

$$\hat{H}_{CM} = \frac{\hat{p}_{CM}^2}{2M} + \frac{1}{2} m\omega_{osc}^2 \vec{R}^2$$

$$\hat{H}_{rel} = \frac{\hat{p}_{rel}^2}{2\mu} + \frac{1}{2} m\omega_{osc}^2 \left|\vec{r} - \Delta z \vec{e}_z\right|^2 + \hat{V}_{\mathrm{int}}(r)$$

Very different scales:

$$k_L z_0 = 0.1 \Rightarrow z_0 \sim 20\ \mathrm{nm}$$

$$R \sim 1\ \mathrm{\mathring{A}}$$

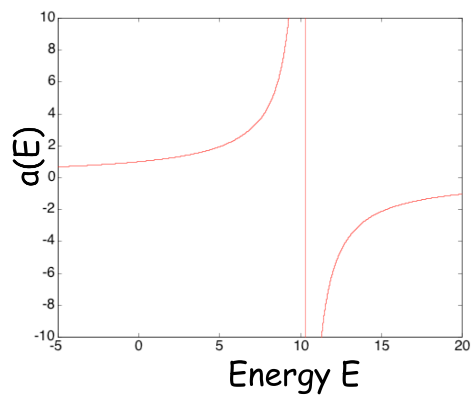
Self-Consistent Scattering Length Model

- Regularized δ-function interaction

$$V_{\text{int}}(\vec{r}) = \frac{2\pi\hbar^2}{\mu}\, a\, \delta^{(3)}(\vec{r}) \frac{\partial}{\partial r} r$$

scattering length: $a = -\lim_{k \to 0} \frac{\tan \delta_0(E)}{k(E)}$

- Near resonance



replace constant a with energy dependent a_{eff}

Bolda et. al, PRA **66**, 2001

$$V_{\text{int}}(\vec{r}) = \frac{2\pi\hbar^2}{\mu}\, a_{eff}(E)\, \delta^{(3)}(\vec{r}) \frac{\partial}{\partial r} r$$

Energy dependent **scattering length** $a_{eff}(E) = -\frac{\tan \delta_0(E)}{k(E)}$

Self-Consistent Eigenvalue Solution

$$\hat{H}_{rel} = \frac{\hat{p}_{rel}^2}{2\mu} + \frac{1}{2} m\omega_{osc}^2 \left(\vec{r} - \Delta z \vec{e}_z\right)^2 + \frac{2\pi\hbar^2}{\mu} a_{eff}(E)\,\delta^{(3)}(\vec{r}) \frac{\partial}{\partial r} r$$

$$\hat{H}_{rel}|\Psi\rangle = E|\Psi\rangle$$

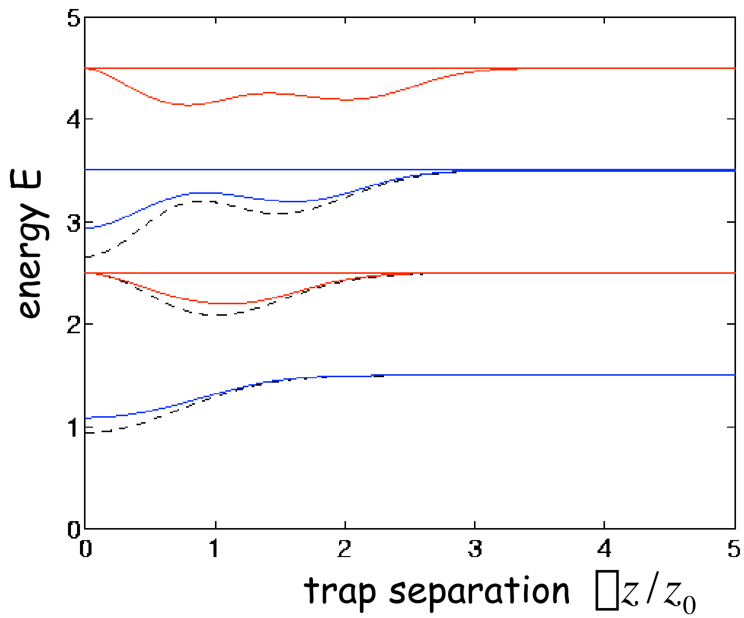
- Solve for eigenvalues as a function of *fixed scattering length:* $E(a_{eff})$

Simultaneous solutions

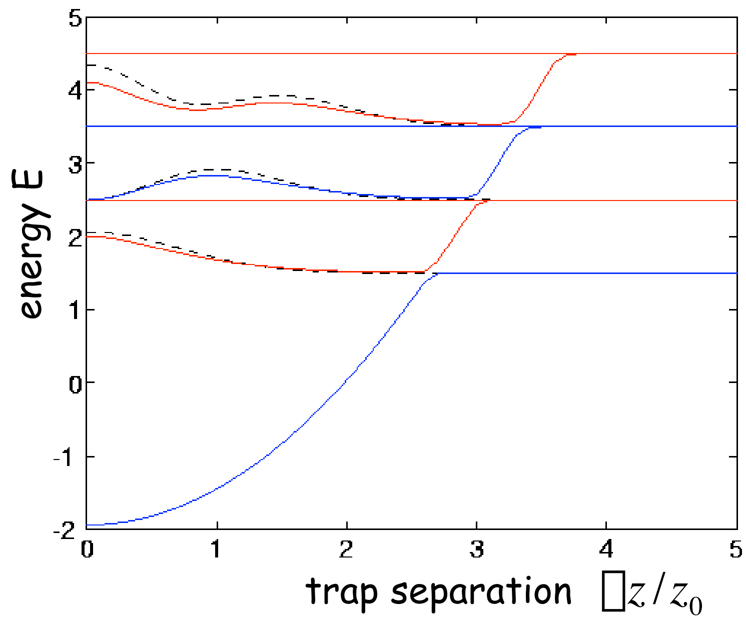
- Solve for scattering length as a function of *fixed scattering length:* $a_{eff}(E)$

Atoms in Separated Traps: Eigenspectrum

$a_{eff} = -z_0 < 0$



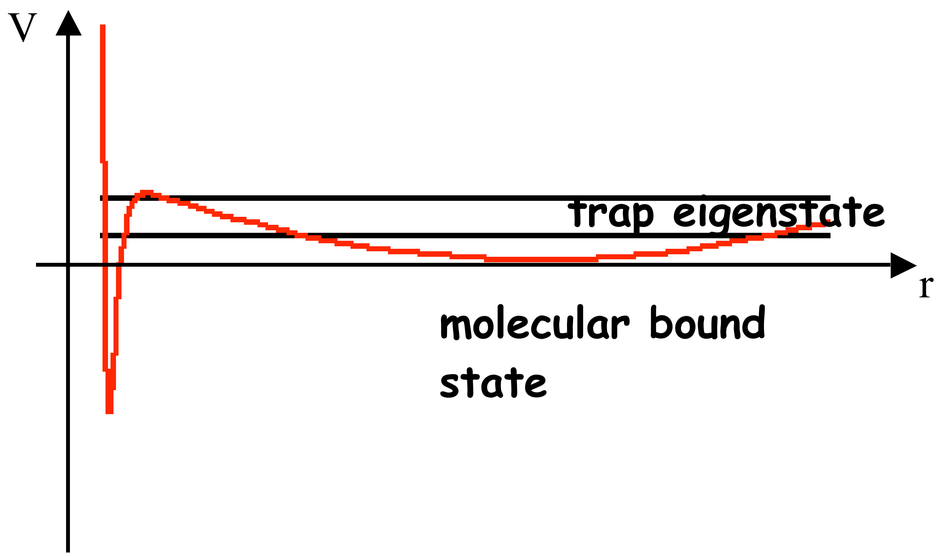
$a_{eff} = 0.5 z_0 > 0$

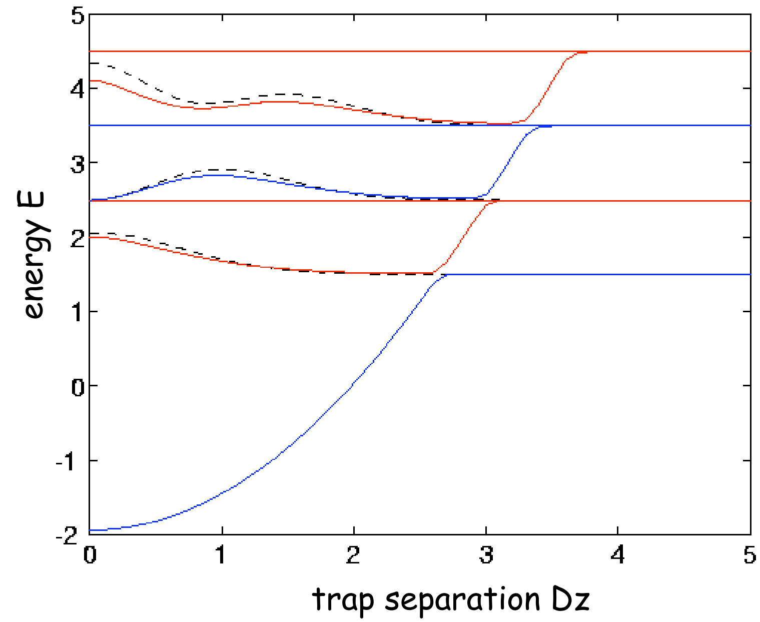


Why is there an "Anti-crossing" for positive scattering lengths?

Atoms in Separated Traps: "Shape Resonance"

- **Total potential**





- **Localization of resonance**

$$E_{bound} + V_{trap} = \frac{3}{2}\hbar\omega \qquad \Rightarrow \qquad \frac{\Delta z}{z_0} = \sqrt{3 + \frac{z_0^2}{a_{scatt}^2}}$$

Energy dependent δ-function interaction: Bound States

- Bound state of actual potential

$$E_b = \frac{\hbar^2 k_b^2}{2m} = -\frac{\hbar^2 \kappa_b^2}{2m} \quad \text{with} \quad k_b = i\kappa_b$$

- Pole in the S-matrix

$$s_l(\kappa_b) = e^{2i\delta_l} \to \infty \qquad \text{therefore} \quad i\delta_l \to \infty$$

$$a_{eff}(\kappa_b) = -\frac{i\tanh(i\kappa_0)}{i\kappa_b} \approx \frac{1}{\kappa_b}$$

- *Reg. δ - function bound state*

$$E_\delta = -\frac{\hbar^2}{2m\, a_{eff}^2} = -\frac{\hbar^2 \kappa_b^2}{2m} = E_b$$

→ **Effective scattering length model contains information about all bound states self-consistently**

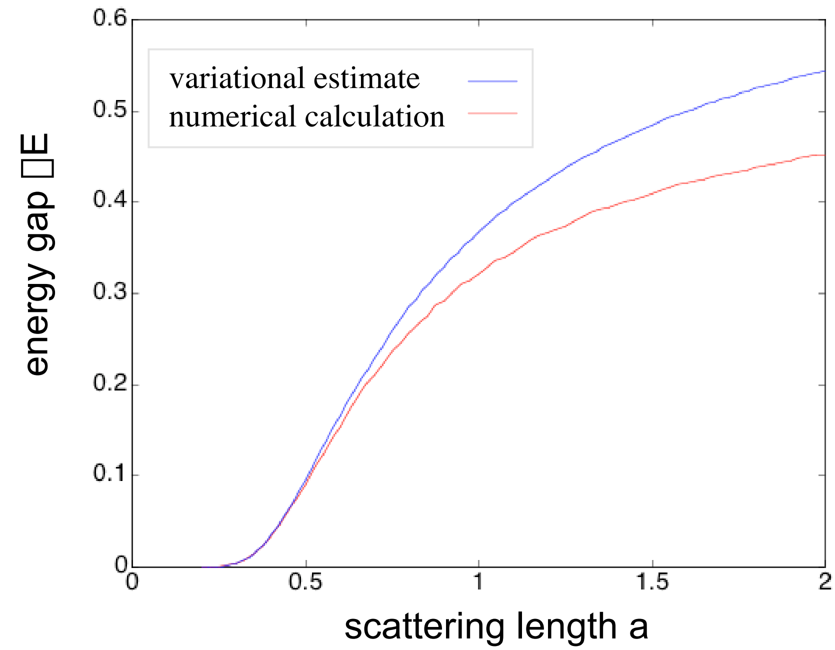
Energy Gap Calculation

- **Example: Collisions in ^{87}Rb**
  singlet $a_s = 93\ a_0 = 0.39\ z_0$
  triplet $a_t = 102\ a_0 = 0.42\ z_0$
  (□ = 789 nm , □ = k z_0 = 0.1)

  □ **□E □ 0.03 ħ□**

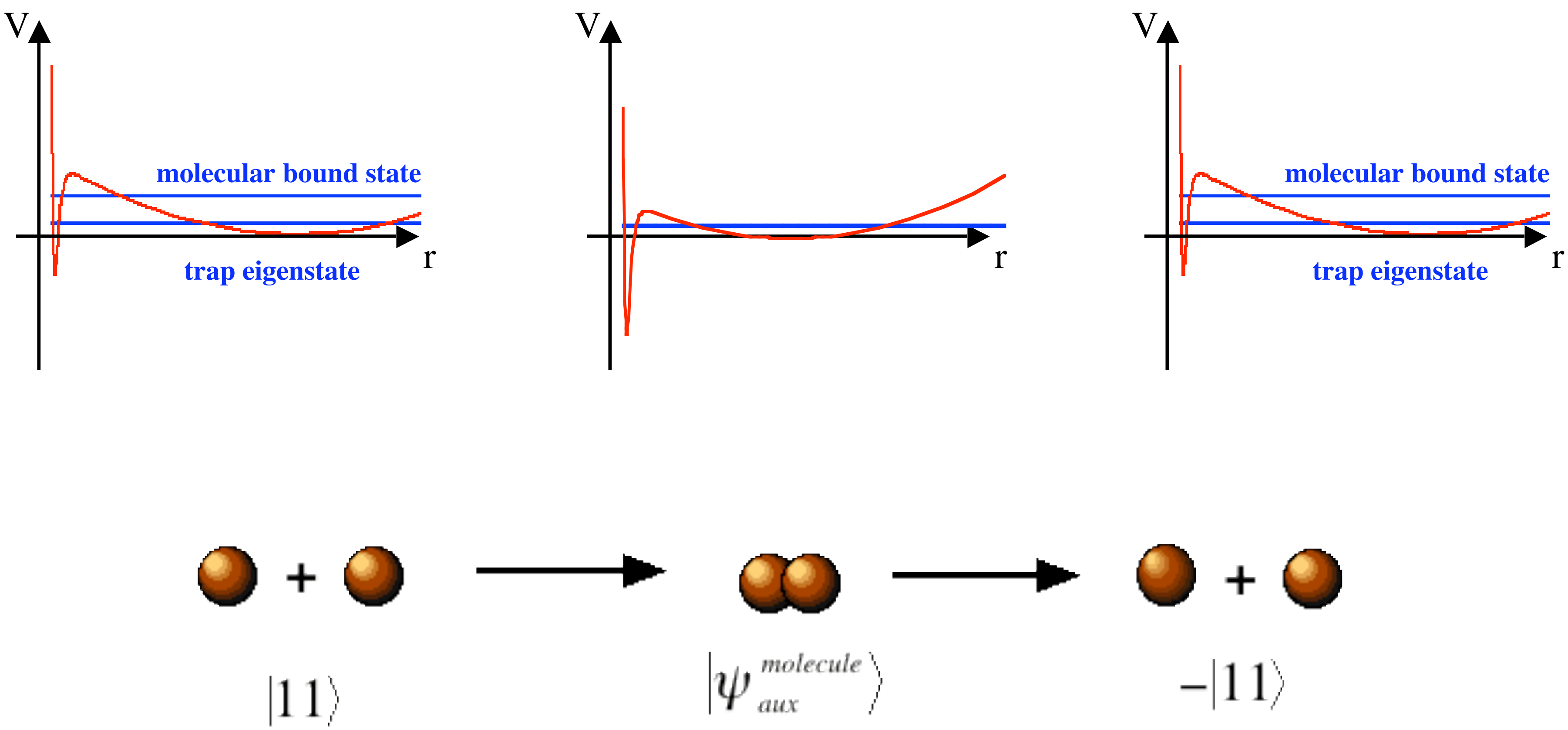
- **Example: Collisions in ^{133}Cs**
  singlet $a_s = 280\ a_0 = 1.2\ z_0$
  triplet $a_t = 2400\ a_0 = 10\ z_0$
  (□ = 852 nm, □ = 0.1)

  □ **□E □ 0.3 - 0.5 ħ□**



"Quantum State Control via a Trap-Induced Shape Resonance in Ultra-Cold Atomic Collisions", R. Stock, I. H. Deutsch, and E. Bolda, quant-ph/0304093

" Shape Resonance" and Conditional Logic



http://info.phys.unm.edu/~deutschgroup

I.H. Deutsch, Dept. Of Physics and Astronomy
University of New Mexico

Collaborators:

- Physical Resource Requirements for Scalable Q.C.

  Carl Caves (UNM), Robin Blume-Kohout (LANL)

- Quantum Logic via Dipole-Dipole Interactions

  Gavin Brennen (UNM/NIST), Poul Jessen (UA), Carl Williams (NIST)

- Quantum Logic via Ground-State Collisions

  René Stock (UNM), Eric Bolda (NIST)

ARDA NSF ONR