Demonstration of Rabi Oscillations in a Josephson Tunnel Junction

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Quantum Computing with Superconducting Qubits

- Basic types of superconducting Josephson devices:

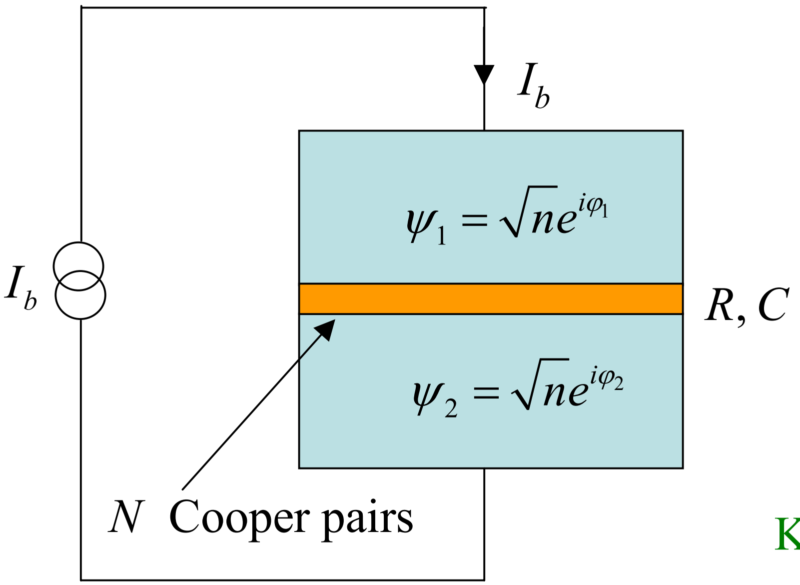
  a. Charge, e.g., Cooper pair box.

  b. Flux, e.g., RF SQUIDs, 3-JJ persistent current qubits.

  c. Phase, e.g., Josephson tunnel junctions.

- As qubits:
  1. Solid state approach leads to better scalability.
  2. Engineered Hamiltonians are easy to control.
  3. Long coherence time (compare to other solid state approaches).
  4. Very easy initial state preparation.
  5. Qubit rotation using microwave.
  6. Adjustable interaction between qubits.
  7. Non-invasive (in a practical sense) single-shot state detection.
  8. Coupling to flying qubits via cavity QED.

Josephson Tunnel Junction



“Potential Energy”:

$$U_J = -E_J \cos\varphi, \quad \varphi \equiv \varphi_2 - \varphi_1$$

$$E_J \equiv \frac{I_c \Phi_0}{2\pi}, \qquad \varphi \Rightarrow \text{"position"}$$

“Kinetic Energy”:

$$\text{KE} = E_c = \frac{Q^2}{2C}, \quad C \Rightarrow \text{"mass"}; \quad Q = 2eN$$

$$Q \Rightarrow \text{"momentum"}$$

Josephson Effect (1962)

$$I_s = I_c \sin\varphi, \qquad \text{where } I_c \equiv E_J / (\Phi_0 / 2\pi)$$

$$V = \frac{\Phi_0}{2\pi} \frac{\partial\varphi}{\partial t}$$

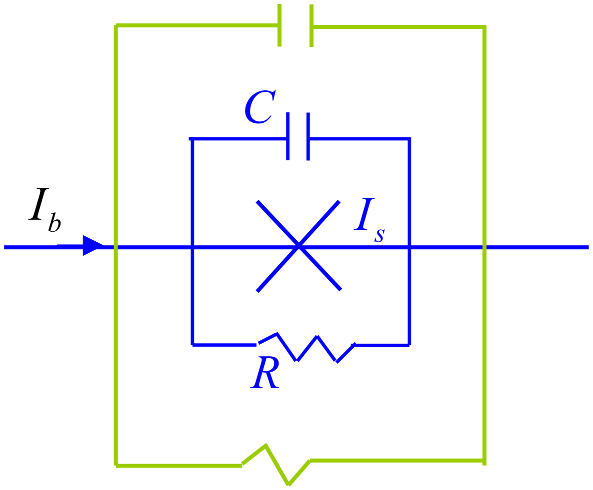
$$[\varphi, N] = i$$

$$H = E_c + U_J + U_{ex}$$

U_{ex} depends on the circuit

Classical Dynamics of a Current Biased Josephson Junction

RSJ model



Current bias: $U_{ex} = -\frac{\Phi_0}{2\pi}\varphi I_b$

Conservation of current:

$$C\frac{dV}{dt} + \frac{V}{R} + I_s = I_b$$

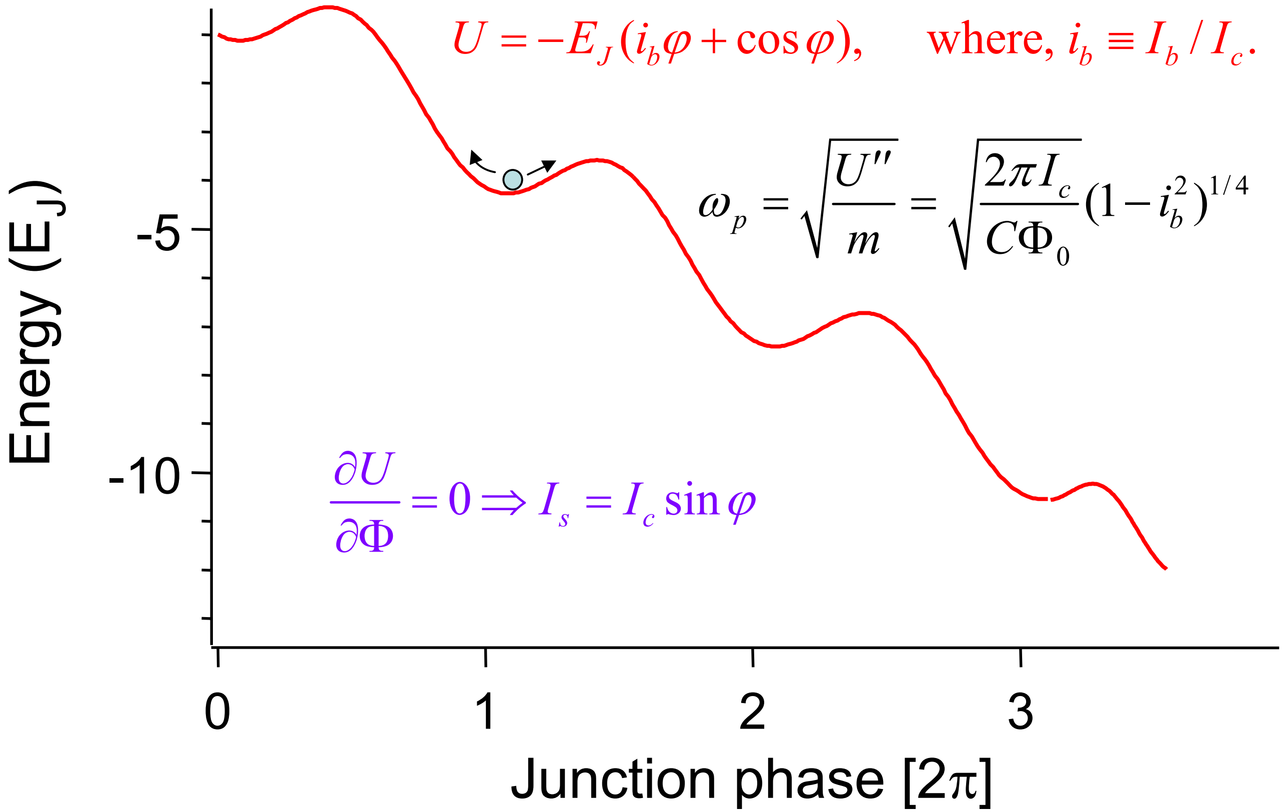
1st and 2nd Josephson equations

$$C\frac{\partial^2\Phi}{\partial t^2} + R^{-1}\frac{\partial\Phi}{\partial t} = I_b - I_c \sin\left(2\pi\frac{\Phi}{\Phi_0}\right)$$

where, $\Phi \equiv (\varphi/2\pi)\Phi_0$

Mechanical model of a Josephson junction
- a "particle" in a tilted washboard potential U(Φ)

$$C\ddot{\Phi} + R^{-1}\dot{\Phi} = -\frac{\partial U}{\partial \Phi},$$

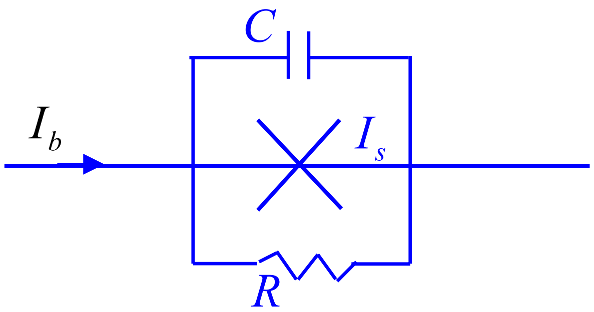


$$U = -E_J(i_b\varphi + \cos\varphi), \quad \text{where, } i_b \equiv I_b / I_c.$$

$$\omega_p = \sqrt{\frac{U''}{m}} = \sqrt{\frac{2\pi I_c}{C\Phi_0}}(1 - i_b^2)^{1/4}$$

$$\frac{\partial U}{\partial \Phi} = 0 \Rightarrow I_s = I_c \sin\varphi$$

Electrical model of a JJ
- a nonlinear LCR resonantor



JJ as a nonlinear inductor:

$$V = L_J \frac{dI_s}{dt} = L_J I_c \cos\varphi \frac{d\varphi}{dt}$$

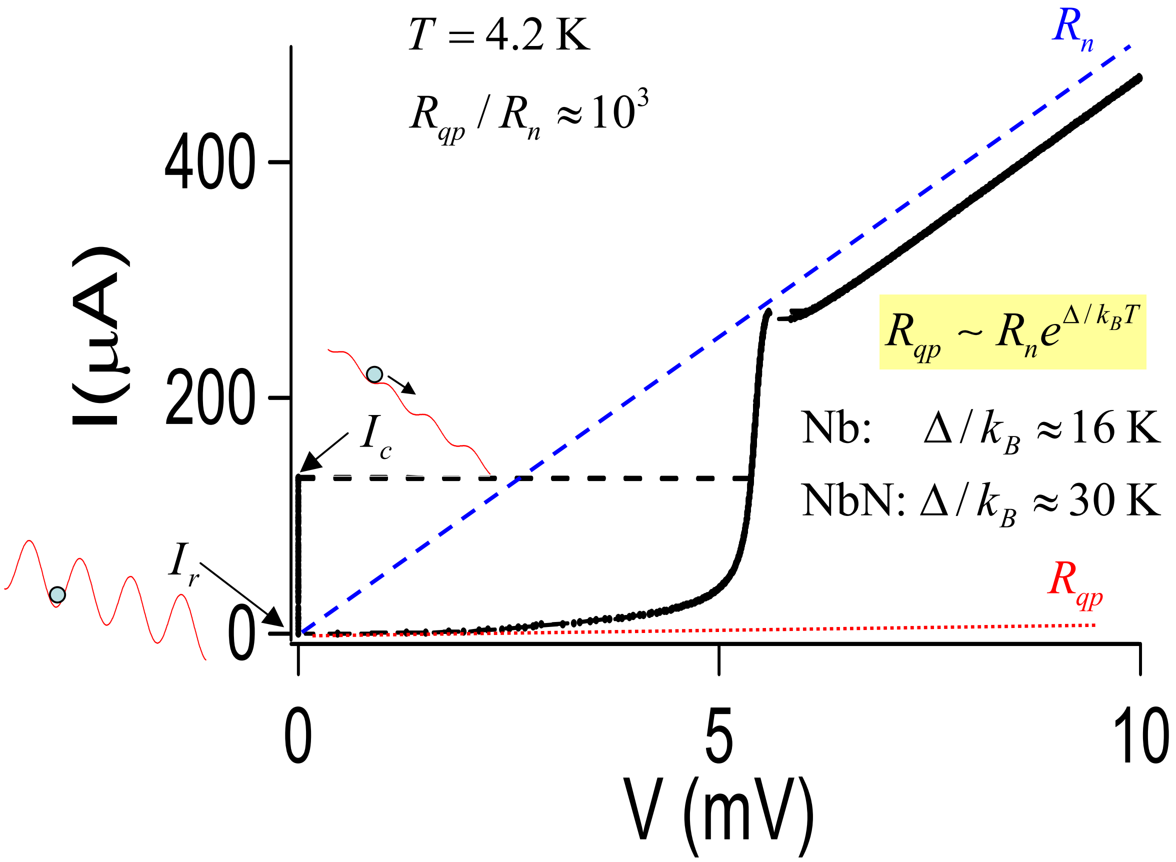
2nd Josephson equation ⟹ $\boxed{L_J = \frac{\Phi_0}{2\pi I_c \cos\varphi}}$

For $I_b < I_c$, $\Longrightarrow$ $I_b = I_s = I_c \sin\varphi$, $\Longrightarrow$ $\sin\varphi = i_b$ $\Longrightarrow$ $\cos\varphi = (1 - i_b^2)^{1/2}$

Resonance frequency: $\omega_p = \frac{1}{\sqrt{L_J C}} = \sqrt{\frac{2\pi I_c}{C\Phi_0}}(1 - i_b^2)^{1/4}$

Quality factor: $\boxed{Q = RC\omega_p}$

I-V Curve of a Josephson Tunnel Junction



$T = 4.2\ \mathrm{K}$

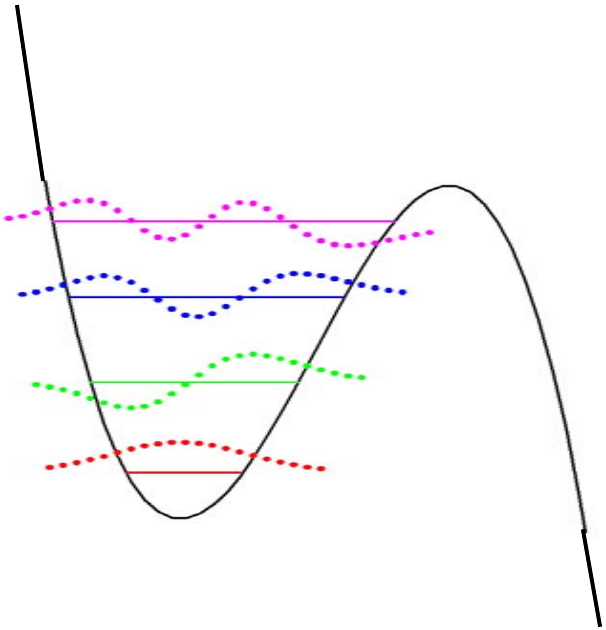
$R_{qp} / R_n \approx 10^3$

$R_{qp} \sim R_n e^{\Delta / k_B T}$

Nb: $\Delta / k_B \approx 16\ \mathrm{K}$

NbN: $\Delta / k_B \approx 30\ \mathrm{K}$

Quantum mechanical description



Phase qubit Hamiltonian:

$$H = \frac{\hat{Q}^2}{2C} - E_J \cos(2\pi\hat{\Phi}/\Phi_0) - I_b(t)\hat{\Phi}$$

$$\begin{aligned} I_b(t) &= I_{dc} + \delta I(t) \\ &= I_{dc} + \delta I_{dc}(t) + I_{mwc}(t)\cos\omega_{10}t + I_{mws}\sin\omega_{10}t \end{aligned}$$

Cubic potential: $\omega_{10} > \omega_{21} > \omega_{32} \ldots$

⬇

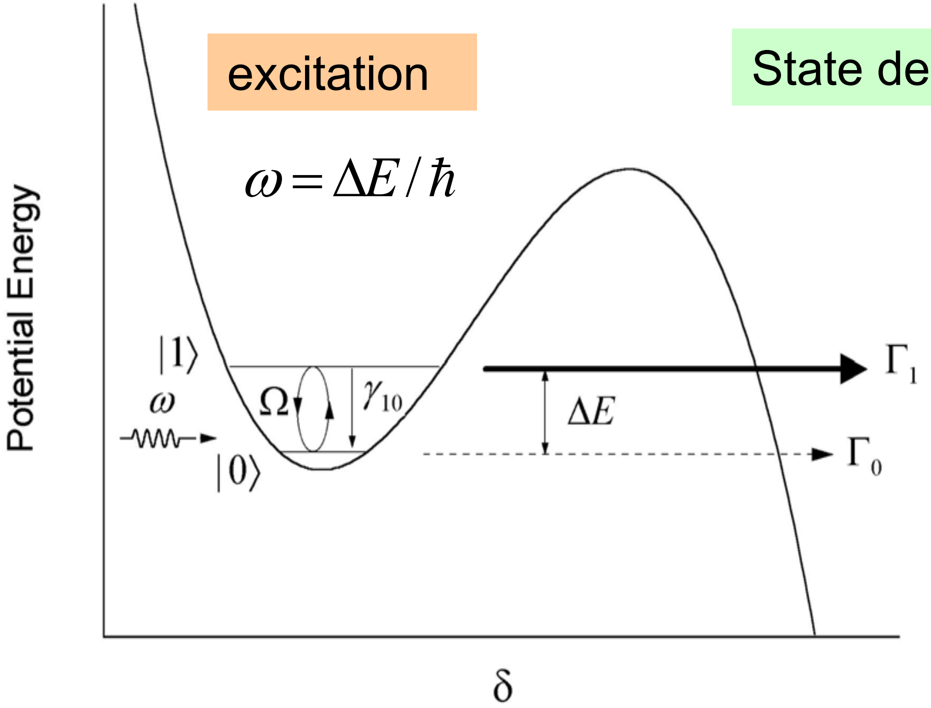
Two-level approximation:

$$H = \begin{pmatrix} E_0 + \hat{\Phi}_{00}\delta I & \hat{\Phi}_{01}\delta I \\ \hat{\Phi}_{10}\delta I & E_1 + \hat{\Phi}_{11}\delta I \end{pmatrix}, \qquad \text{where, } \hat{\Phi}_{ij} \equiv \langle i | \hat{\Phi} | j \rangle$$

Rabi oscillation in a metastable two-level system

Excitation and detection

- Microwave excites Rabi oscillations between the two levels.
- Excited state detected by monitoring tunneling events.



excitation

$$\omega = \Delta E / \hbar$$

State detection: escape from V=0 state

$$\Gamma_{n+1} / \Gamma_n \simeq 10^3$$

Γ_n – tunneling rate from level $|n\rangle$

Fidelity of state detection

$\sim 99.9\%$

Liouville equation: $i\hbar\dot{\rho}(t) = [\hat{H}(t), \rho(t)] + i\hbar[\hat{R}\rho(t)]$

$$H(t) = \begin{pmatrix} E_0 & \hbar\Omega_0 \cos\omega t \\ \hbar\Omega_0 \cos\omega t & E_1 \end{pmatrix},$$

$$[\hat{R}\rho(t)]_{\alpha\alpha} = -\Gamma_\alpha \rho_{\alpha\alpha} + \sum_{\alpha\neq\beta} \gamma_{\beta\alpha}\rho_{\beta\beta} \qquad (\mathrm{T}_1 \text{ process})$$

$$[\hat{R}\rho(t)]_{\alpha\beta} = -\Gamma_{\alpha\beta}\rho_{\alpha\beta}, \qquad \alpha \neq \beta \qquad (\mathrm{T}_2 \text{ process})$$

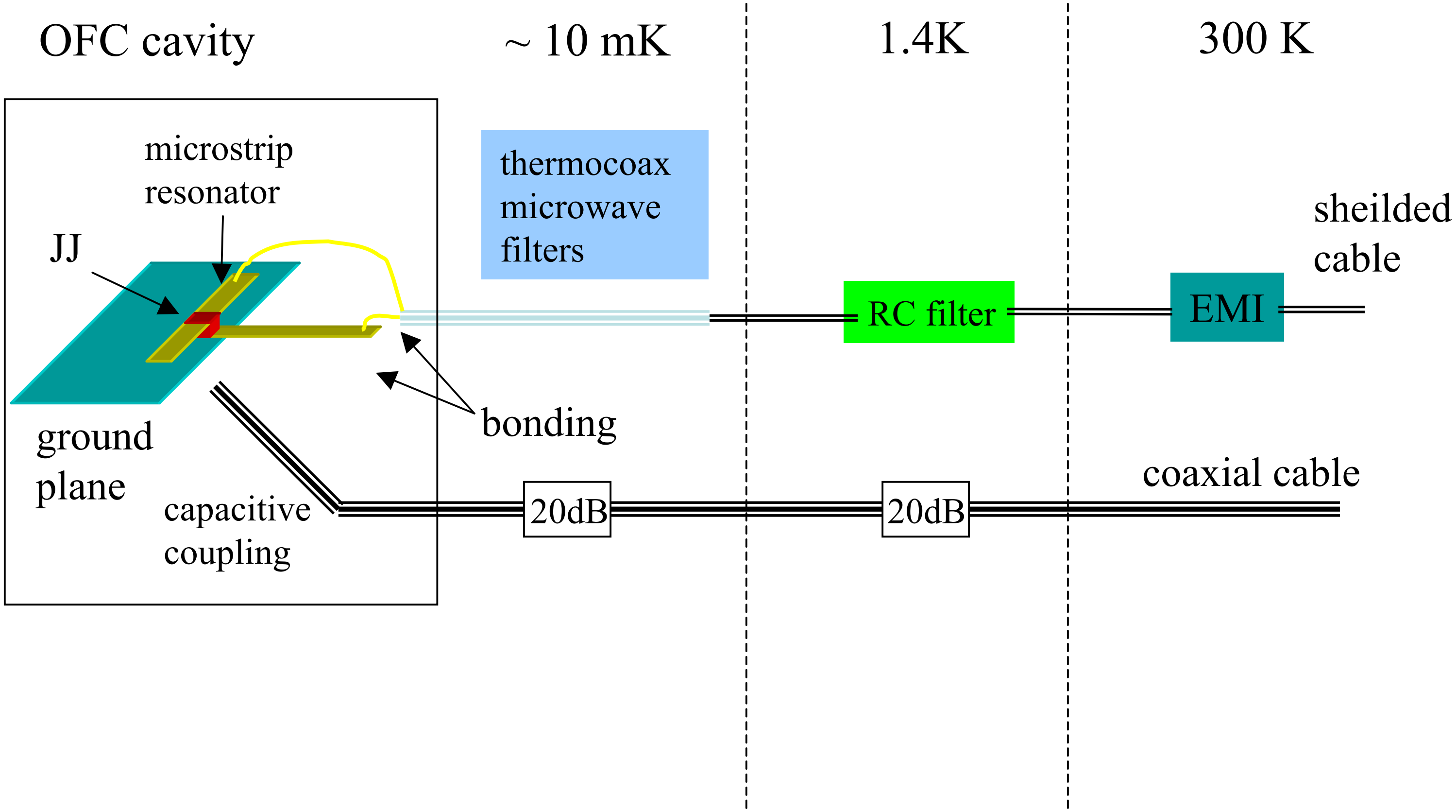
$$U = \rho_{01} + \rho_{10}, \quad V = i(\rho_{10} - \rho_{01}), \quad W = \rho_{11} - \rho_{00}, \quad S = \rho_{00} + \rho_{11}.$$

$$\begin{aligned}
\dot{U} &= \Delta V(t) - \Gamma U(t), \\
\dot{V} &= -\Delta U(t) - \Gamma V(t) + \Omega W(t), \\
\dot{W} &= -\tfrac{1}{2}(\Gamma_1 + 2\gamma_{10} + \Gamma_0)W(t) - \Omega V(t) - \tfrac{1}{2}(\Gamma_1 + 2\gamma_{10} - \Gamma_0)S(t), \\
\dot{S} &= -\tfrac{1}{2}(\Gamma_1 + \Gamma_0)S(t) - \tfrac{1}{2}(\Gamma_1 - \Gamma_0)W(t),
\end{aligned}$$

$\Omega \equiv \sqrt{\Omega_0^2 - (\Gamma - i\Delta)^2}$, Rabi frequency. Total off-diaganol decay rate: $\Gamma_{\alpha\beta} = \frac{1}{2}(\Gamma_\alpha + \Gamma_\beta + \gamma_{\alpha\beta}) + \gamma_\varphi$

Upper level population: $\rho_{11}(t) = e^{-\Gamma_{10}t}\dfrac{\Omega_0^2}{|\Omega^2|}\left|\sin\left(\dfrac{\Omega t}{2}\right)\right|^2, \quad \text{for } \Gamma_1 > \gamma_{10} > \Gamma_0, \gamma_\varphi$

Experimental Setup



OFC cavity

~ 10 mK

1.4K

300 K

microstrip
resonator

thermocoax
microwave
filters

sheilded
cable

JJ

RC filter

EMI

bonding

ground
plane

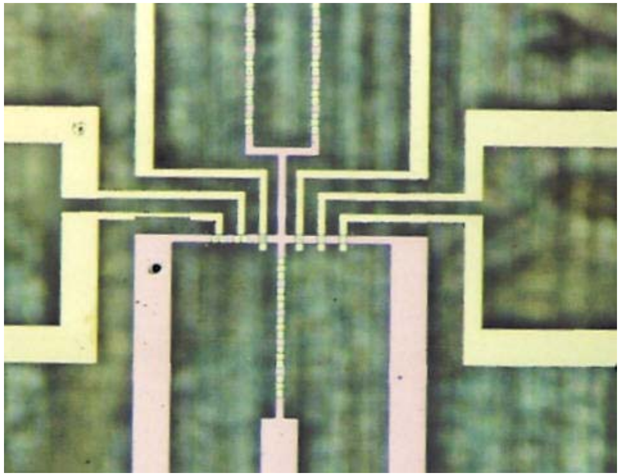
coaxial cable

capacitive
coupling

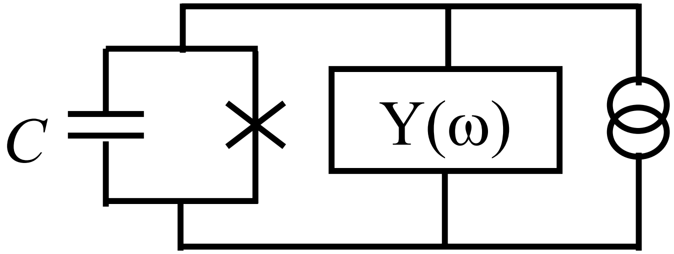
20dB

20dB

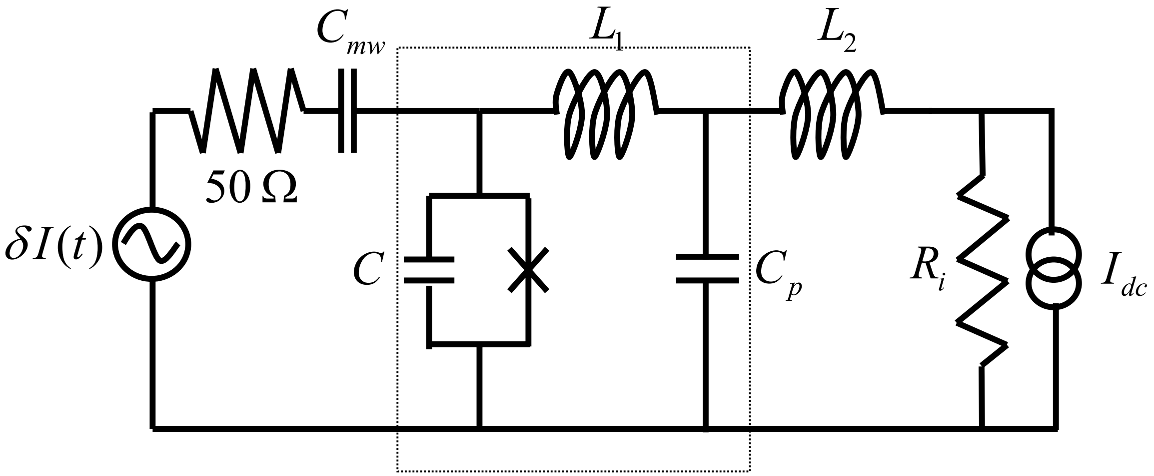
Sample and Equivalent Circuit



$10 \times 10\ \mu\text{m}^2$,
$I_{c0} \simeq 150\ \mu\text{A}$,
$C \approx 5.8$ pF.



Energy relaxation time:
$T_1 = C / \mathrm{Re}[Y(\omega_{10})]$



$L_1 \approx 6$ nH, $C_p \approx 0.4$ pF,
$L_2 \approx 10$ nH, $R_i \approx 100$ kΩ

→

$\mathrm{Re}[Y(\omega)] \simeq 10^{-7} - 10^{-8}\ \Omega^{-1}$

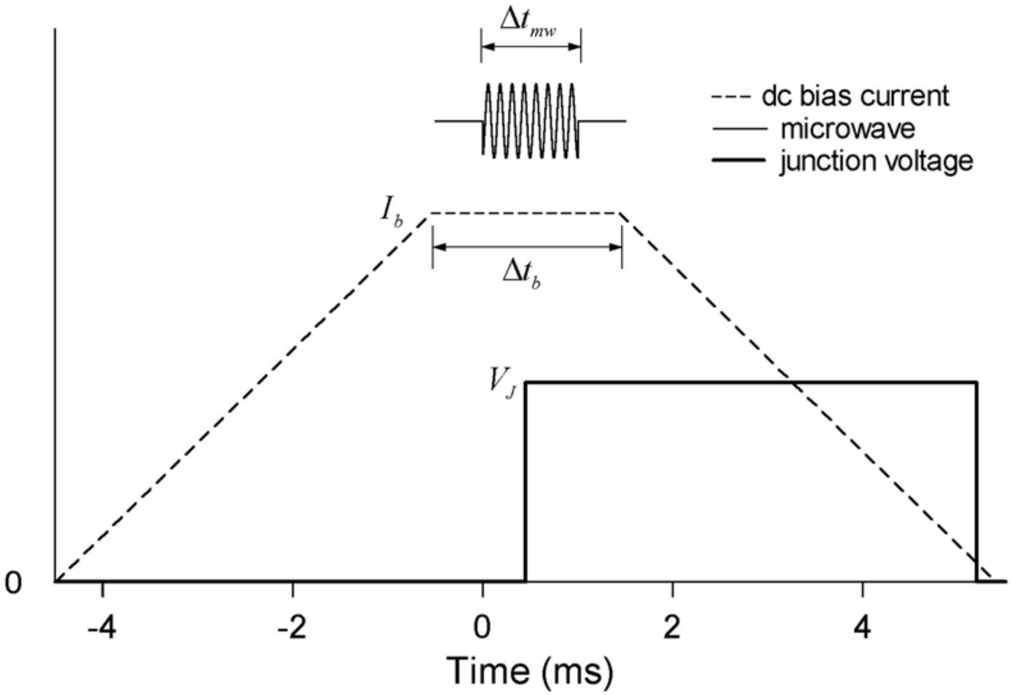
→

$T_1 \approx 50\ \mu\text{s}$

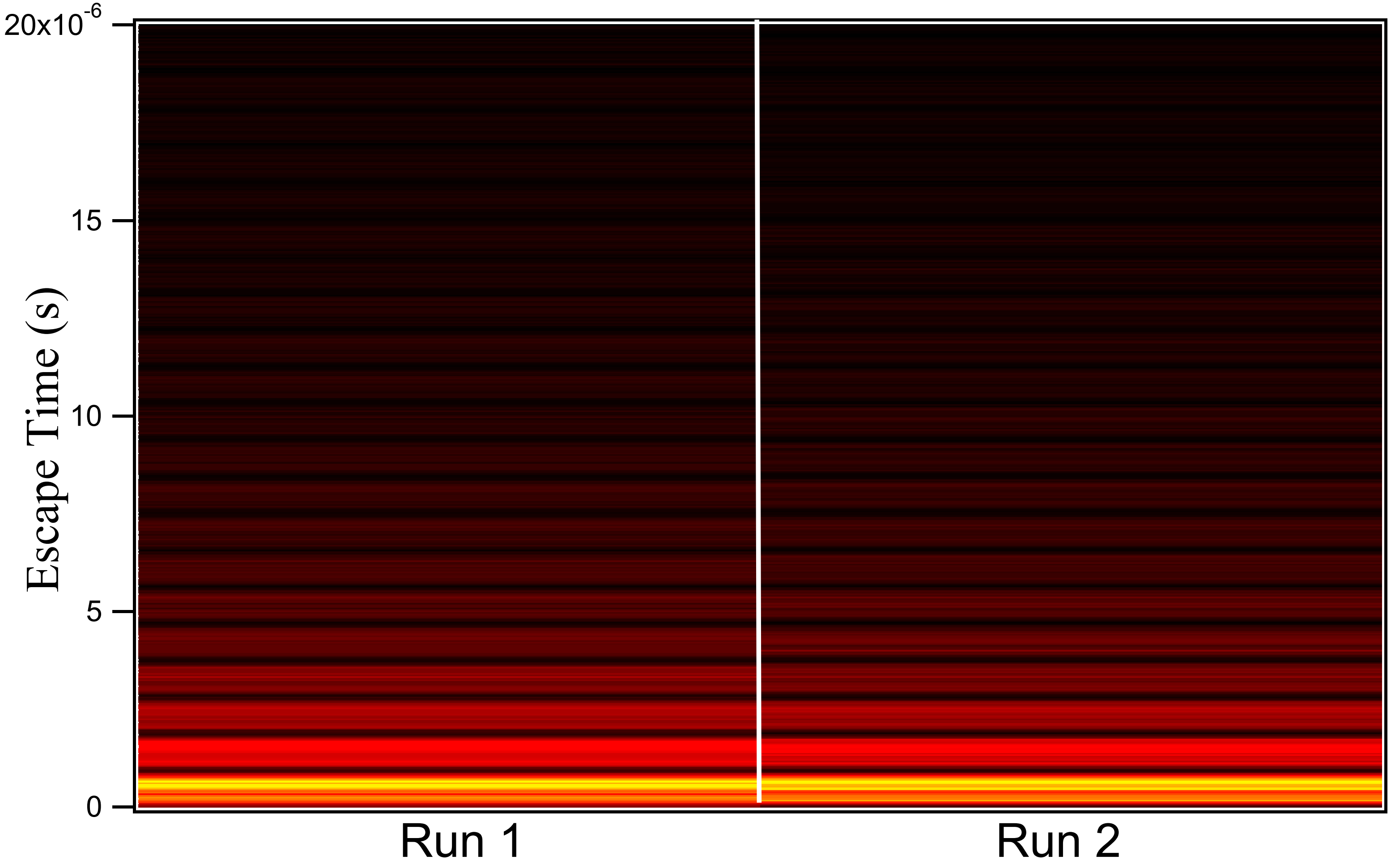
Experimental Procedure

$\Gamma_1 \Delta t_{mw} \gg 1$ → CW (continuous wave)

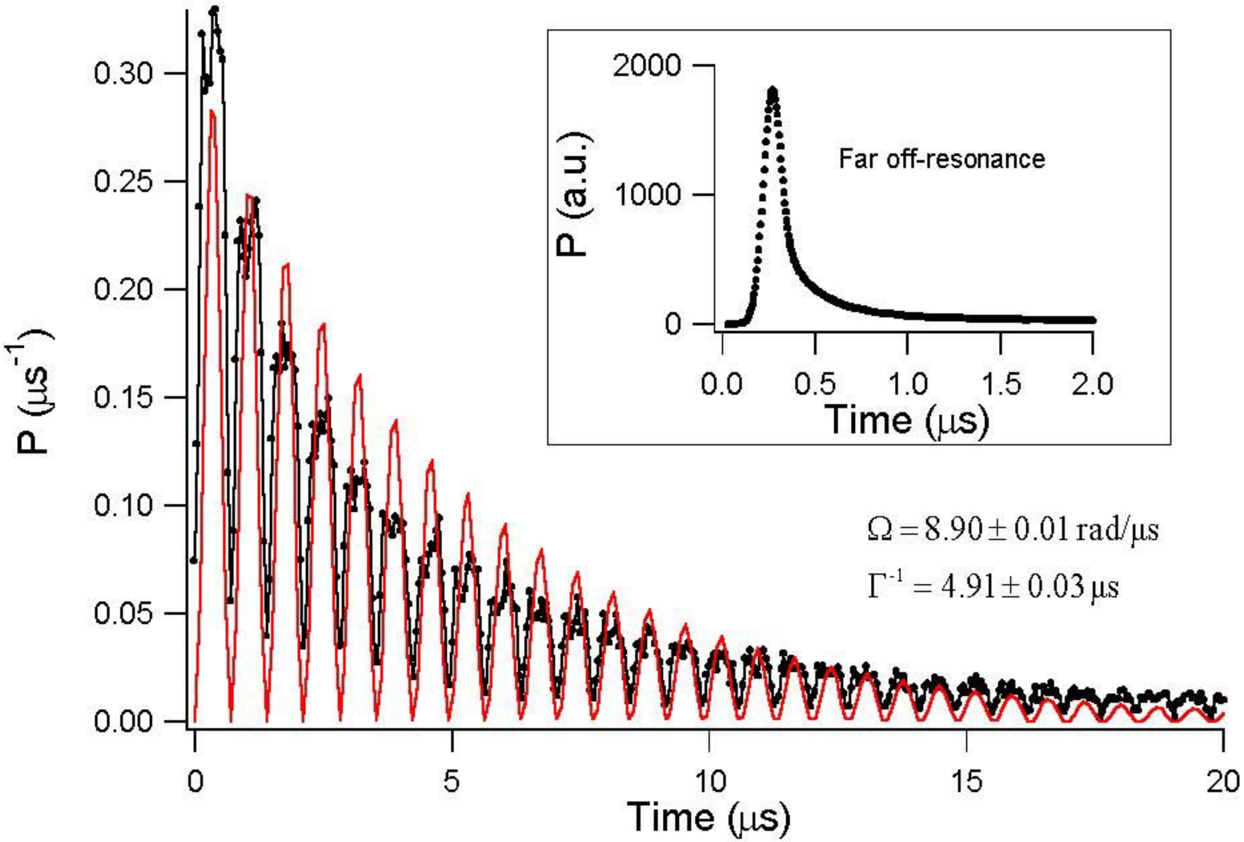
Always-on detection



Escape Time Distribution



Rabi Oscillations with Long Coherence Time



$\Omega = 8.90 \pm 0.01$ rad/μs

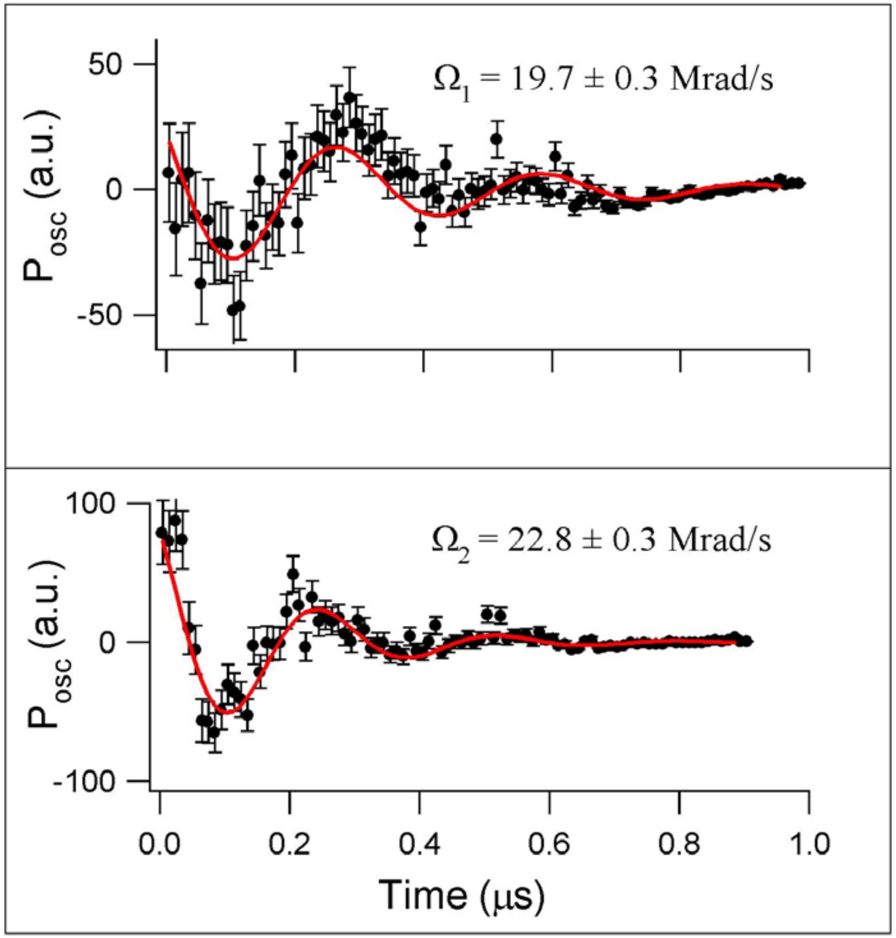
$\Gamma^{-1} = 4.91 \pm 0.03$ μs

$$\Gamma_{10} = \frac{1}{2}(\Gamma_0 + \Gamma_1 + \gamma_{10}) + \gamma_\varphi$$

⇒

$decoherence\ \ time > 4.9\ \mu s$

Power Dependence



Theory :

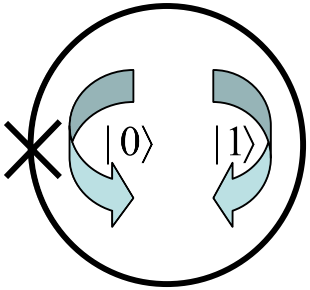
$$\Omega_1 / \Omega_2 \leq \sqrt{P_1 / P_2}$$

Experiment :

$$\sqrt{P_1 / P_2} = 0.88$$

$$\Omega_1 / \Omega_2 = 0.86 \pm 0.03$$

Superconducting flux qubit (SQUID)

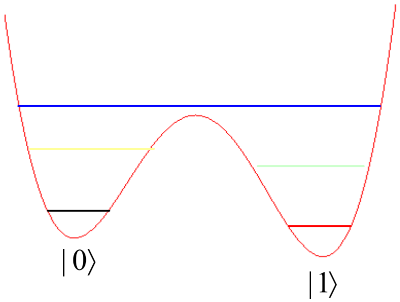


$$\beta_L \equiv 2\pi L I_c / \Phi_0$$

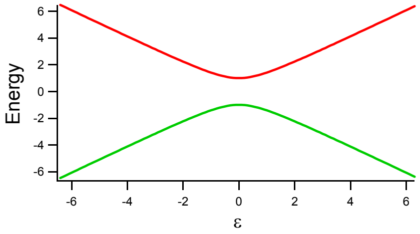
$$Z_0 \equiv \sqrt{L/C}$$

$$\omega_{LC} \equiv 1/\sqrt{LC}$$

An rf SQUID flux qubit

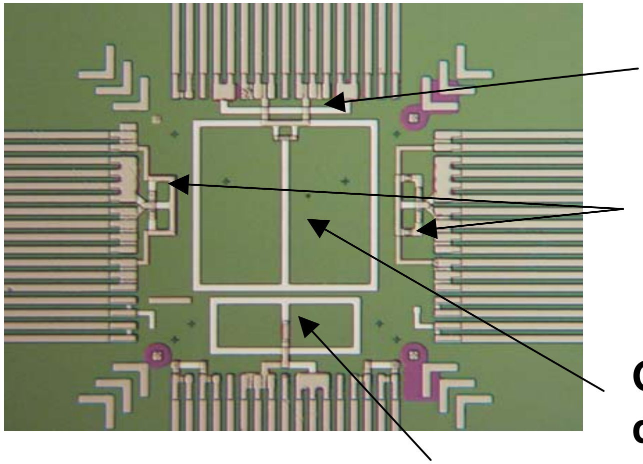


SQUID potential



Two-level approx.

$$H = \frac{1}{2}(\varepsilon\sigma_z + \Delta\sigma_x)$$



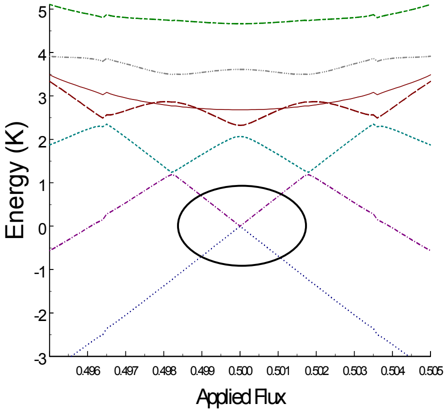
Barrier modulation Δ

Switching magnetometer

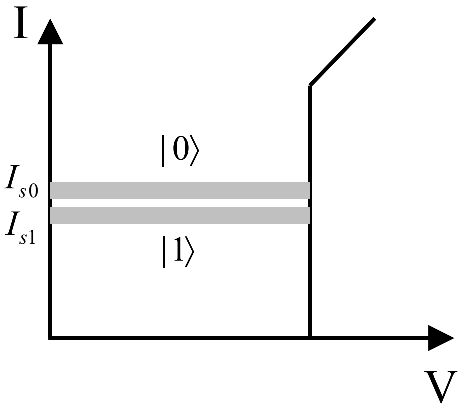
Gradiometer qubit

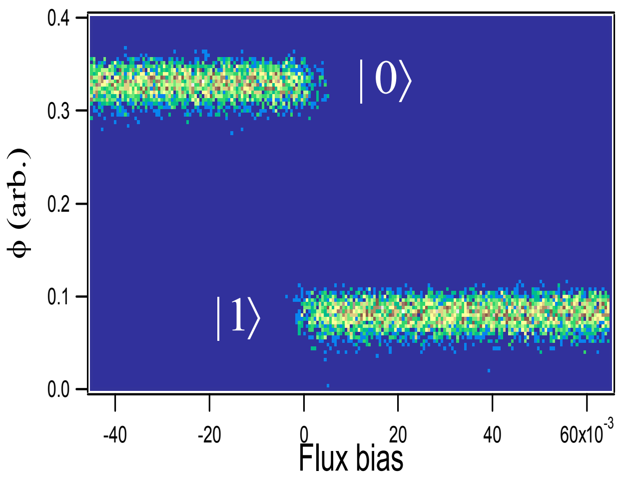
Flux bias ε

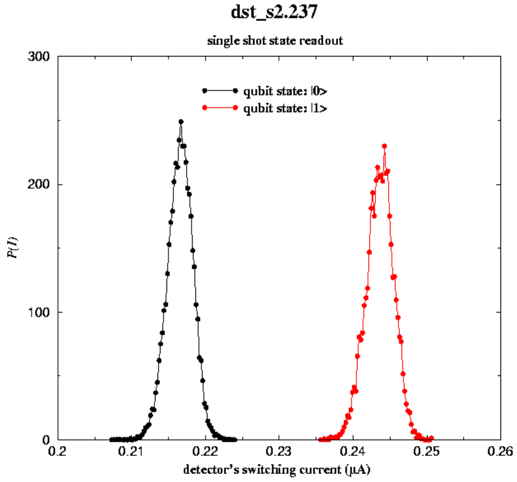
Oversimplified!



High fidelity, very small back-action, single-shot readout





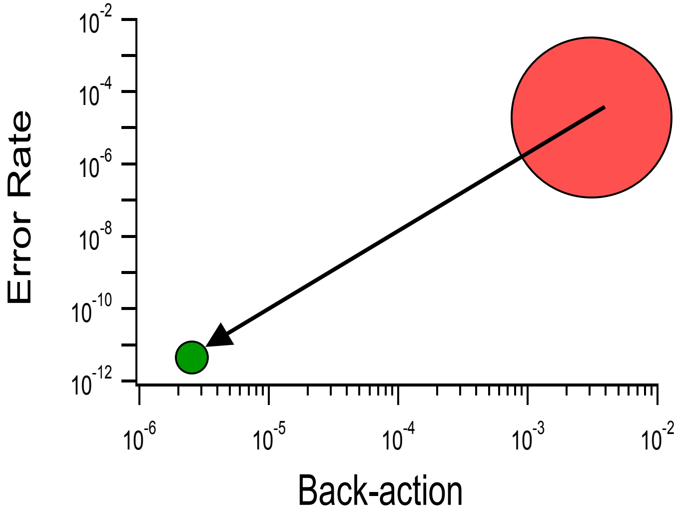


$$\delta I_s \approx 3\ \text{nA} \ll \Delta I_s \approx 30\ \text{nA}$$

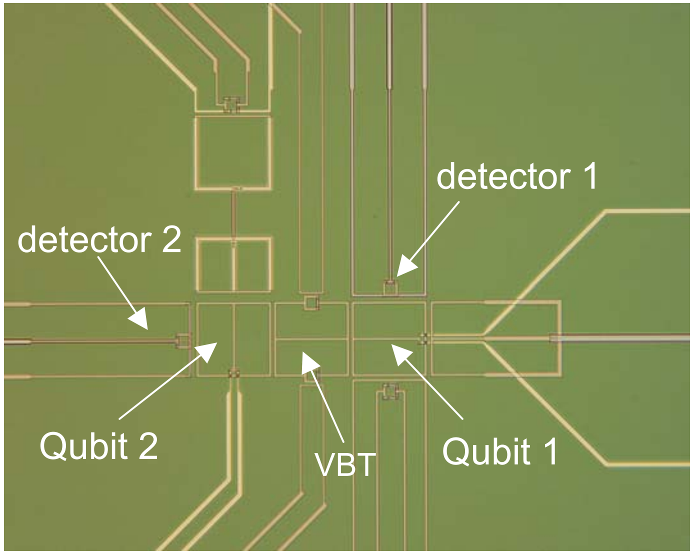
$$M \approx 0.7\ \text{pH},\quad L_Q \approx 165\ \text{pH}$$

$$\frac{M}{L_Q} \approx 4\times10^{-3},\qquad \left(\frac{L_Q}{M}\right)^2 \approx 5\times10^{4}$$

Back-action: $\phi_{bk} < 10^{-5}$, $R_{eff}(0) > 10\ \text{M}\Omega$.



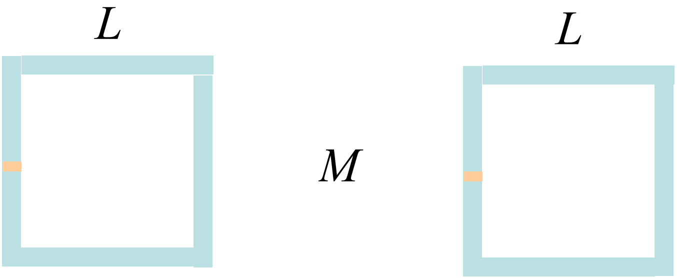
Coupled SQUID qubits for demonstration of CNOT and two-bit entanglement



Design and measurement: KU
Fabrication: Northrop Grumman (TRW)

Experiment status:
High temperature characterizations close to completion.

Qubits: $\beta_{L\,\max} \approx 4.5$, detectors: $2I_0 \simeq 13.5\ \mu\mathrm{A}$.



Two-level Approximation:

$$H = \sum_{i=1}^{2} \frac{1}{2}(\varepsilon_i \sigma_{zi} + \Delta_i \sigma_{xi}) + \frac{1}{2}\lambda \sigma_{z1}\sigma_{z2}$$

$$\text{where, } \lambda \approx \frac{2M}{L}\Phi_m^2$$

It is NOT that simple!

Dynamics depends on level structure in a non-trivial way.

Decoherence due to Critical Current Fluctuations

- 1/f power spectrum: $S_I(f) = S_I(1\,\mathrm{Hz})/f.$

- Affects all Josephson qubits.

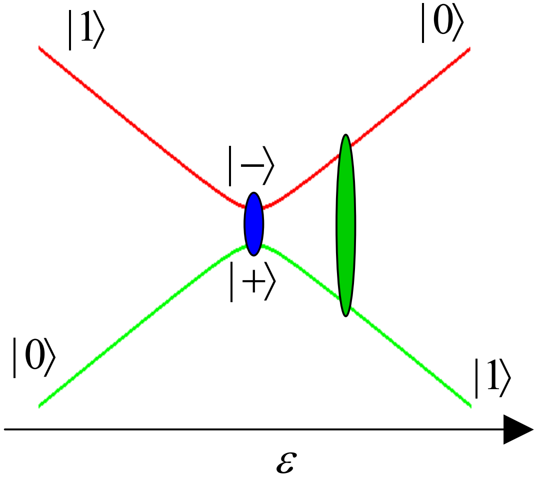
- Nearly universal $\dfrac{\delta I_c \cdot A^{1/2}}{I_c} \equiv \left[\dfrac{S_I(1\,\mathrm{Hz})\cdot A}{I_c^2}\right]^{1/2} \approx 10^{-5}\ \mu\mathrm{m}$

- Qubit type dependent $\Lambda \equiv \dfrac{\delta\omega_{01}/\omega_{01}}{\delta I_c/I_c}$

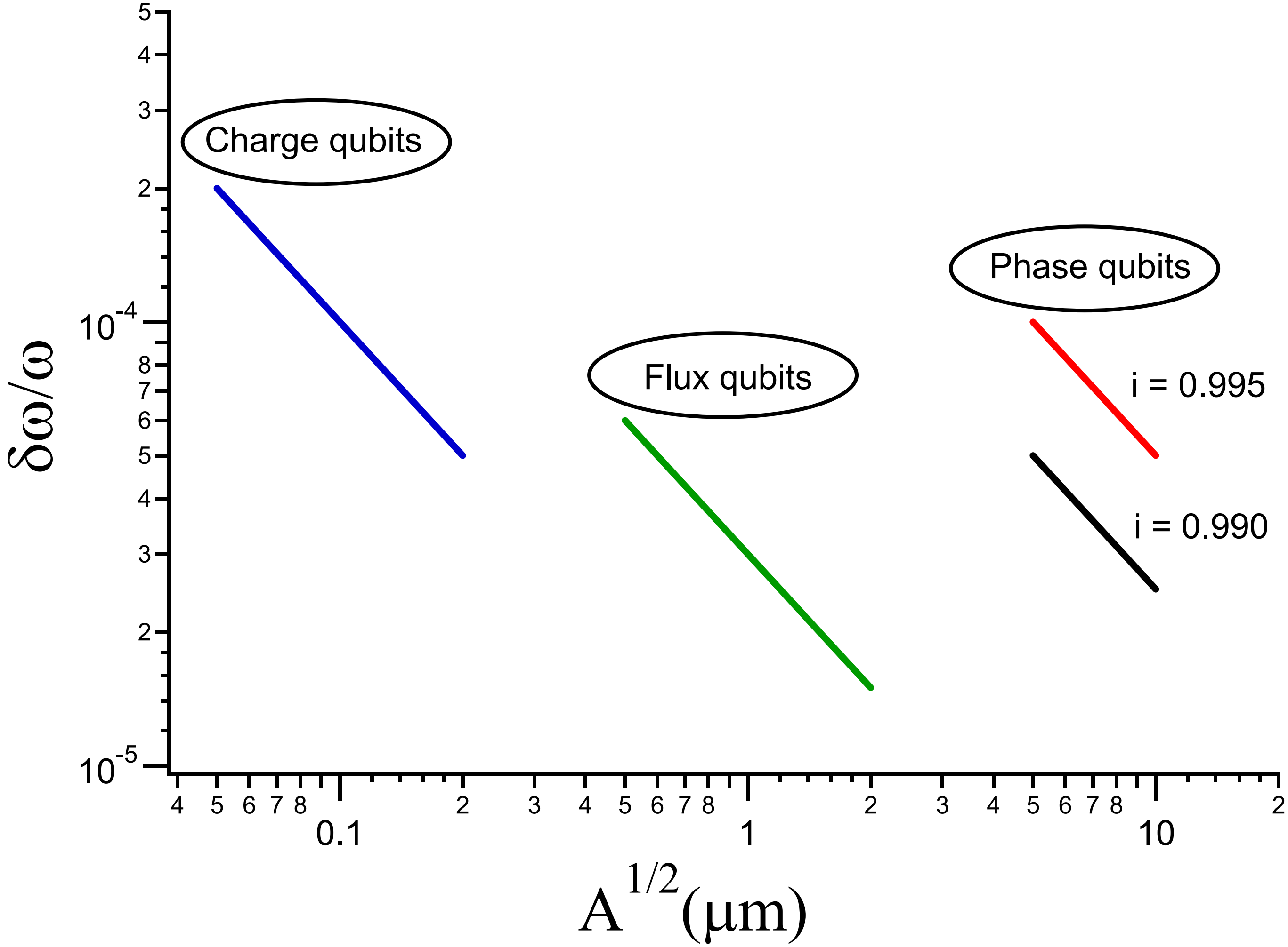
$$\frac{\delta\omega}{\omega} = \frac{\Lambda}{A^{1/2}}\left(\frac{\delta I_c A^{1/2}}{I_c}\right)$$

$E_J/E_c \propto A^2$

Qubit type	Λ	$A^{1/2}$ (μm)	E_J/E_c
charge	1	0.05 — 0.2	0.01 — 1
flux	~3	0.5 — 2	10^2 — 10^4
phase	25-50	5 — 20	10^6 — 10^8



Fractional detuning due to I_c fluctuations



Conclusions

- Phase qubit rotation demonstrated.
- Long coherence time observed.
- Effect of 1/f critical current fluctuation on dephasing is similar to all types of Josephson qubit.
- Superconducting approach to QC is very competitive.

Acknowledgement

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