Entanglement in Spin Systems

Quantum Phase transitions & Dynamical Properties

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NEST

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OUTLINE

- ✓ **Entanglement**
  - Measure of entanglement
  - entanglement in many body systems

- ✓ **Quantum vs Classical Phase Transitions**
  - Long range correlations
  - Ising model in a transverse field
  - Entanglement scaling & universality

- ✓ **Dynamics of entanglement in spin chains**

ENTANGLEMENT

Non local correlations without a classical analog

$$|\phi\rangle = |00\rangle + |11\rangle$$

- ✓Entanglement cannot be created locally
- ✓Absence of entanglement IS NOT absence of correlations. A separable state (e.g. |00>) is highly correlated

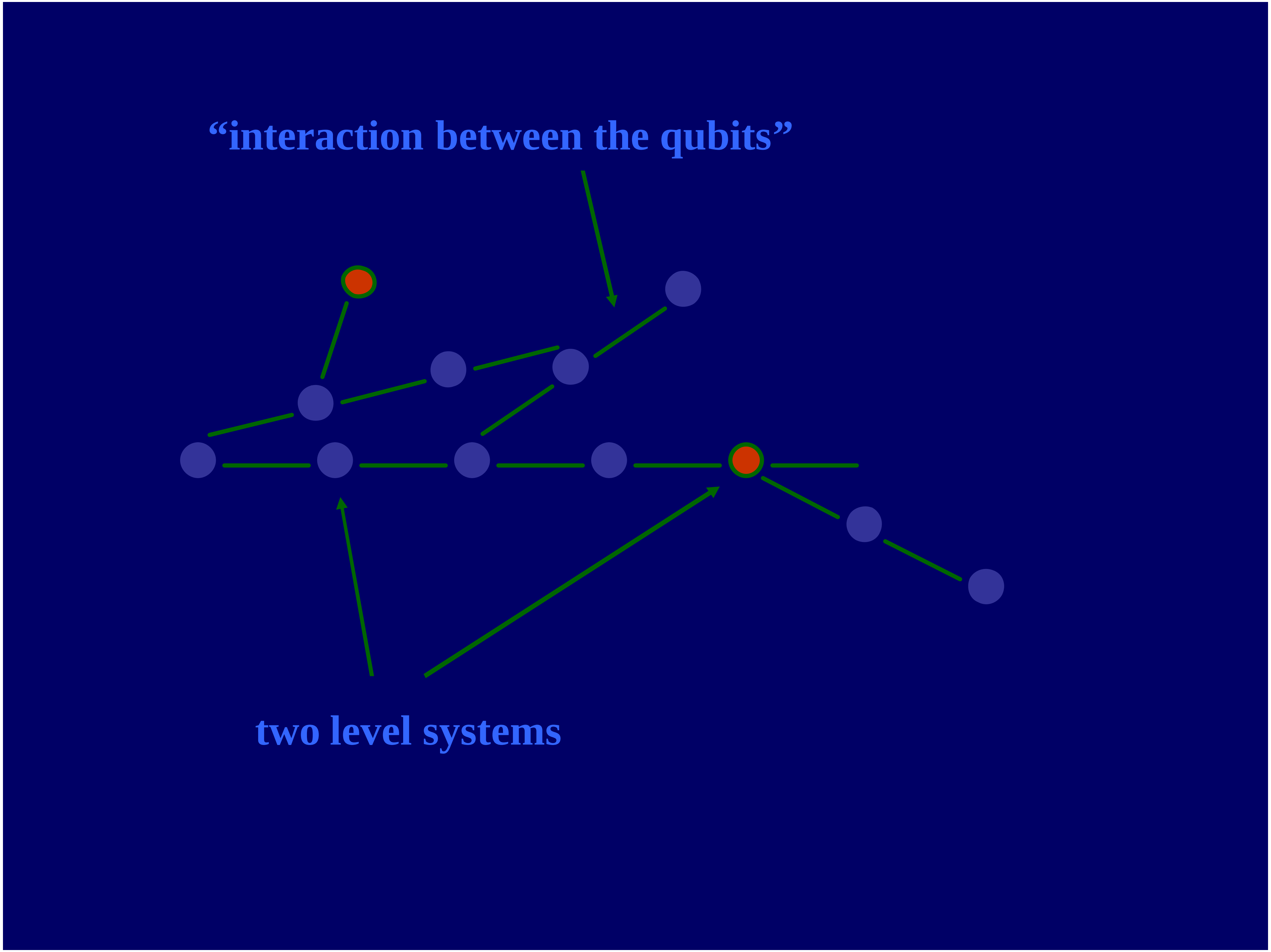
Entanglement between two spins in the mixed state ...

because

- Two state systems are coupled to an environment

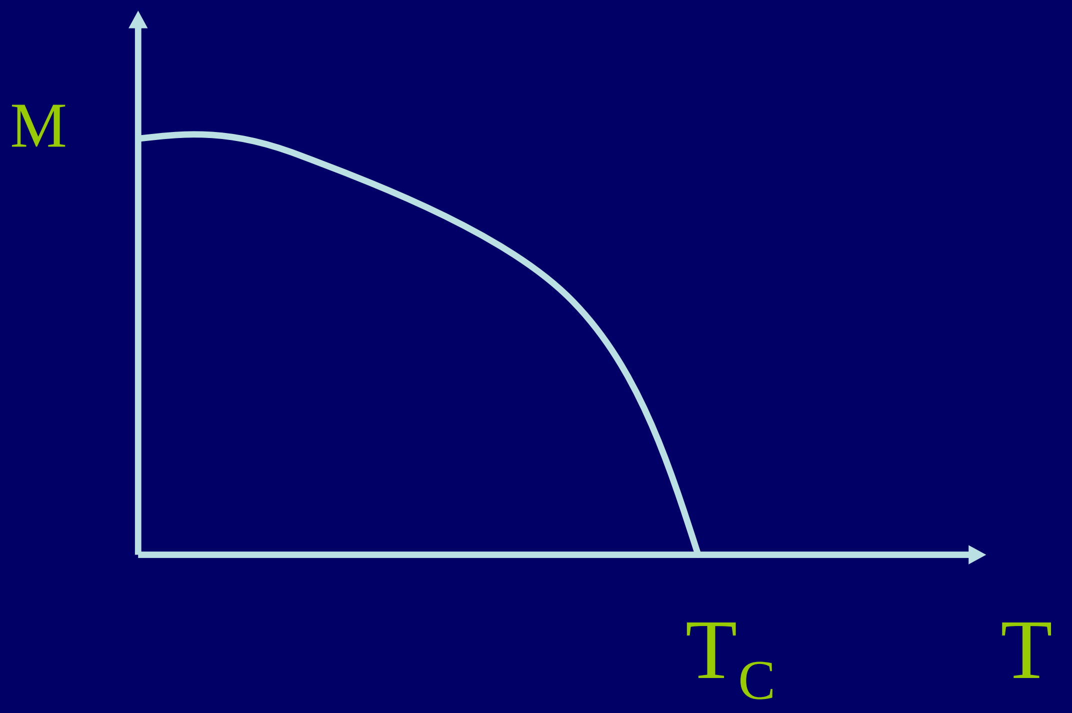
- Many body states

O'Connor and Wootters '00
Arnesen, Bose and Vedral '01
Gunlycke, Bose, Kendon and Vedral '01
Wang '01
Wang and Zanardi '02
Osborne and Nielsen '02
...



“interaction between the qubits”
two level systems

Classical Phase Transitions



e.g. Ferromagnet

- Long range correlations (ξ diverges at T_C)
- Universality

Quantum Phase Transitions

- T=0 phase transition driven by a coupling constant "g" of the system
- Correlations diverge as $\xi \sim |g - g_c|^{-\nu}$
- The phase transition reflects in a change of the ground state wavefunction

?? Change in the entanglement properties of the ground state ??

Measure of Entanglement

- Separable state

  $|a\rangle = |00\rangle$

- "Not"

  $|b\rangle = |11\rangle$

$$\langle a|b\rangle = 0$$

- Entangled state

  $|\phi\rangle = |00\rangle + |11\rangle$

- Not

  $|\phi\rangle = |00\rangle + |11\rangle$

$$\langle \phi|\phi\rangle = 1$$

Concurrence

- Measure of the entanglement between two spins

- ρ is the reduced density matrix

- Construct $s_y \otimes s_y ?^* s_y \otimes s_y$

- $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$ are the sqare roots of the eigenvalues of $?s_y \otimes s_y ?^* s_y \otimes s_y$

$$\boxed{C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}}$$

Wootters '98

- Long range correlations and entanglement?

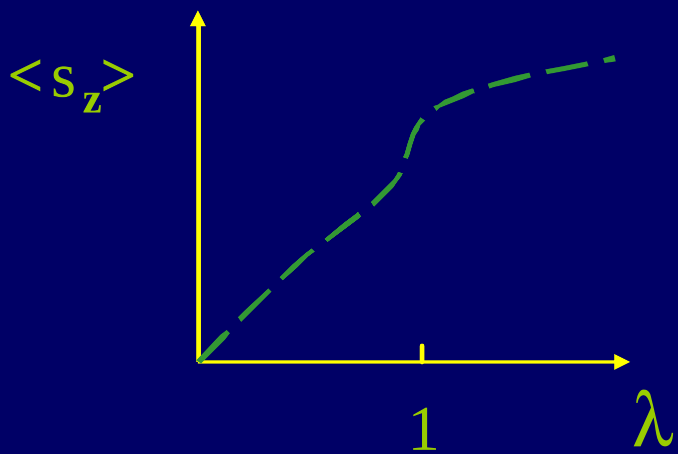
- Critical properties (and critical exponents)?

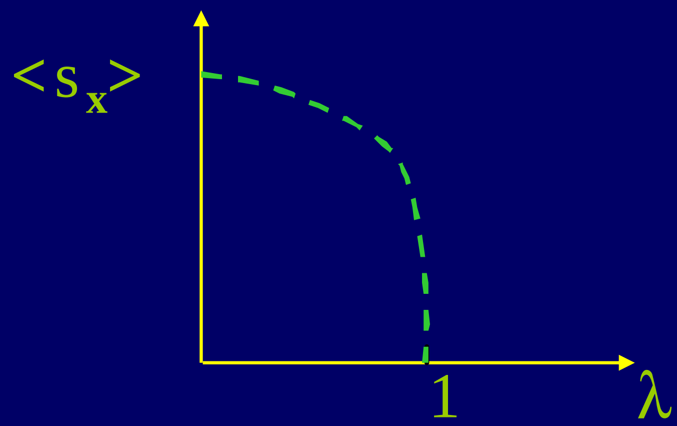
- Universality?

Ising model in a transverse field

$$H = -J\sum_i s_i^x s_{i+1}^x - h\sum_i s_i^z$$

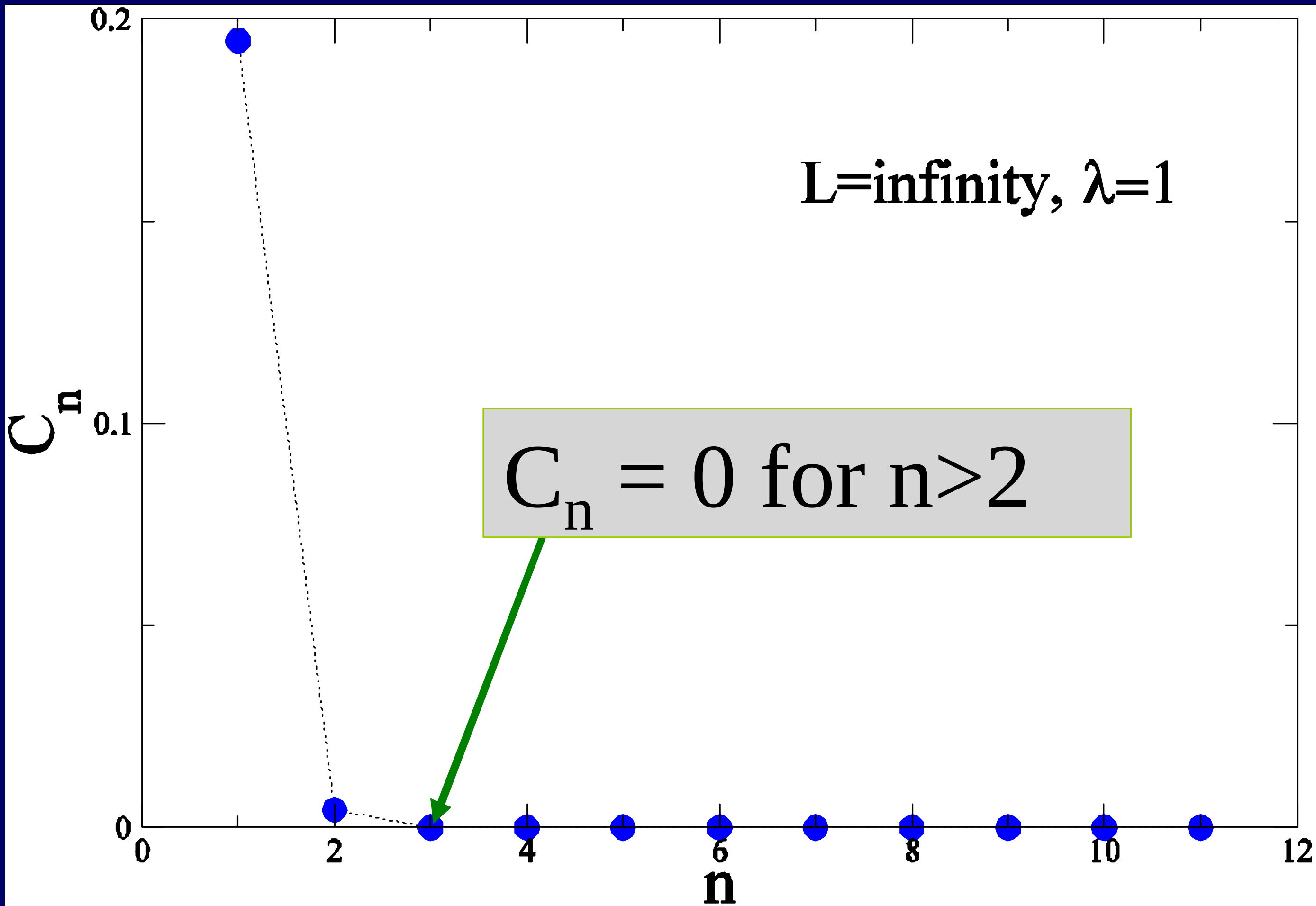
- Critical point at $\lambda = J/2h = 1$



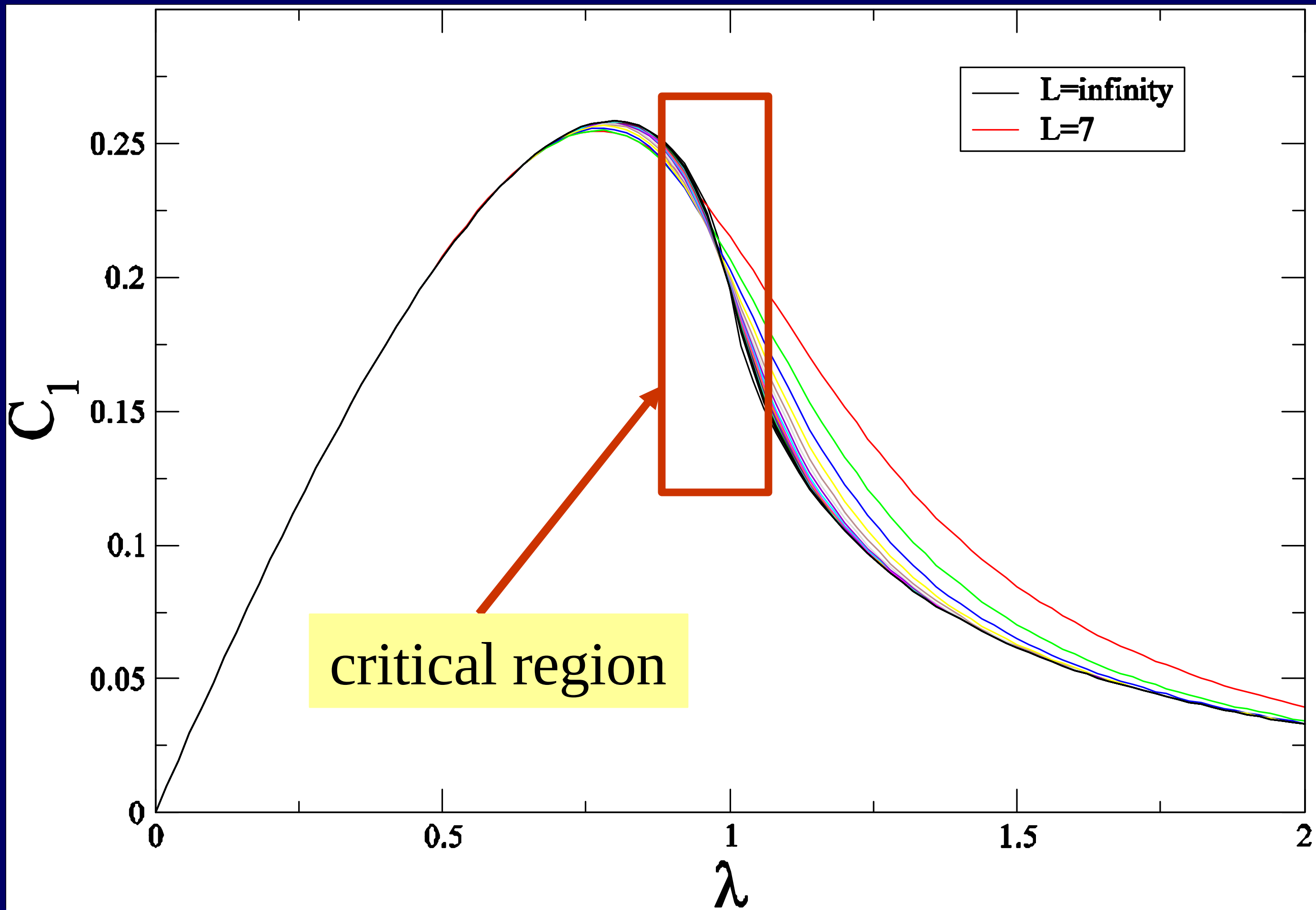


Pfeuty '70

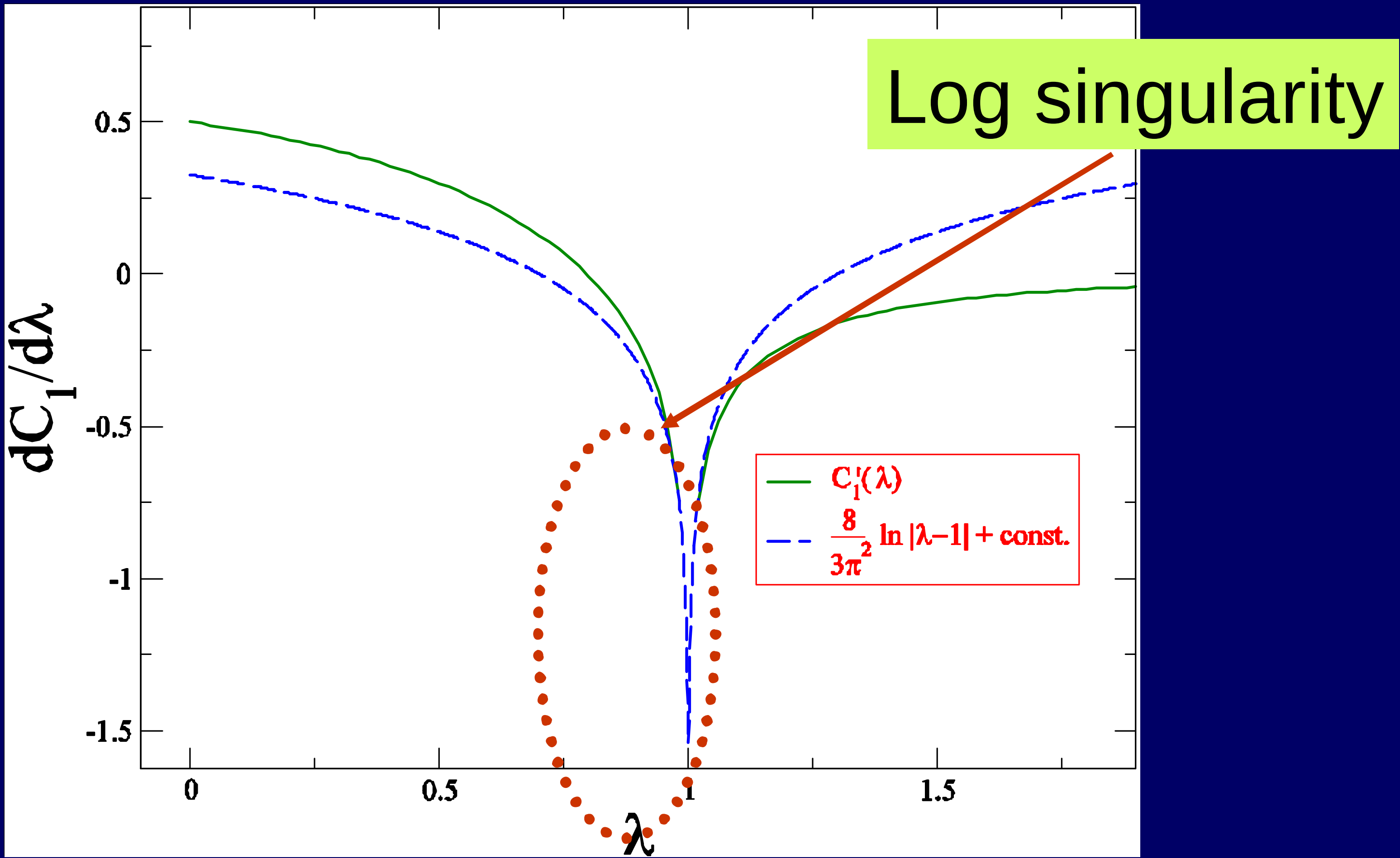
NO long range entanglement



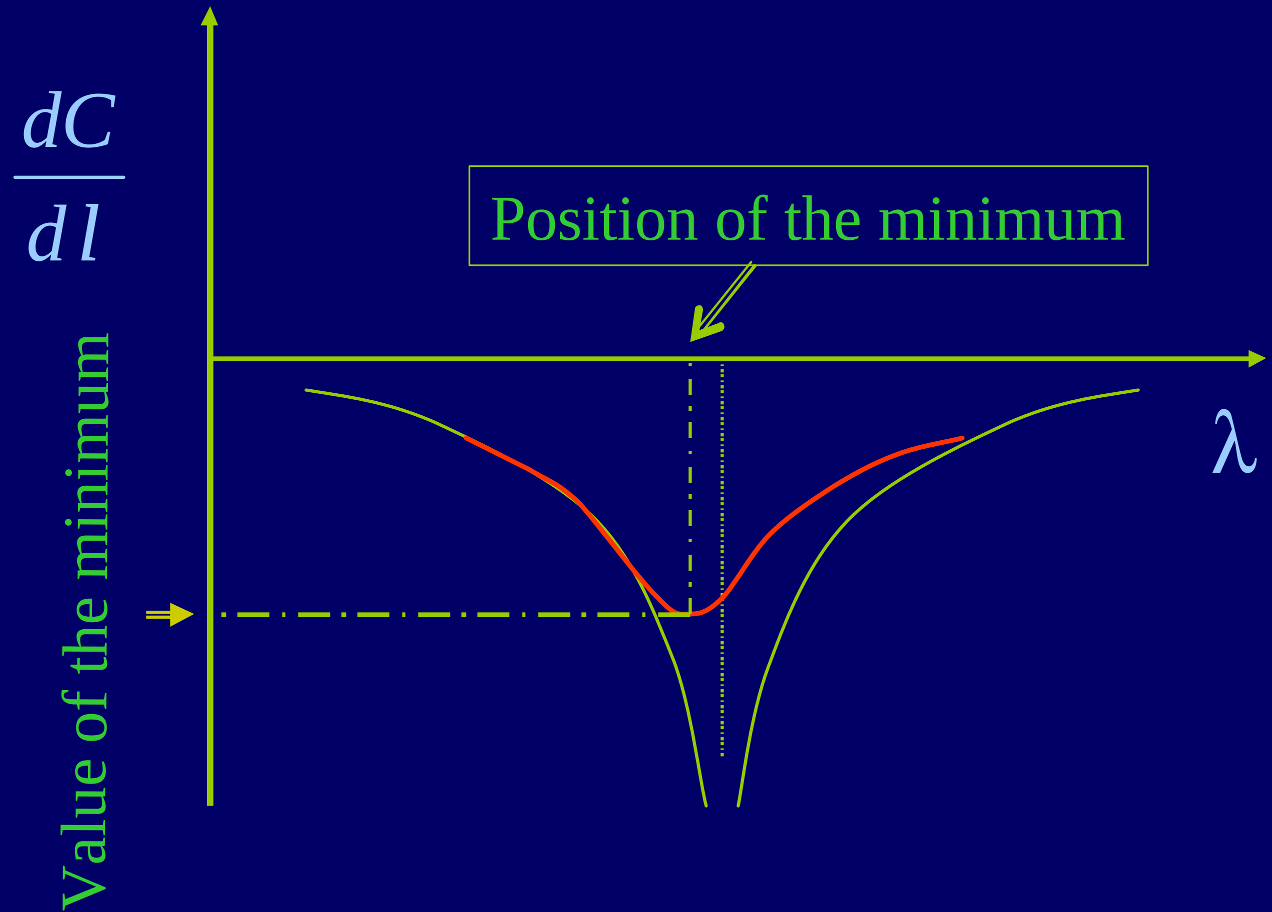
Next-neighbour concurrence as a function of the system size



Properties of $\dfrac{dC}{dl}$ (L= ∞)



Properties of $\dfrac{dC}{d\lambda}$



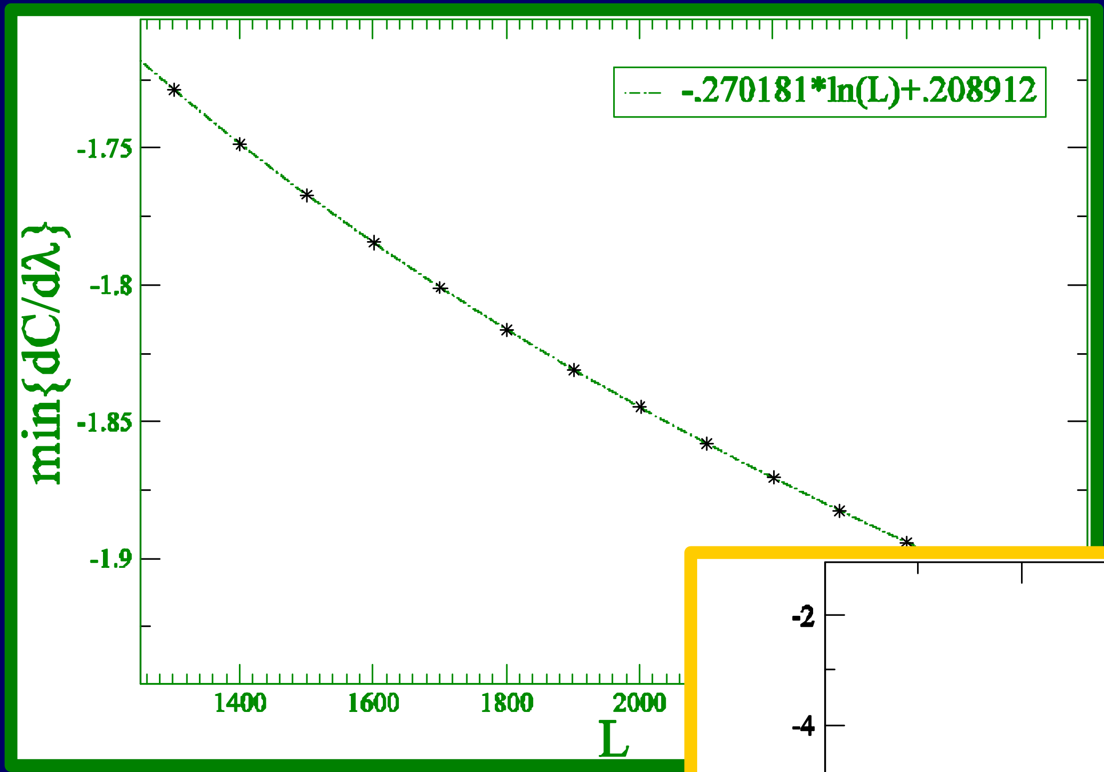
Finite Size Scaling for log singularities

$$\sim A \ln | l - l_c |$$

Log divergence in the thermodynamic limit

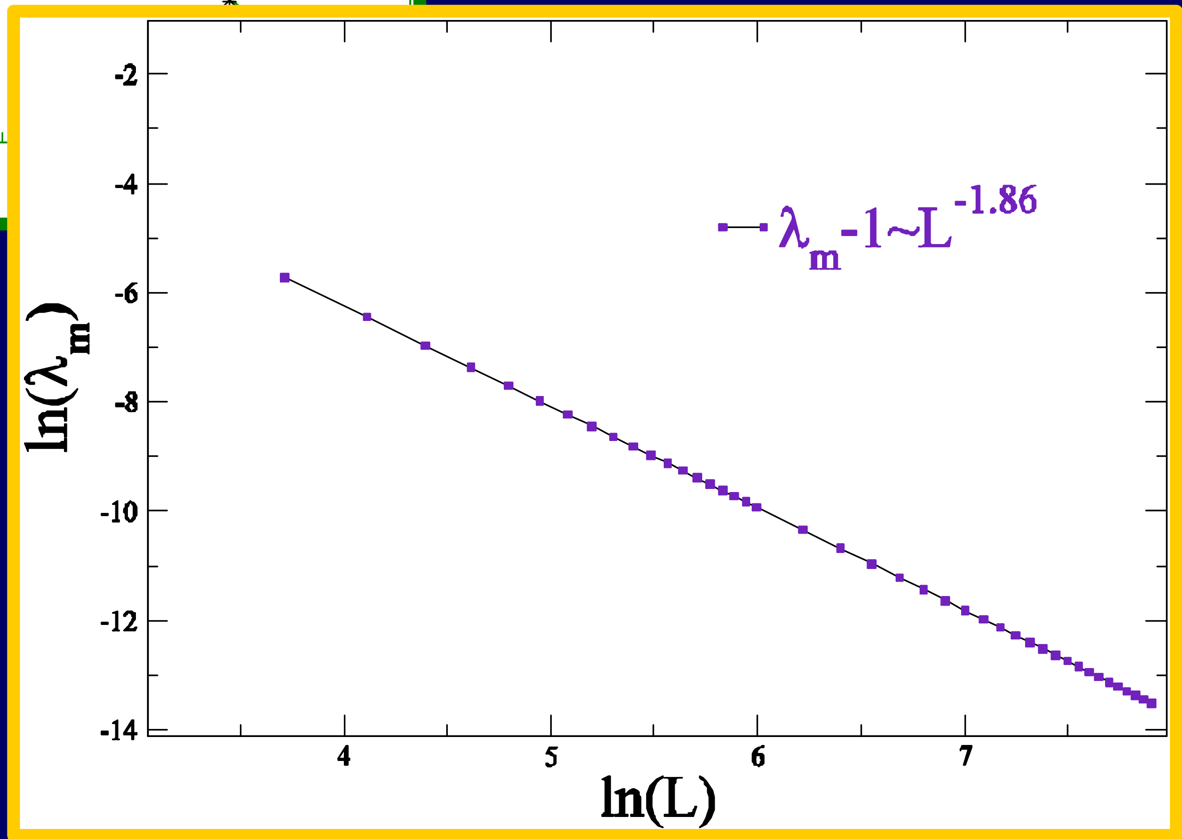
$$\sim \frac{A}{n} \ln N$$

Log divergence of the minimum as a function of the system size



Value of the minimum

Position of the minimum



Properties of $\frac{dC}{d\lambda}$

- For the infinite system it diverges as $\boxed{\sim \frac{8}{3\pi^2} \ln |\lambda - 1|}$

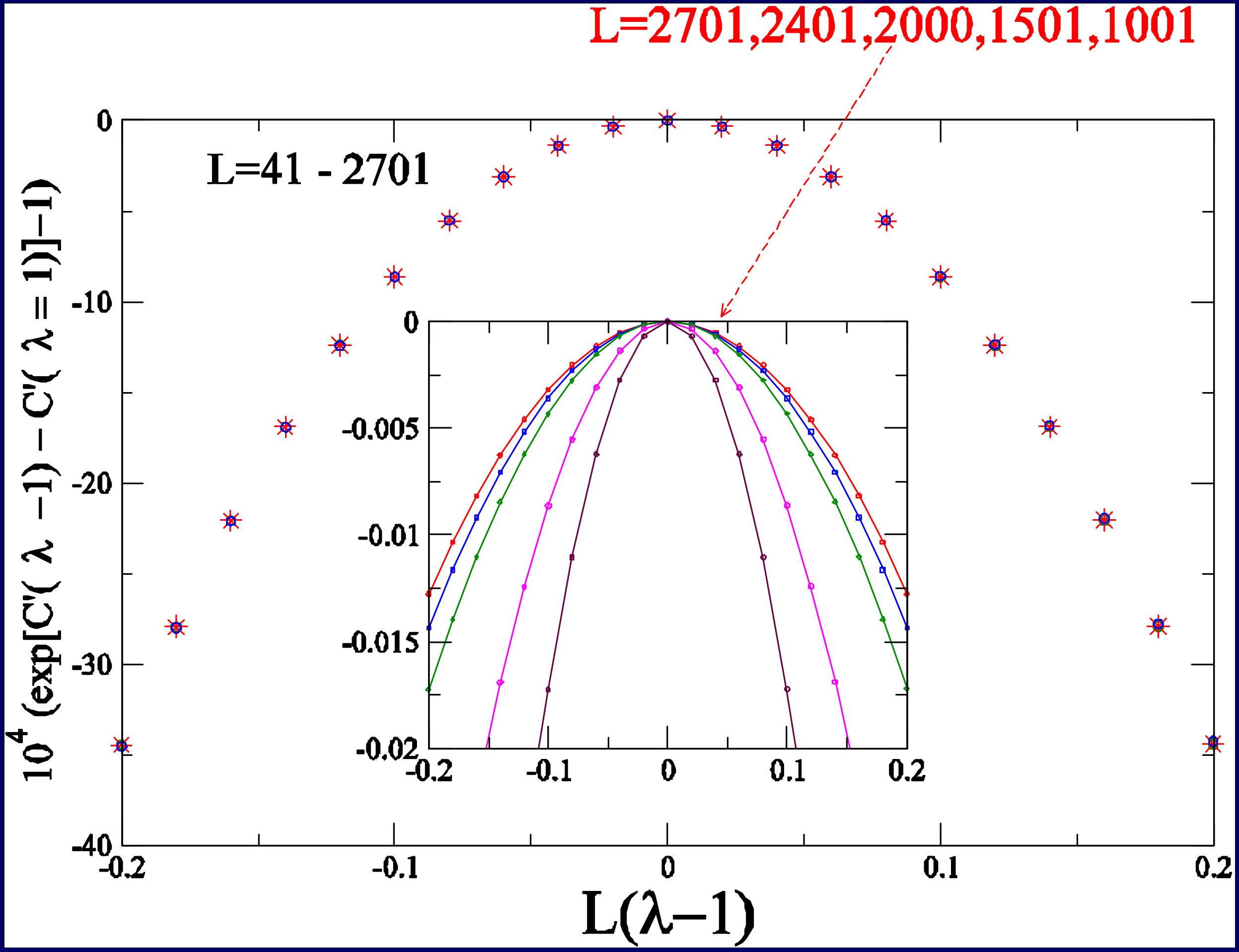
- The value of the minimum diverges as $\boxed{\sim \frac{8}{3\pi^2} \ln L}$

$$\boxed{\nu = 1}$$

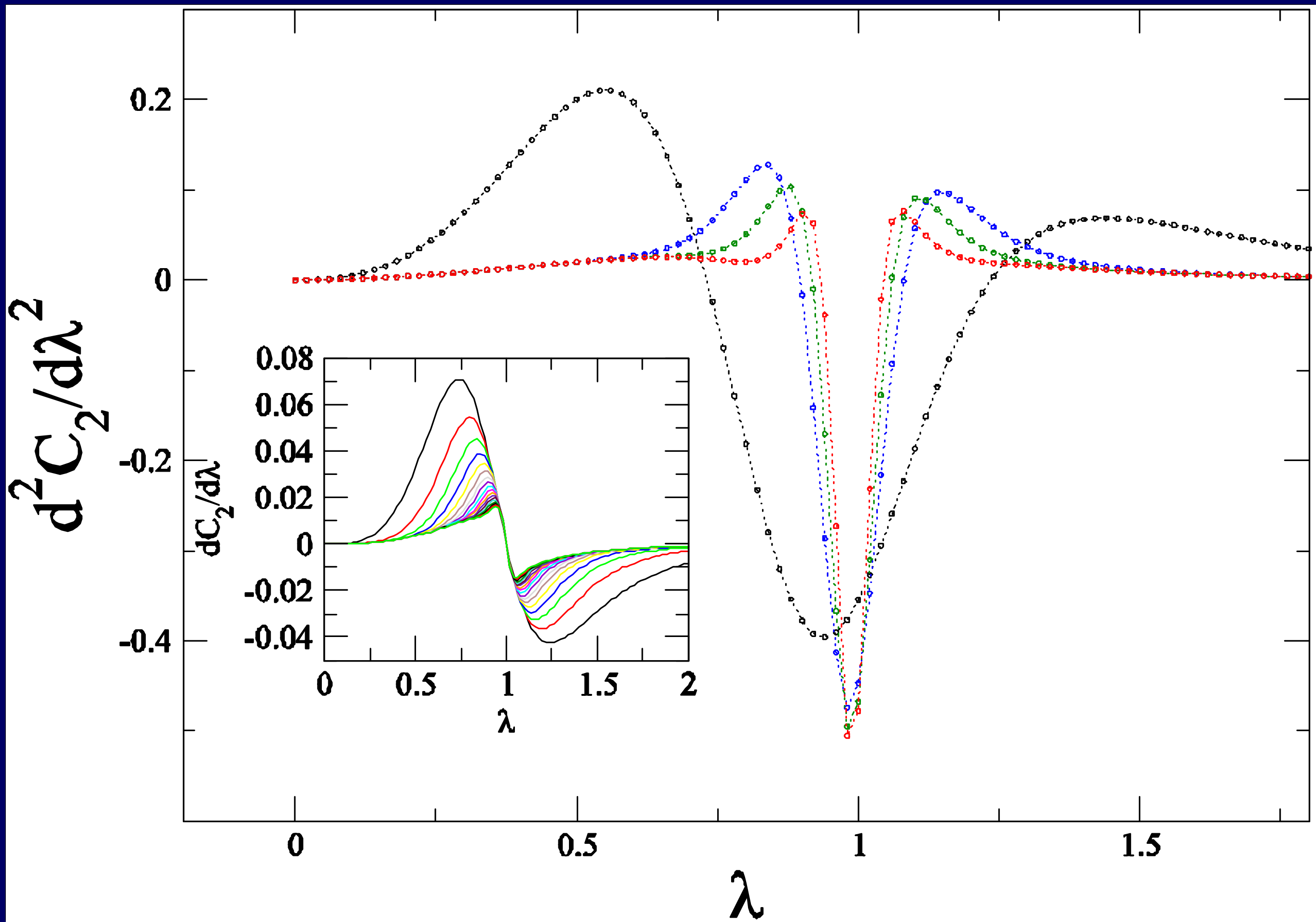
Data collapse

$$\frac{dC}{dl} = \ln\{H(L)K[L(l-1)]\}$$

$$H(L) = \frac{8}{3p^2}\ln(L) + a \qquad K(x) = b + cx^2$$



Next nearest neighbor concurrence



Universality

Check for the model

$$H = -J\left(\sum_i (1-g) s_i^x s_{i+1}^x + \sum_i (1+g) s_i^y s_{i+1}^y\right) - h\sum_i s_i^z$$

All the critical properties of concurrence C(λ) are the same for these models

Dynamics of entanglement

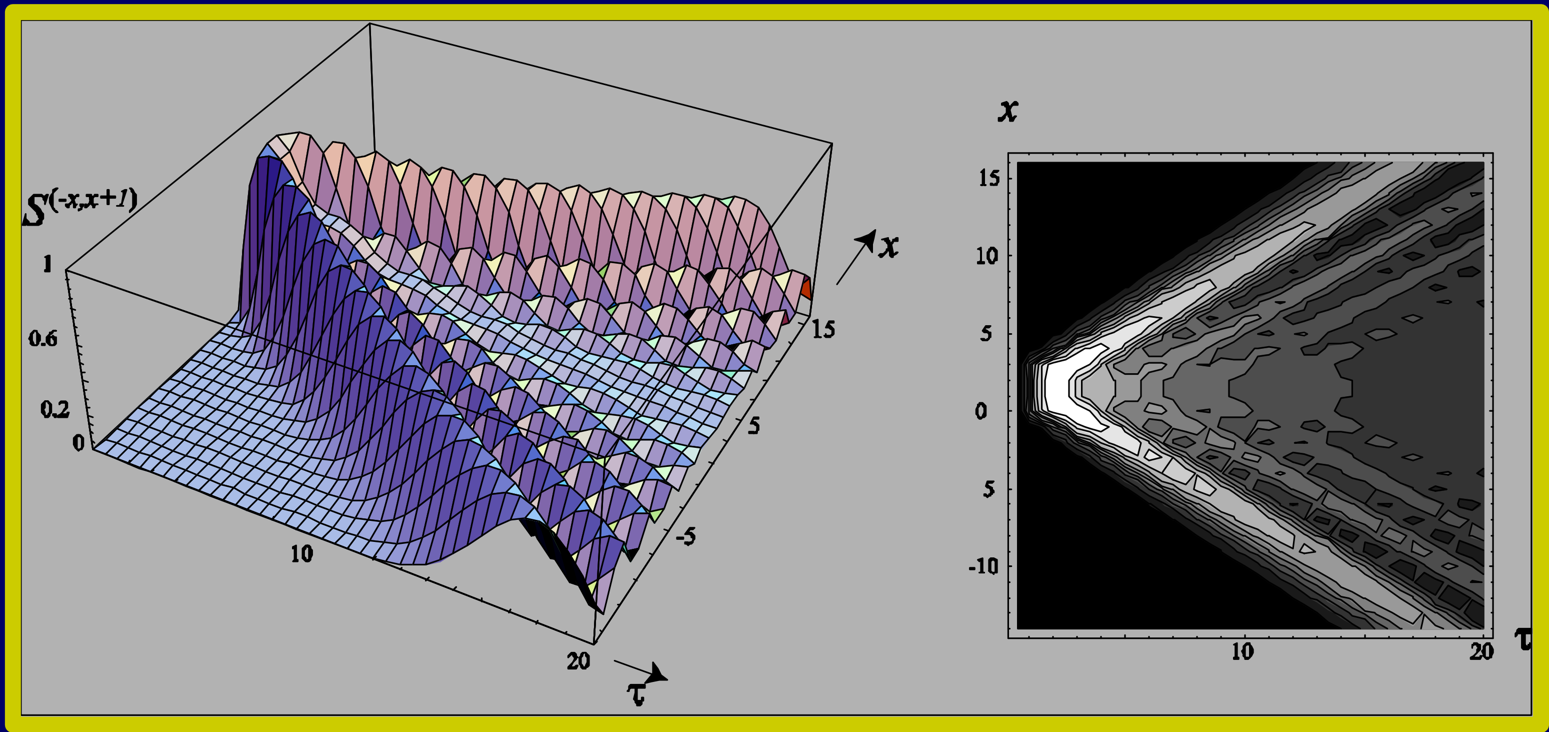
Prepare a Bell state at t=0 in a system described by

$$H = -J\left(\sum_i (1-g)s_i^x s_{i+1}^x + \sum_i (1+g)s_i^y s_{i+1}^y\right) - h\sum_i s_i^z$$

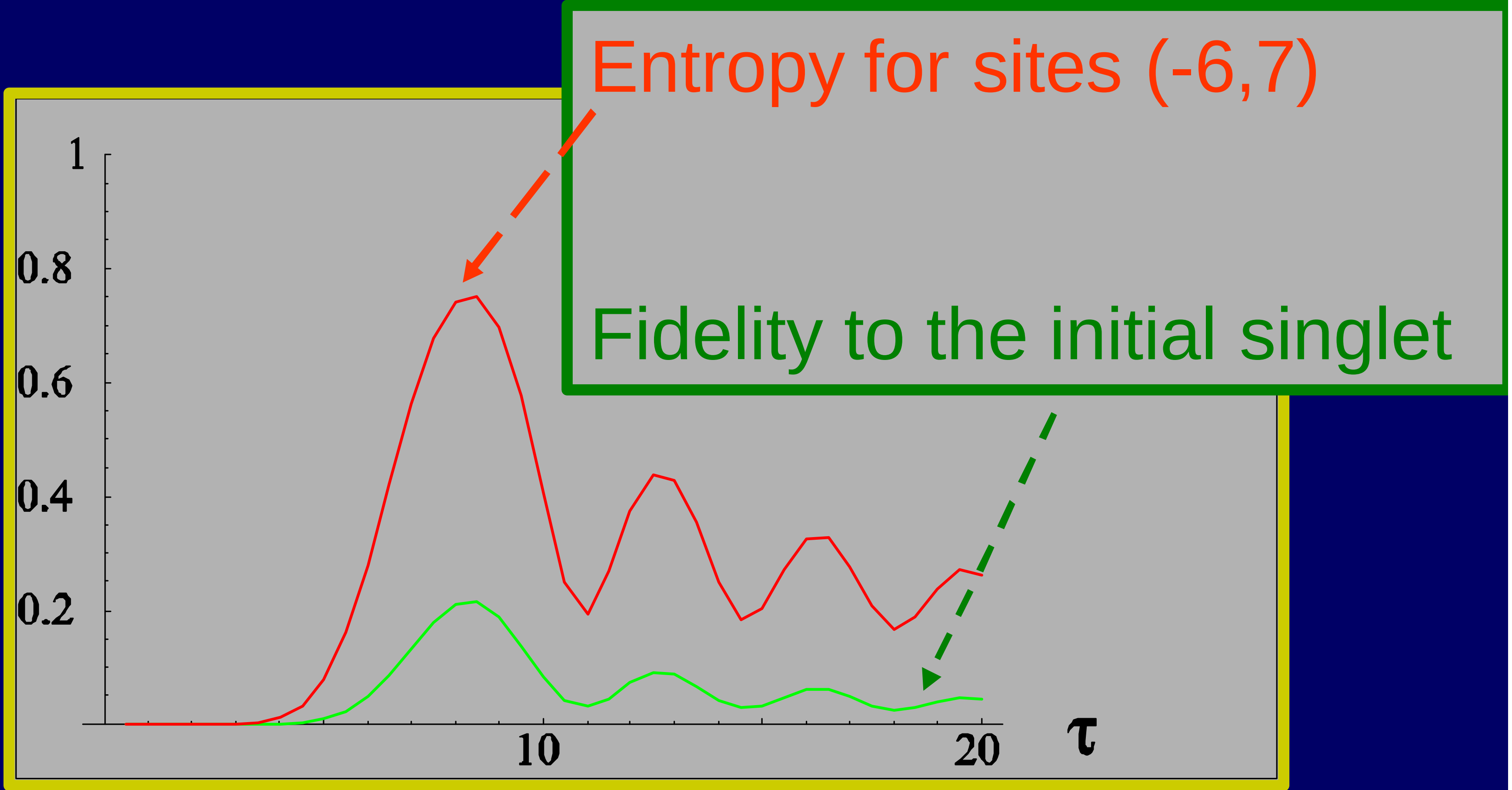
and study the evolution of the entanglement as a function of time and the position of the spins

γ=0 - XY model

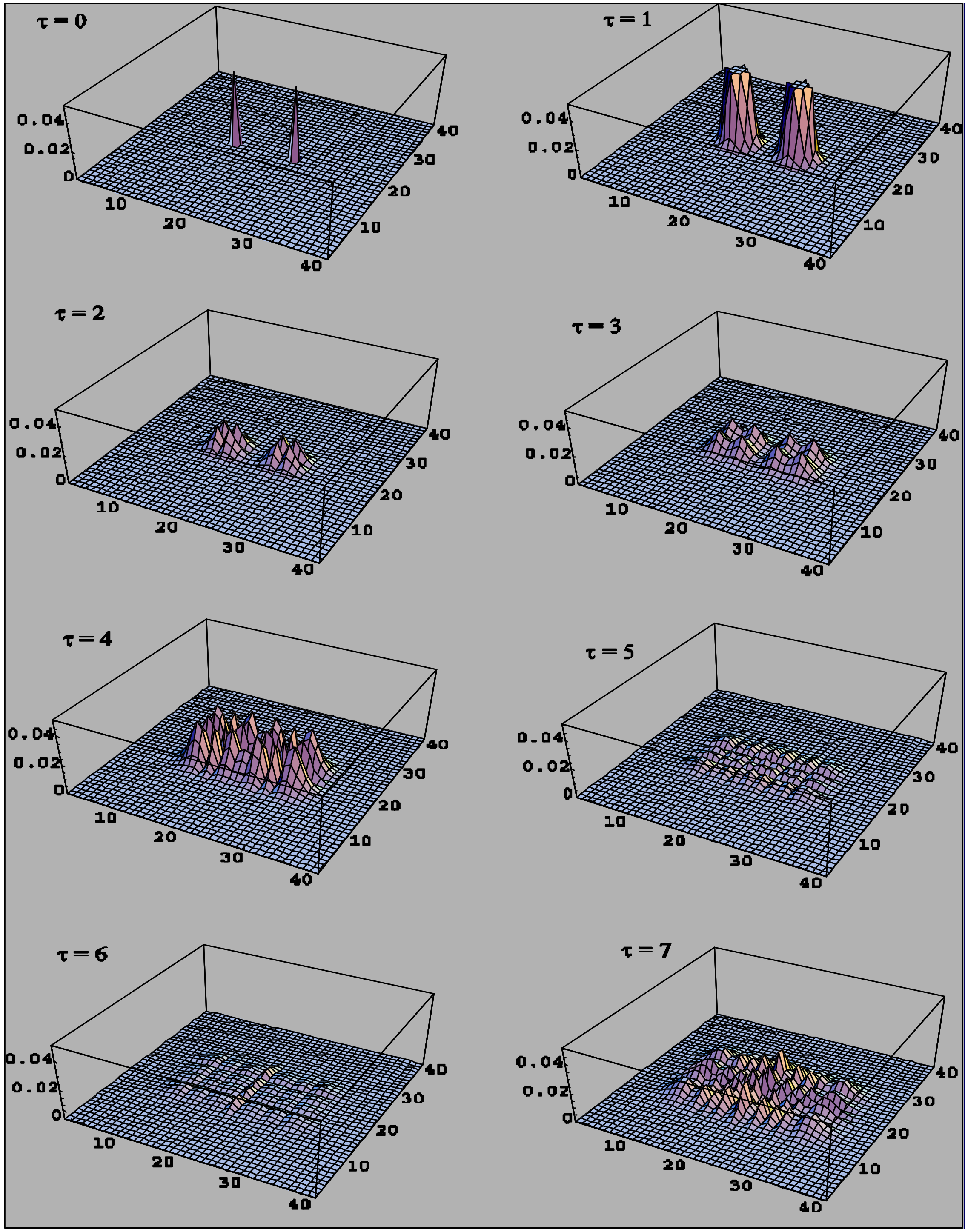
Entropy for symmetric sites



γ=0 - XY model

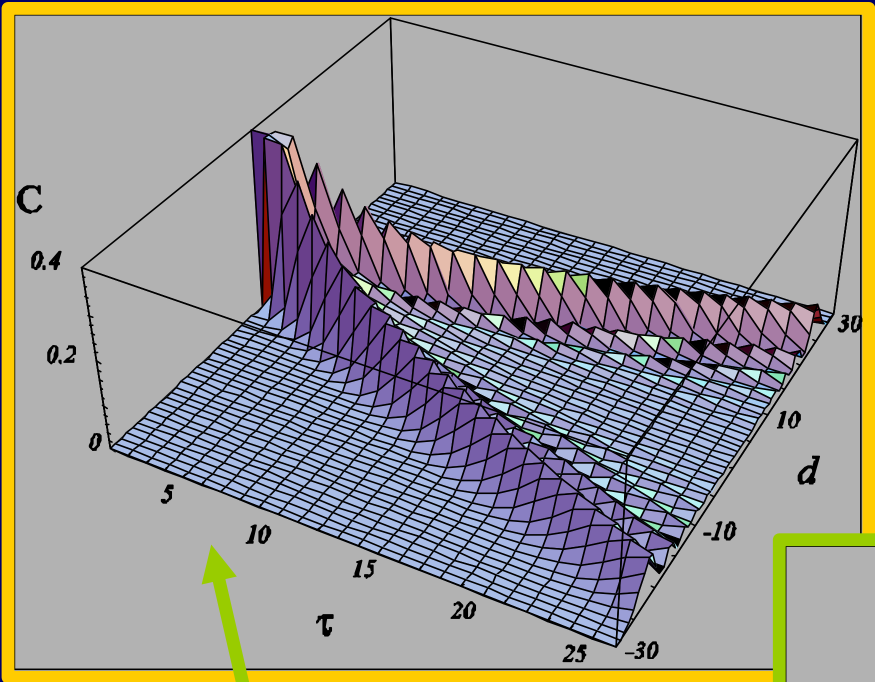


Entropy for sites (-6,7)
Fidelity to the initial singlet
1
0.8
0.6
0.4
0.2
10
20
τ



2D-XY model

γ=0 - XY model (singlet initial state)



Nearest neighbors

Symmetric sites