Coherent Control of Fractional Charges in Quantum Antidots

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thanks to:

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Bob Laughlin
Horst Stormer
Dan Tsui ...

Geometric Phase

- when Hamiltonian H(t) changes with time adiabatically:

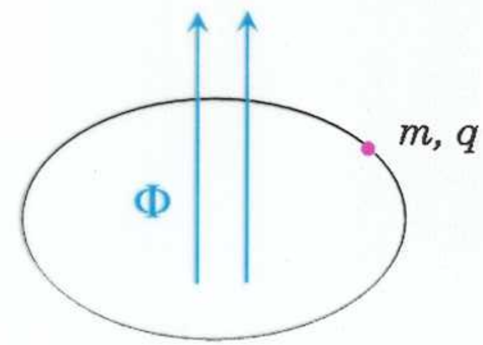
$$\Psi(\mathbf{r},t) = \psi(\mathbf{r},t)\exp\{i\theta(t)\}\exp\{i\gamma(t)\}$$, where $$H(t)\psi(\mathbf{r},t) = E(t)\psi(\mathbf{r},t)$$

dynamic $\theta(t) = -\frac{i}{\hbar}\int_0^t E(t')dt'$ geometric (Berry's) $\gamma(t) = i\int_{R(0)}^{R(t)} \langle\psi|\nabla_R\psi\rangle\cdot dR$

- Aharonov-Bohm effect:

$$H = \frac{1}{2m}(i\hbar\nabla + qA)^2 + qV$$

$$\gamma(T) = \frac{q}{\hbar}\oint A(r)\cdot dr = \frac{q}{\hbar}\int_\Sigma B\cdot dS = 2\pi\frac{q}{h}\Phi$$



magnetic flux Φ

define: flux quantum $\Phi_0 = h/e$, then $\gamma(T) = 2\pi\Phi/\Phi_0$

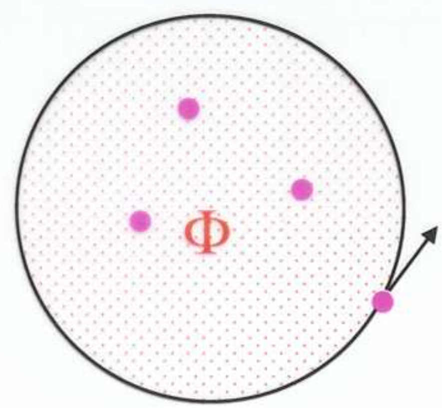
q = e

Fractional Statistics

- electrons: Ψ acquires phase factor $\exp\{i\pi\Theta\}$
  upon exchange $\Rightarrow$ $\Theta_e = 1$
  $(\text{exchange})^2$ = complete loop

Ψ acquires phase factor $\exp\{i2\pi\Theta N_e\}$ upon loop

$$\gamma(T) = \frac{e}{\hbar}\Phi + 2\pi\Theta_e N_e = 2\pi\left(\frac{\Phi}{\Phi_0} + \Theta_e N_e\right)$$



- ν=1/3 Laughlin quasihole:

$$\gamma(T) = \frac{q}{\hbar}\Phi + 2\pi\Theta_{qh} N_{qh}$$

since $q = e/3$, when flux changes by Φ_0 acquires phase $2\pi/3$

$\Rightarrow$ need $\Theta_{qh} = 1/3$ for single-valued Ψ

Leinaas, Myrheim 1977; F. Wilczek 1982

Halperin 1984; Arovas, Wilczek, Schrieffer 1984; W.P. Su 1986

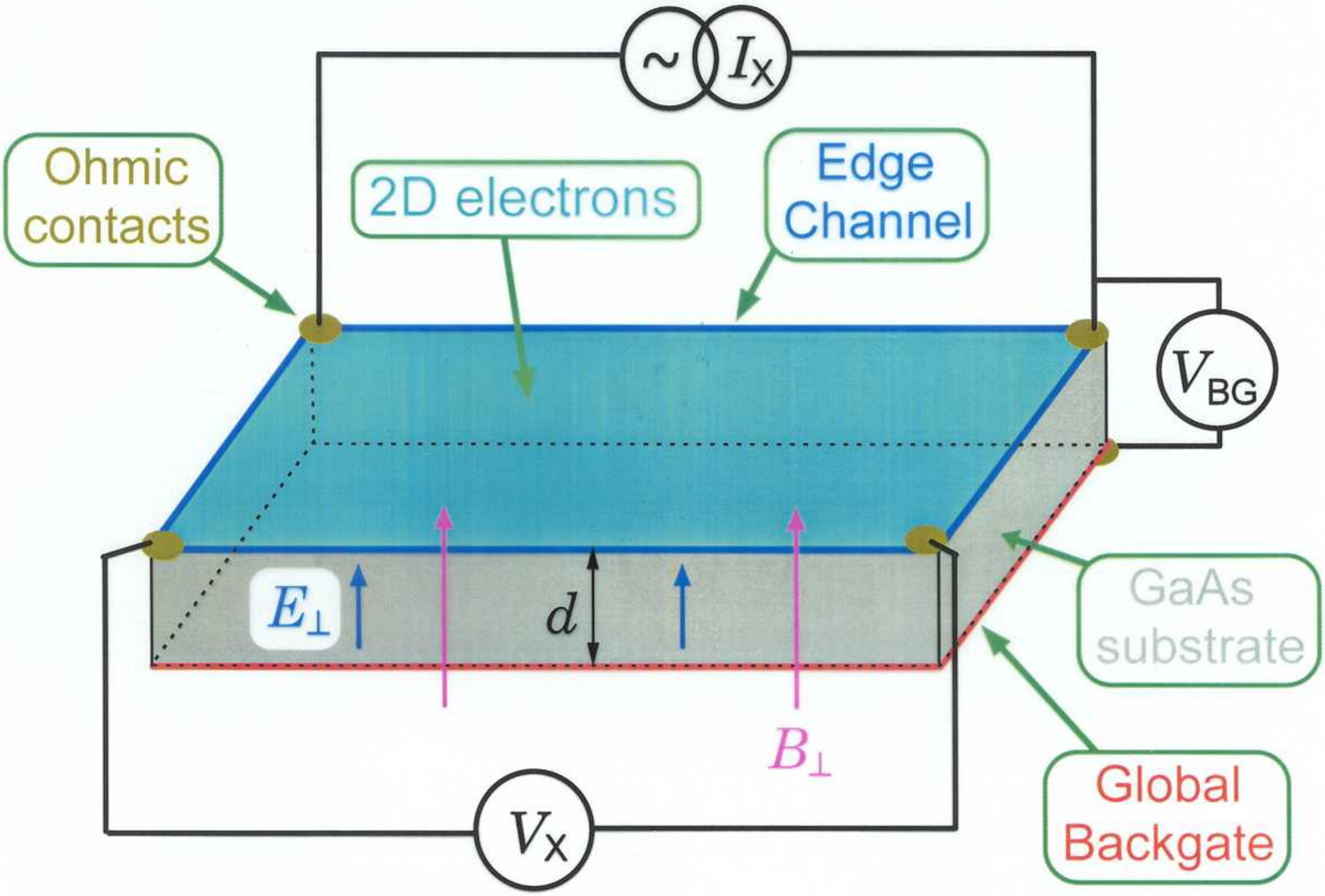
⇒ wish to explore *topological quantum computation* in 2D electron system in strong B

- adiabatic transport of particles leads to a unitary transformation of Ψ , which can be used to perform quantum logic

- topological nature of such quantum logic is expected to be more robust to environmental decoherence of Ψ , possibly be fault-tolerant

Vladimir J Goldman

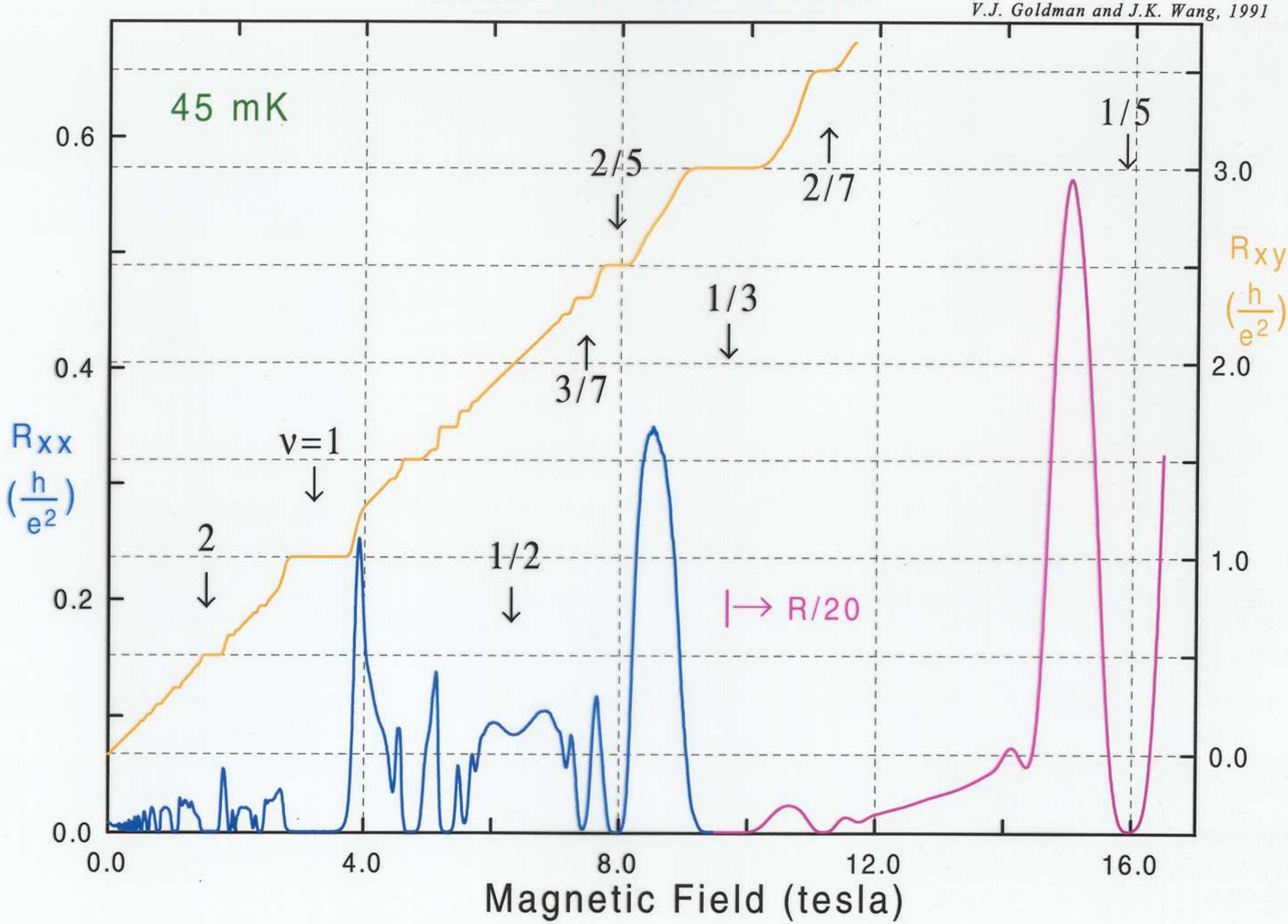
2D Electron System (2DES)



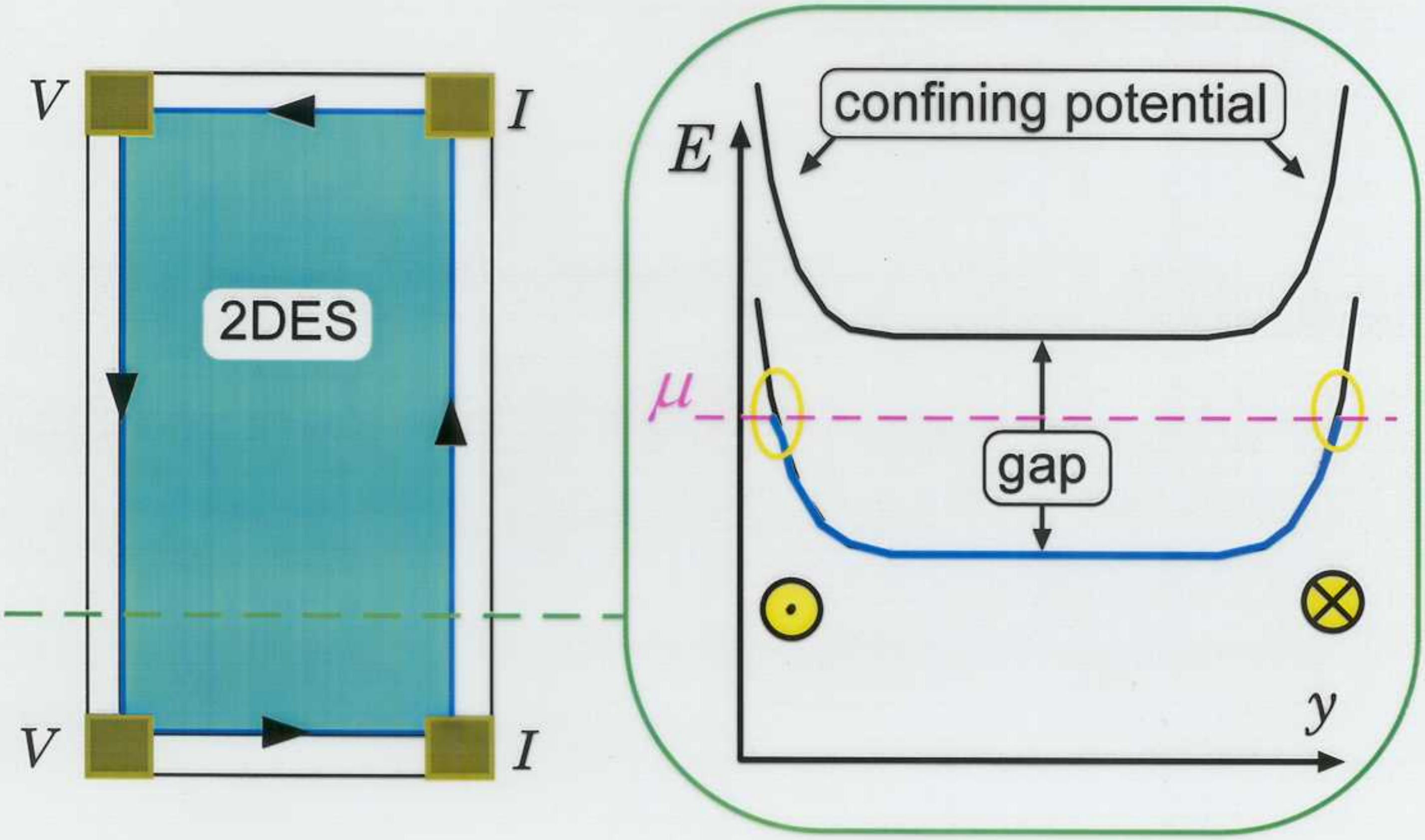
LL energy: $E_i = (i + ½)\,\hbar\omega_C$ *Landau '30*

LL filling factor: $\nu = \dfrac{h\,n}{e\,B} = \dfrac{n}{B\,/\,\phi_0} = \dfrac{\text{density of } e^-}{\text{density of } \phi_0}$

Quantum Hall Effect



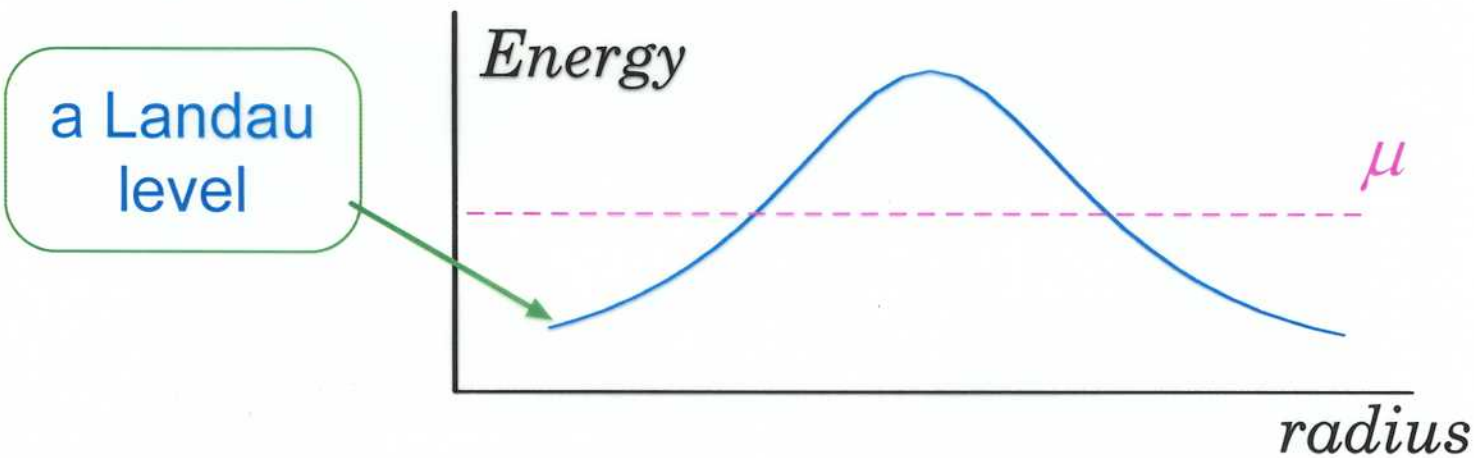
QHE Edge States



- on a QH plateau, in the limit $V \to 0$, $T \to 0$, no light, etc., all current is carried by edge states (2D bulk is localized)
- QH edge states are 1D chiral Luttinger liquids (χLL)
- edge excitation spectrum can be studied by tunneling

Halperin '82, Streda & MacDonald '86, Jain & Kivelson '88, Buttiker '88
X.G. Wen '91, Kane & Fisher '92, Moon et al. '93, Fendley et al. '95

Quantum Antidot

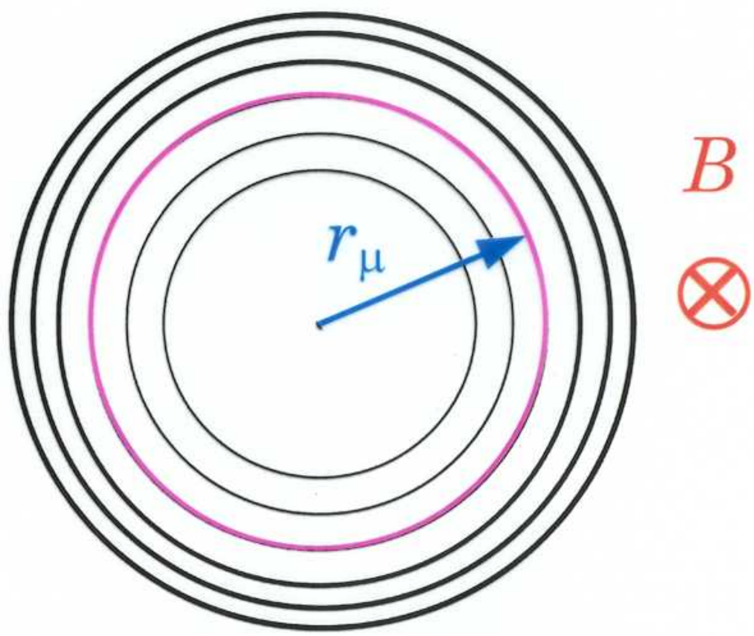


Aharonov-Bohm quantization

$$\oint A\cdot \mathrm{d}l = B\,S_\mu = m\,\varphi_0$$

$$m = 0,\ 1,\ 2,\ 3,\ \ldots$$

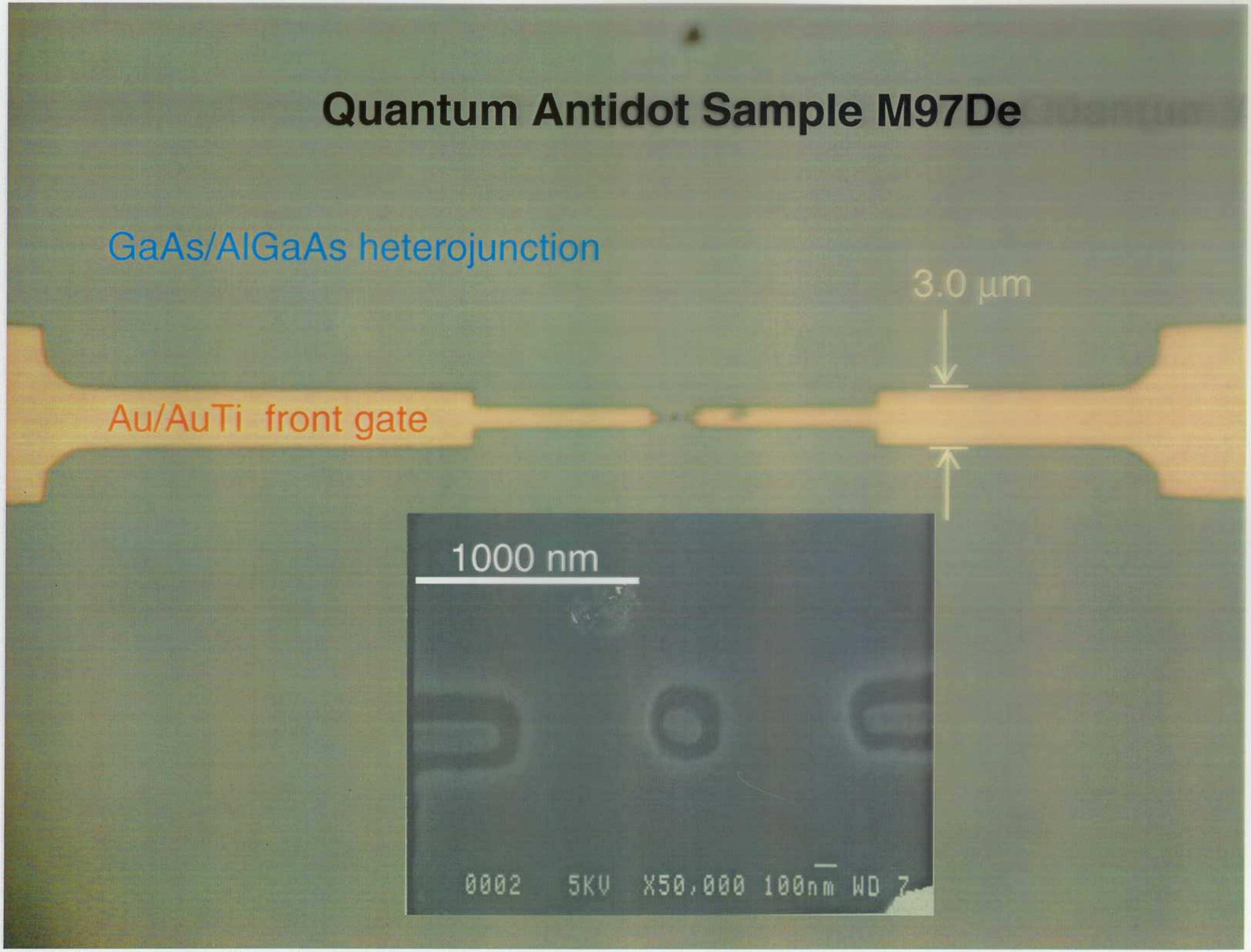
$$\varphi_0 = h/e$$



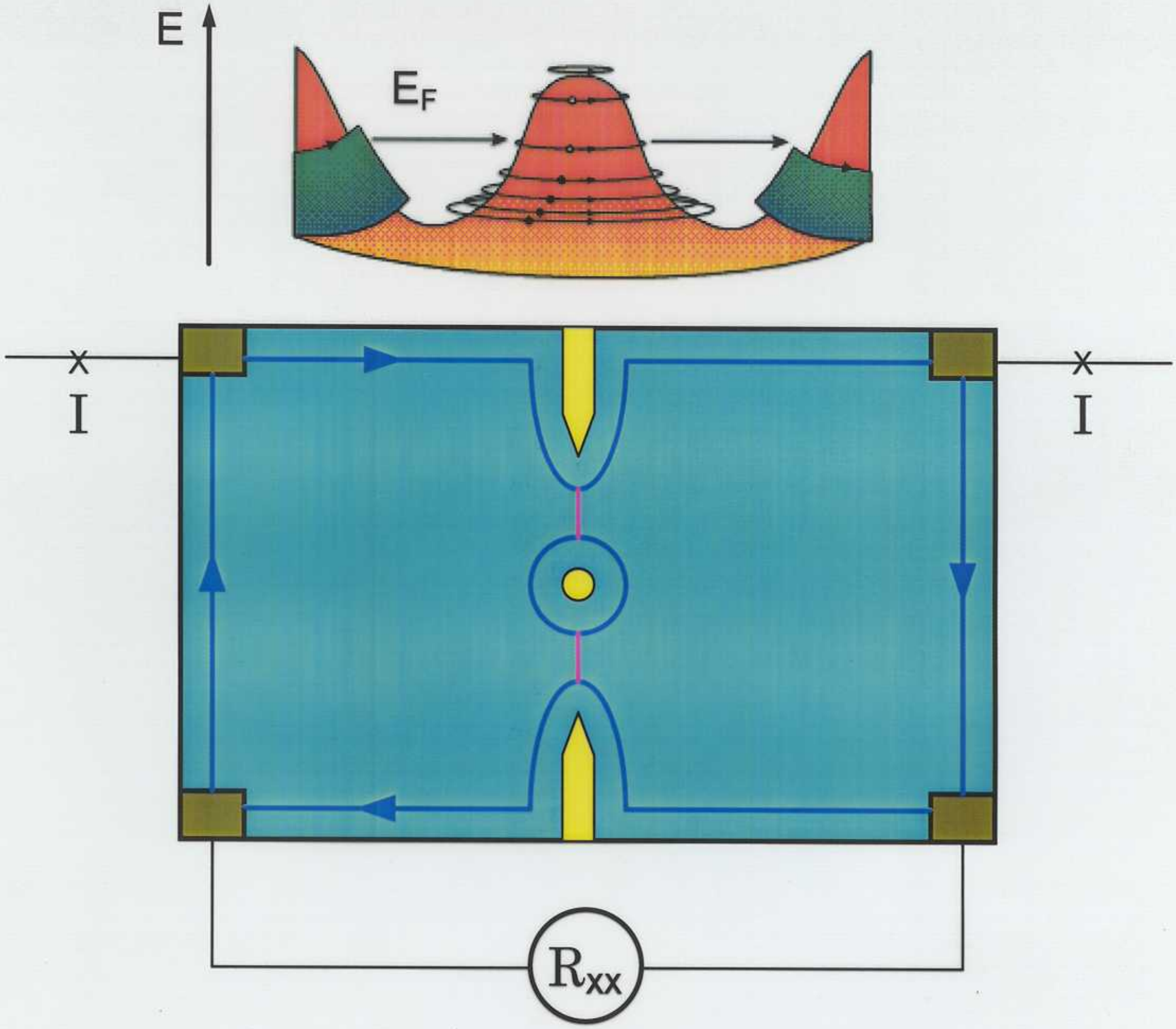
B-sweep: one state thrugh μ when $\Delta B = \varphi_0 / S_\mu$

$$\psi_\mathrm{m}(z) = \frac{z^\mathrm{m}}{(2\pi\, 2^\mathrm{m}\, m!)^{1/2}}\ \exp\left(-\frac{|z^2|}{4l_0^{\,2}}\right),\ z = r\mathrm{e}^{i\theta} = x + iy$$

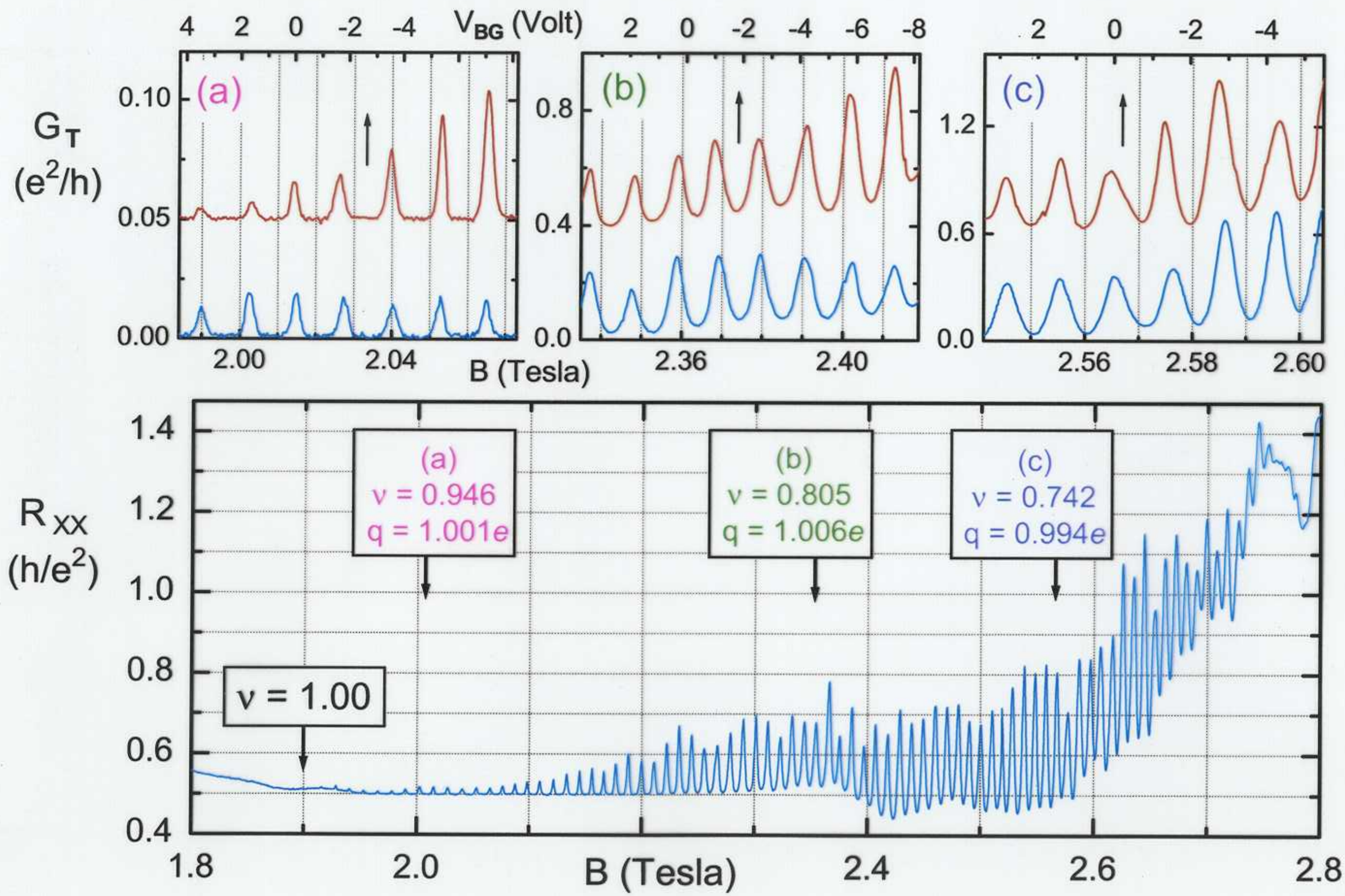
$$2\pi l_0^{\,2} B = \varphi_0$$


Quantum Antidot Sample M97De
GaAs/AlGaAs heterojunction
3.0 μm
Au/AuTi front gate
1000 nm
0002 5KV X50,000 100nm WD 7

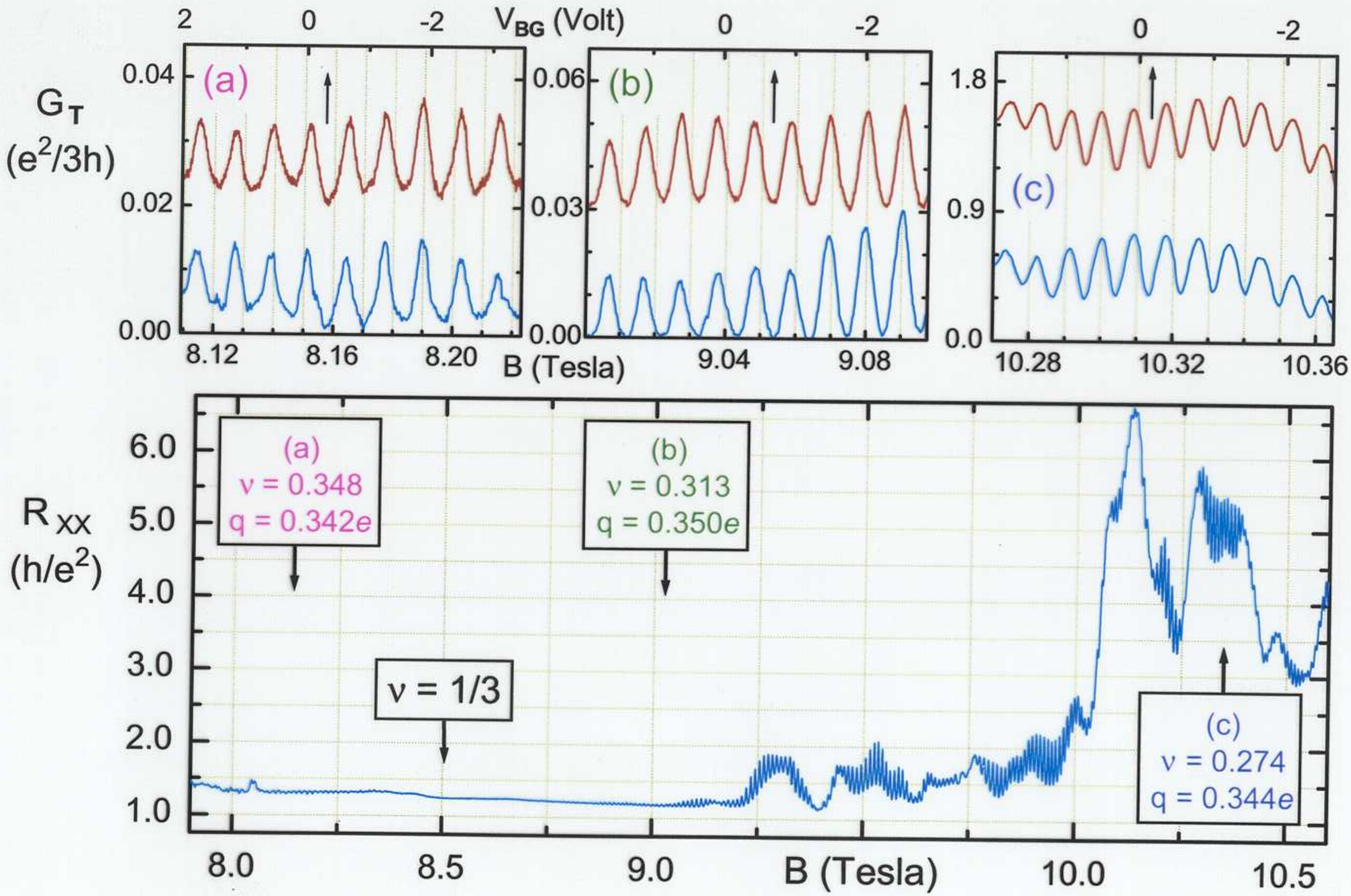
Resonant Tunneling via Quantum Antidot



$$G_{TUN} \approx \frac{R_{XX}}{R_{XY}^{2} - R_{XX} R_{XY}}$$

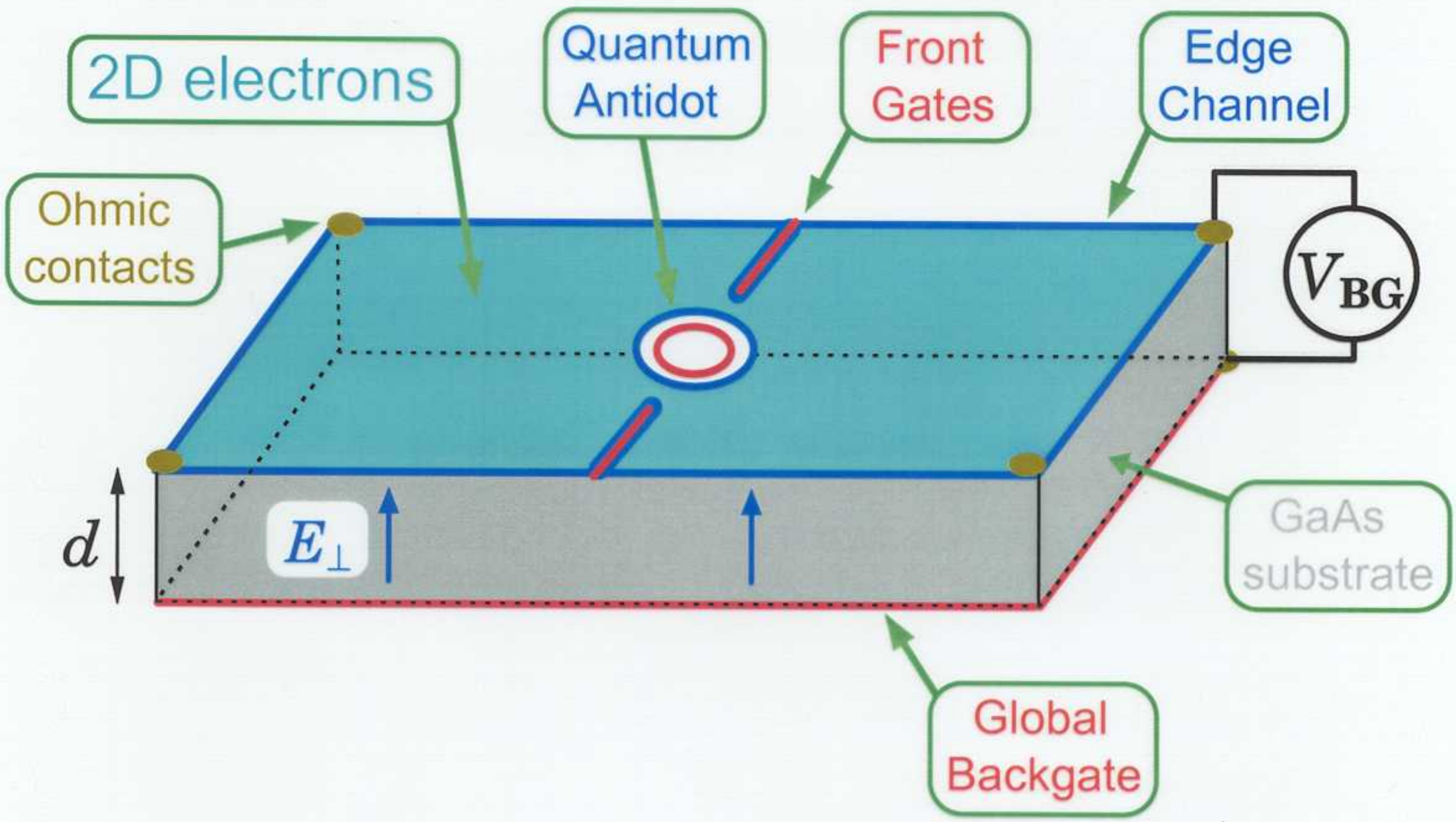


V_{BG} (Volt)
G_T (e^2/h)
(a)
(b)
(c)
B (Tesla)
R_{xx} (h/e^2)
(a) ν = 0.946 q = 1.001e
(b) ν = 0.805 q = 1.006e
(c) ν = 0.742 q = 0.994e
ν = 1.00
B (Tesla)

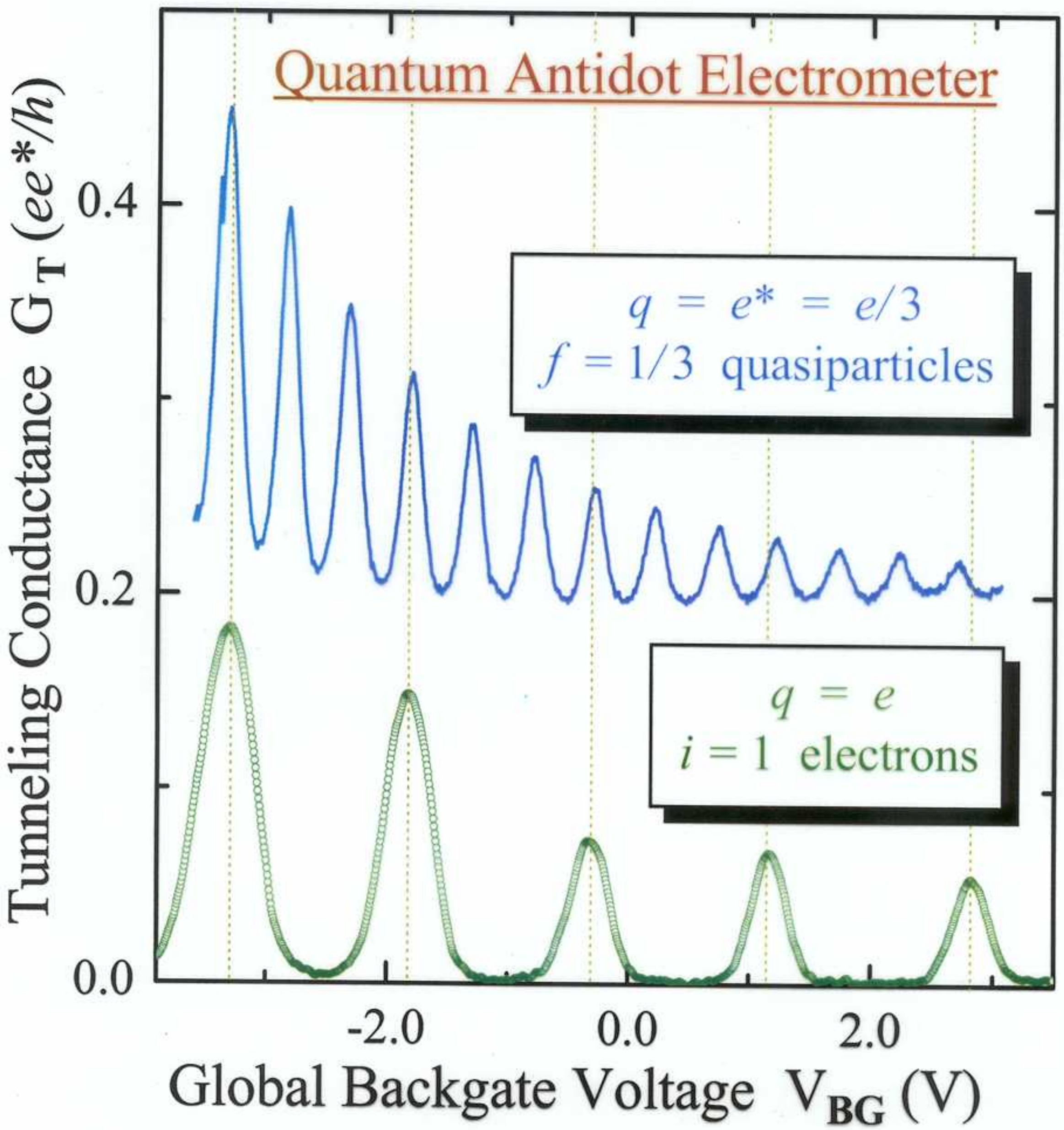


V_{BG} (Volt)
2
0
-2
G_T
($e^2/3h$)
0.04
0.02
0.00
(a)
8.12
8.16
8.20
B (Tesla)
0.06
0.03
0.00
(b)
9.04
9.08
1.8
0.9
0.0
(c)
10.28
10.32
10.36
R_{XX}
(h/e^2)
6.0
5.0
4.0
3.0
2.0
1.0
(a)
ν = 0.348
q = 0.342e
(b)
ν = 0.313
q = 0.350e
ν = 1/3
(c)
ν = 0.274
q = 0.344e
8.0
8.5
9.0
B (Tesla)
10.0
10.5

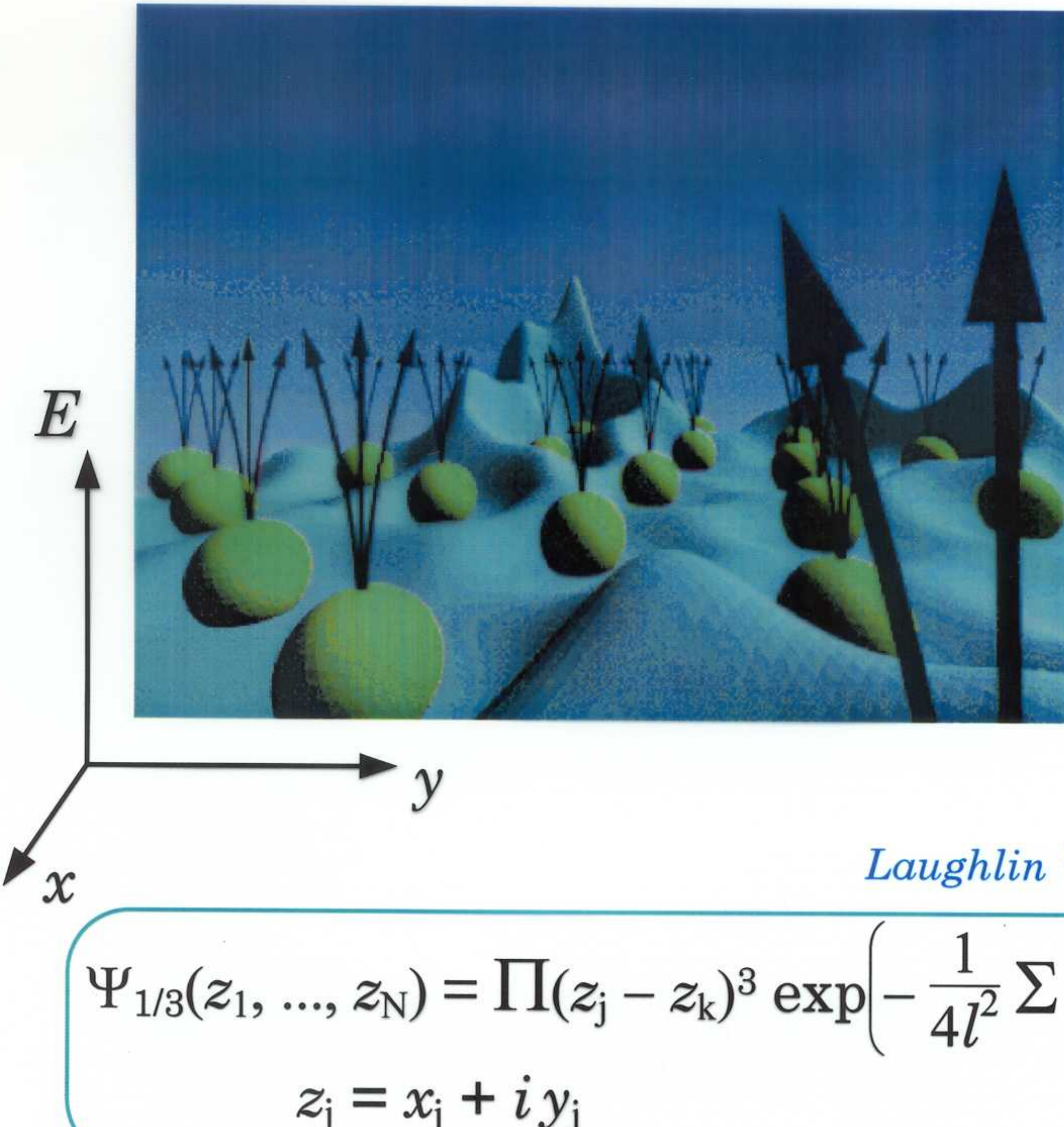
Quantum Antidot Electrometer



Goldman & Su '95
Goldman '96
Goldman '97


Quantum Antidot Electrometer
q = e* = e/3
f = 1/3 quasiparticles
q = e
i = 1 electrons
Tunneling Conductance G_T (ee*/h)
0.0
0.2
0.4
-2.0
0.0
2.0
Global Backgate Voltage V_BG (V)

2D Electrons in ν = 1/3 FQH State

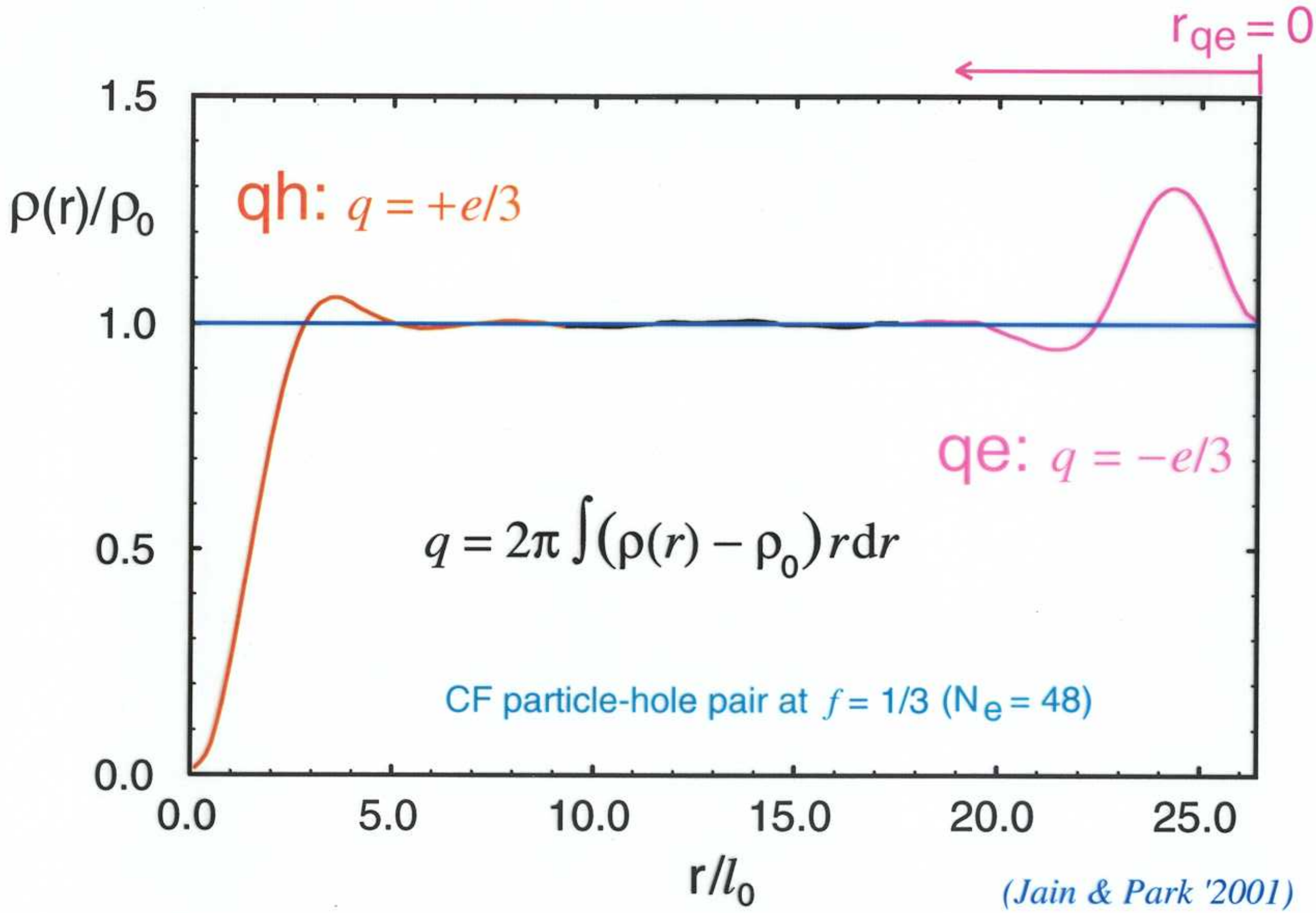


Laughlin '83

$$\Psi_{1/3}(z_1, \ldots, z_N) = \prod (z_j - z_k)^3 \exp\left(-\frac{1}{4l^2}\sum |z_j|^2\right)$$

$$z_j = x_j + i\,y_j$$

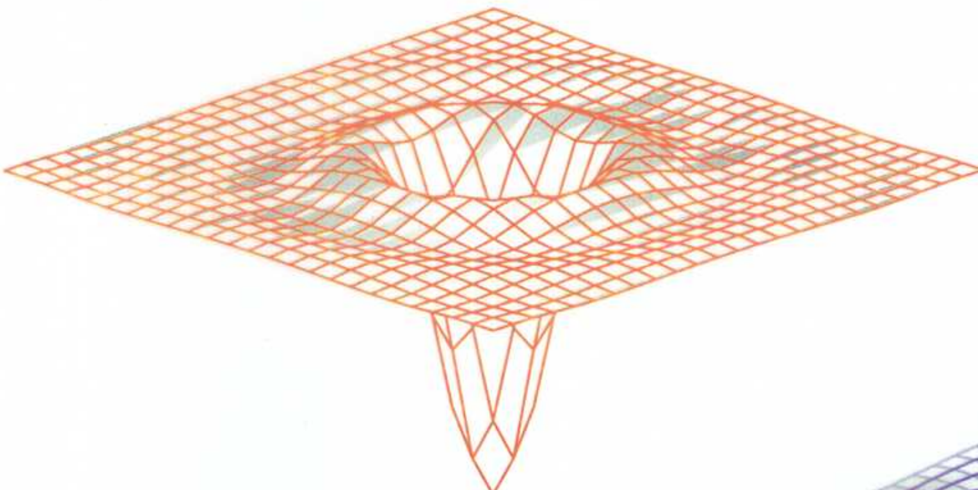
Quasiparticle and Quasihole Charge Density Profile



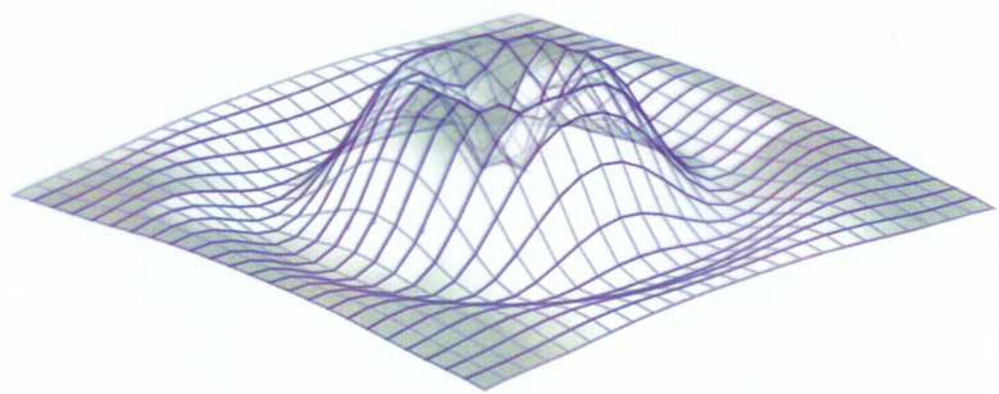
(Jain & Park '2001)

Quasihole and Quasielectron in f = 1/3 FQH Condensate

$q = +e/3$

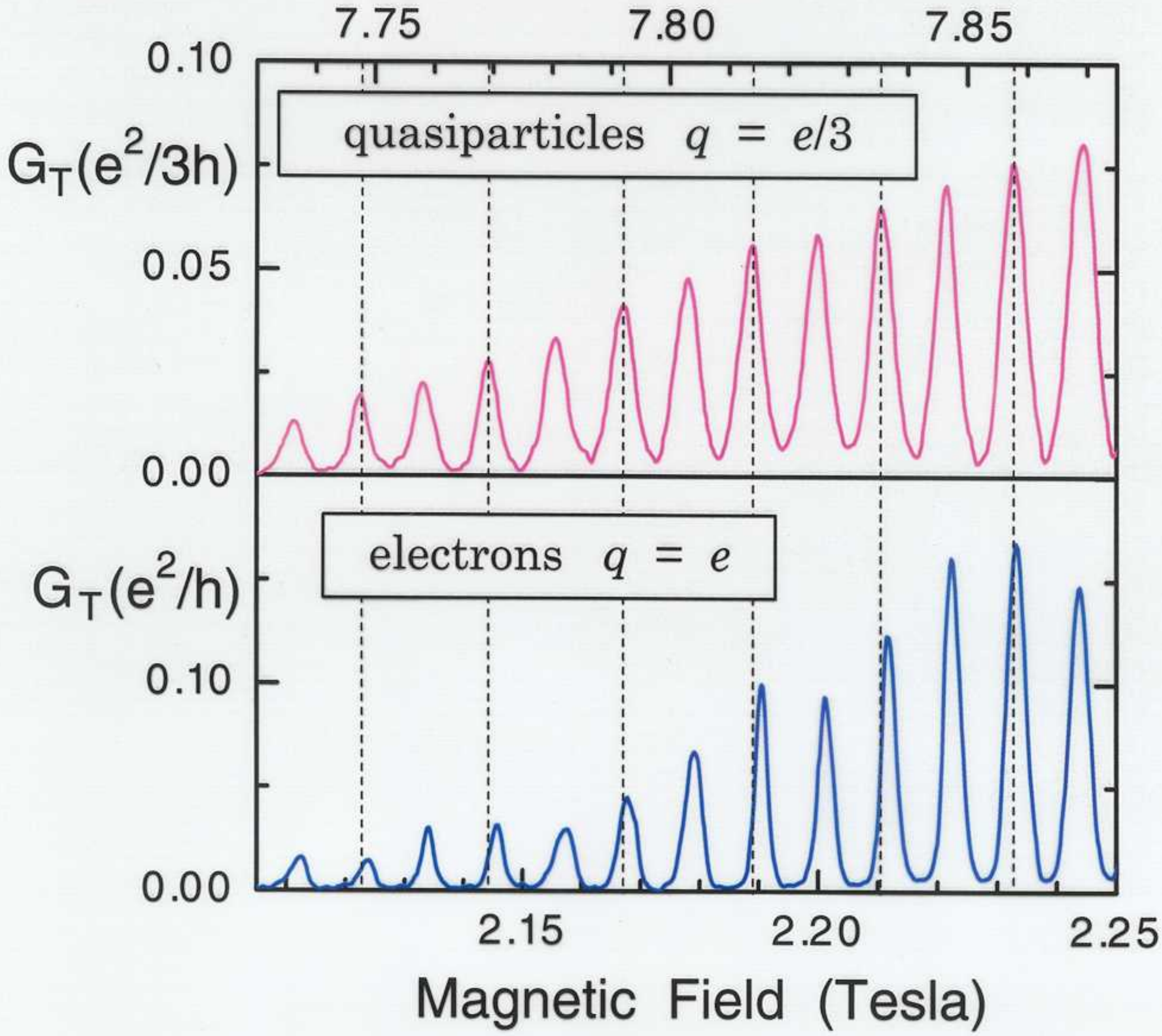


$q = -e/3$



(Park and Jain '01)

Aharonov-Bohm Periodicity



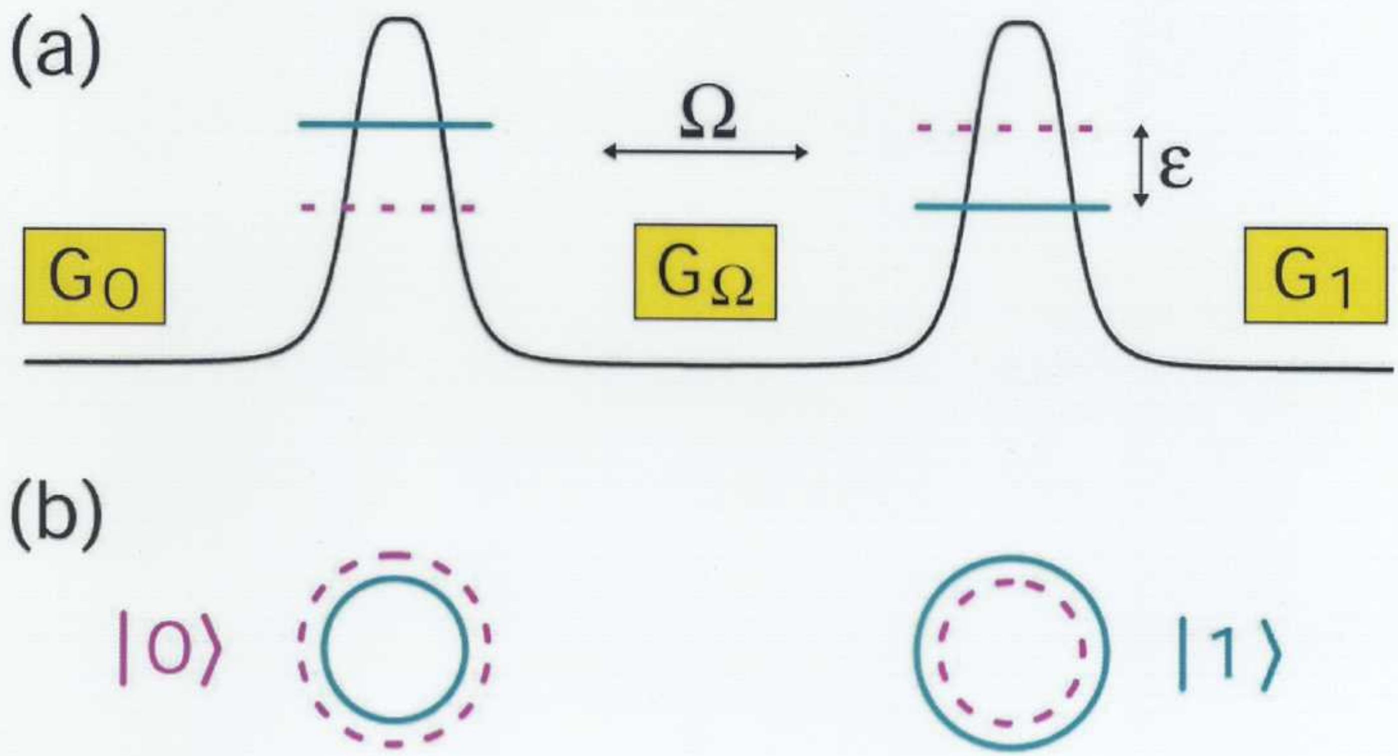
$\Delta B(\nu = 1) = \Delta B(\nu = 1/3)$ $\Rightarrow$ AB quantization $\varphi_0 = h/e$ for both

$\Rightarrow$ need fractional statistical phase for Laughlin qh's

Arovas, Schriefer, Wiczek '84, *Kivelson* '91, *Kjonsberg, Leinas, Myrheim* '99

Quantum Antidot Qubit

Proposed: Quantum AntiDot "molecule" qubit
(Averin and Goldman, 2001)



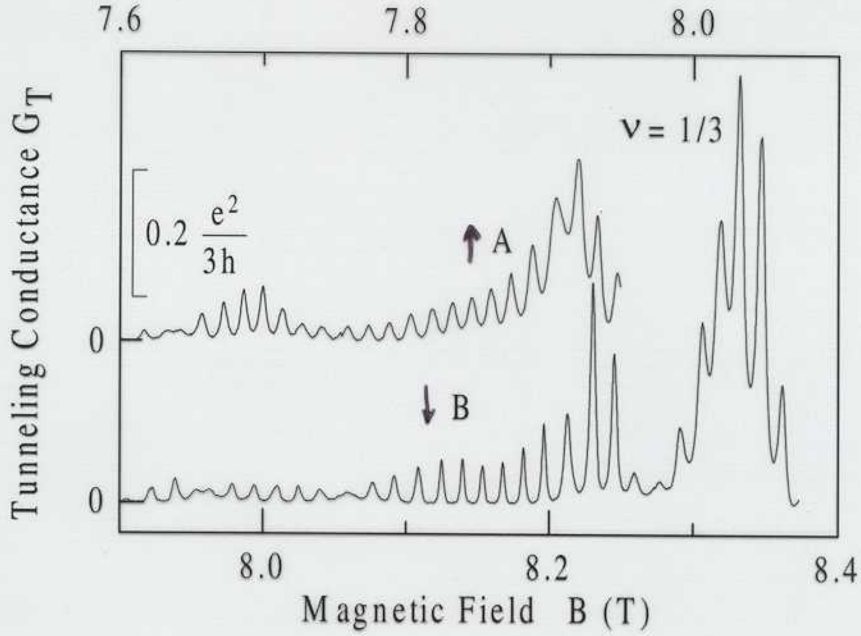
G_j -- are control gates

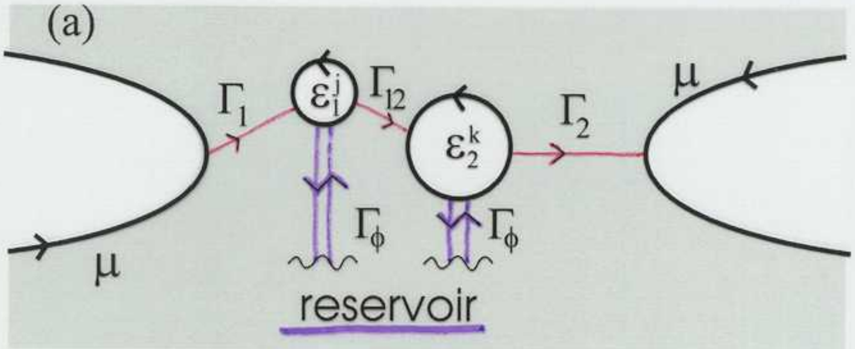
Ω -- inter QAD tunneling rate

ε -- quasiparticle "localization" energy

need: $T << \Omega, \varepsilon << \Delta E$, qp excitation gap

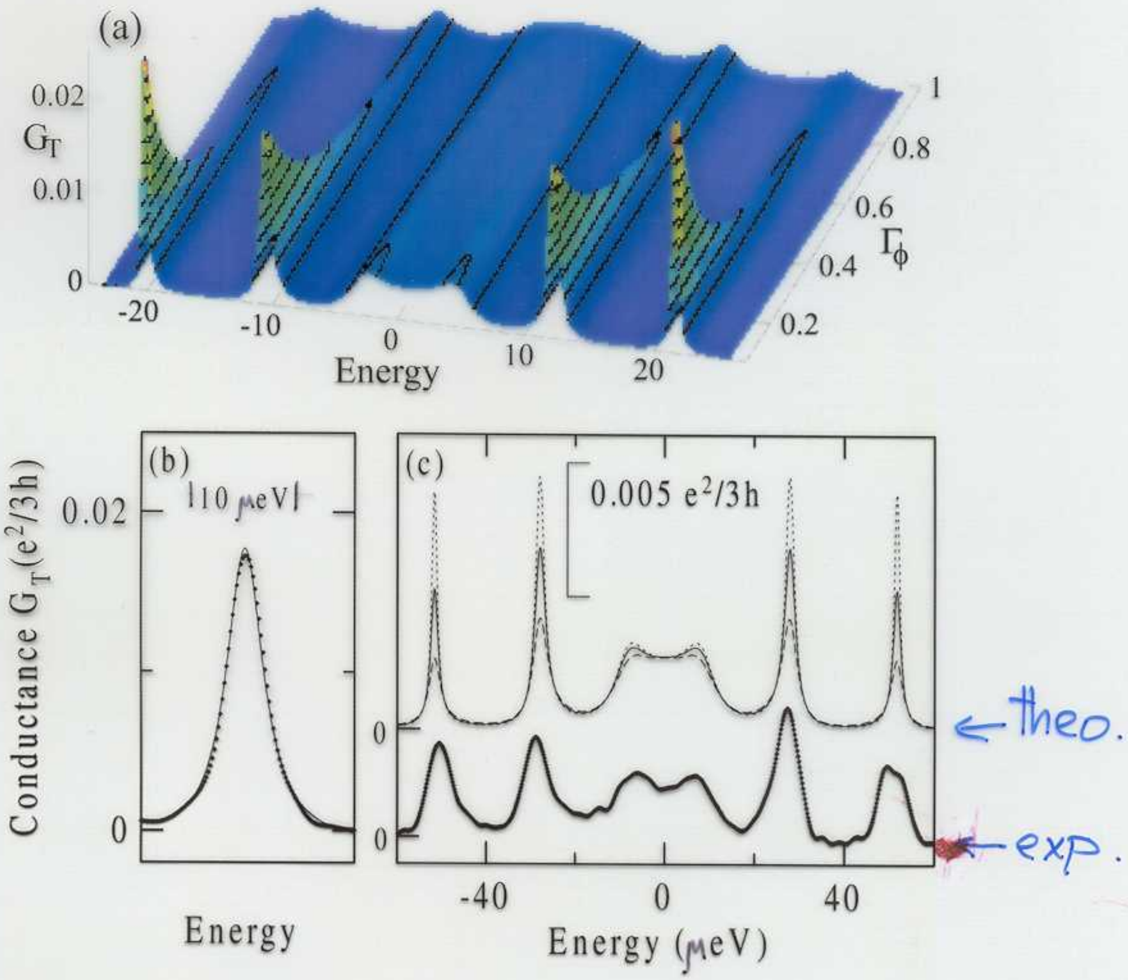
Quantum Antidot Molecule





Maasilta and Goldman, 2000

Quantum Antidot Molecule



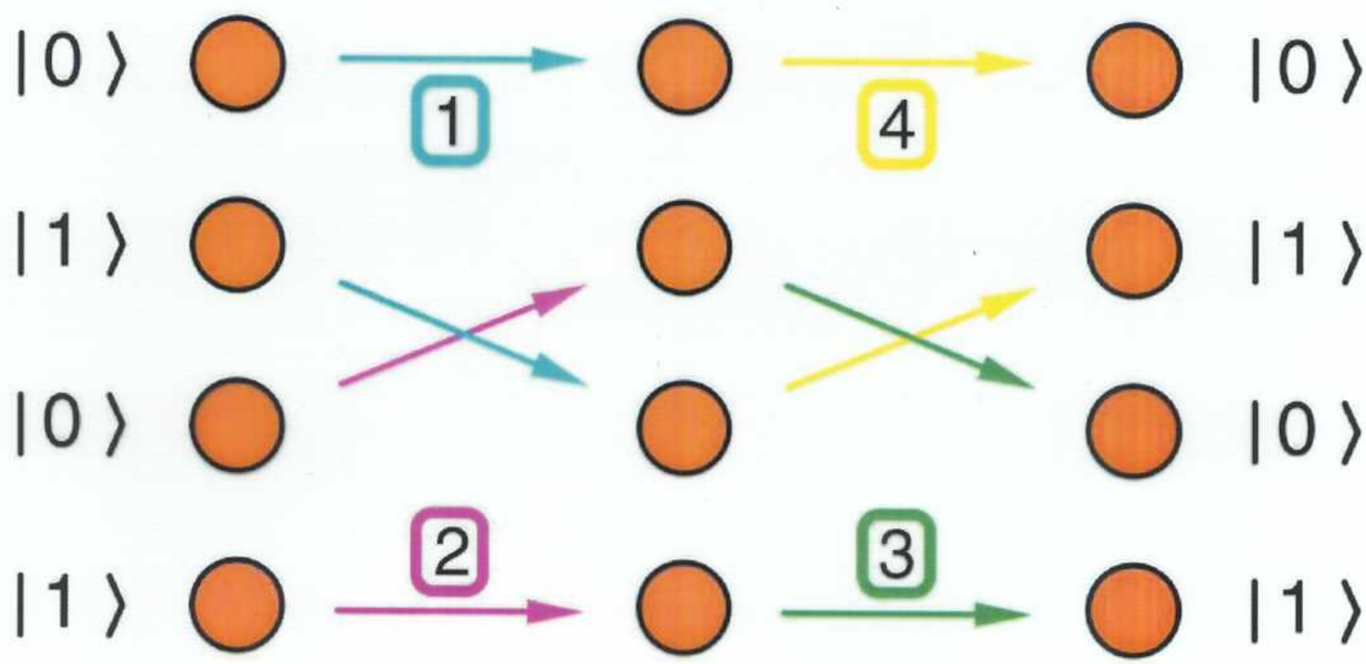
FQH quasiparticles coherence time: $\tau_\varphi \approx 1$ ns

coherence length: $L_\varphi \approx 30\ \mu m$

Maasilta and Goldman, 2000

Quantum Antidot Logic

Proposed: QAD controlled-phase two-qubit gate based on adiabatic transport of FQH quasiparticles (Averin and Goldman, 2001)



transformation matrix: $P = \mathrm{diag}\{1,1,1,\exp(i2\pi/3)\}$

- C-NOT gate can be obtained from C-phase gate and one-qubit transformations