

# Coupled Superconducting Phase Qubits

*Simons Conference on Quantum and Reversible Computing, May 2003*

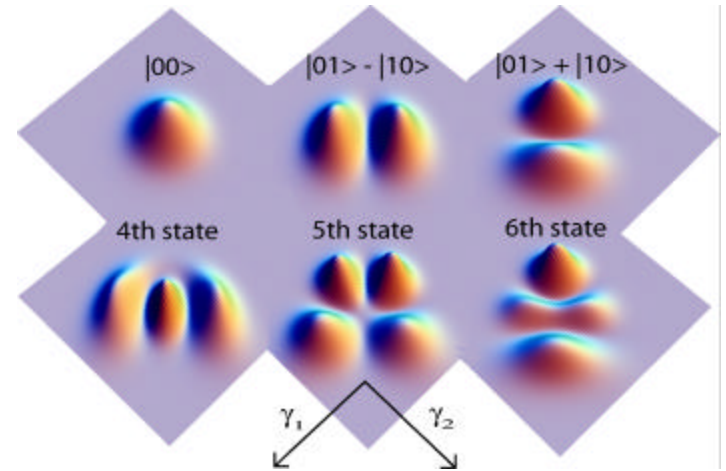
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Center for  
Superconductivity Research



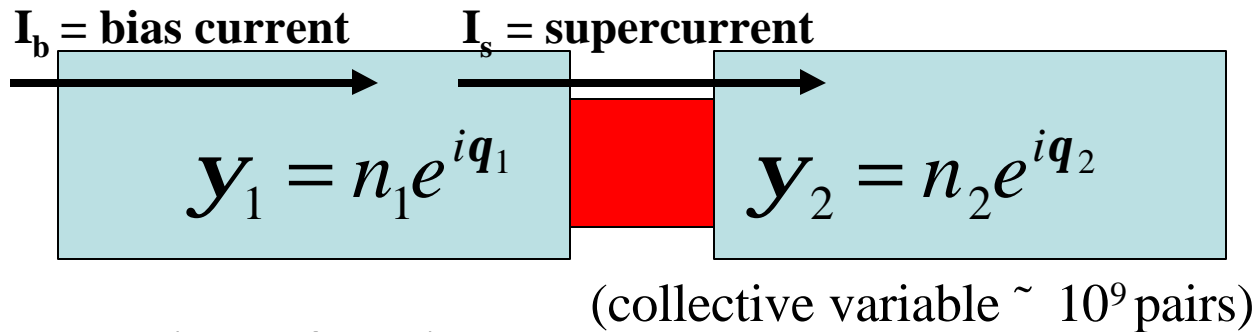
# Outline- Coupled Phase Qubits

- Single **large** (e.g.  $10 \mu\text{m}^2$  junction,  $C_J \sim 5 \text{ pF}$ ) current-biased Josephson-junction (CBJJ) **phase** qubits. (NIST, Kansas, Maryland)
- What happens when you capacitively couple two CBJJ qubits separated by  $\sim 1 \text{ mm}$ ?
- Next challenges: Longer coherence times!, measuring coherent oscillations, measuring correlations.

# Motivation

- What makes CBJJ phase qubits attractive?
  - Single phase qubit QM established, MQT/MQC
  - Solid state scalability.
  - Tunable coupling and energy levels.
  - Easily controlled (wires).
  - Prepare in ground state.
  - High fidelity gates possible in principle.
  - Easily measured.
  - **Good experiments testing important fundamental physics.**
- Serious challenges to meet:
  - Match (in large phase qubit) the impressive *charge* and *flux* qubit results (NEC, Yale, Saclay, SUNY, Delft, MIT, IBM, etc...)
  - Much longer coherence time (*better isolation*, less noise).
  - Measuring coherent oscillations and quantum correlations.

# Dynamics of tunnel junction phase:



## Josephson relations:

$$I_s(t) = I_c \sin g(t)$$

$$V(t) = \frac{Q(t)}{C_J} = \left( \frac{\Phi_0}{2p} \right) \frac{dg(t)}{dt}$$

## Equations of motion:

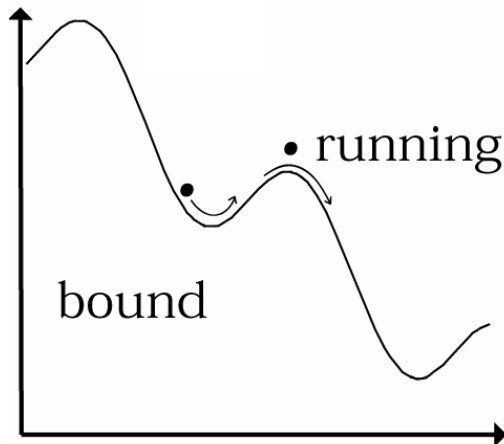
$$\frac{d^2 g(t)}{dt^2} = - \left( \frac{8E_c E_J}{\hbar^2} \right) \frac{d}{dg} U(g)$$

$$g = (q_1 - q_2) + \frac{2e}{\hbar} \int A \cdot dx$$

$$U(g) = -E_J (\cos g(t) + Jg)$$

$$E_J = I_c \Phi_0 / 2p \gg E_c = e^2 / 2C_J$$

$$\Phi_0 = \frac{h}{2e}$$



**Bound:**  $g(t)$  oscillates at plasma frequency  $\omega_p$ :  
AC voltage across junction but no DC voltage.

$$\omega_p(J) = \sqrt{8E_J E_c} (1 - J^2)^{1/4} / \hbar$$

**If  $g(t)$  makes it past barrier** it switches to a finite DC voltage running state.

# Single junction control and Hamiltonian

$$N \cong \frac{\Delta U_{\text{barrier}}}{\hbar \omega_p(J)} = \frac{2^{3/4}}{3} \left( \frac{E_J}{E_C} \right)^{1/2} (1-J)^{5/4}$$

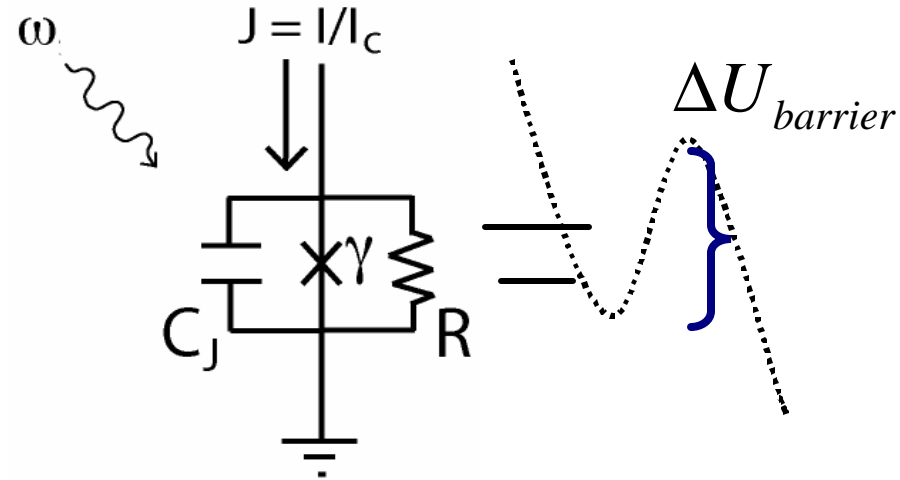
When  $N > 1$  there exist metastable bound states in each well; phase qubit regime.

## Single junction control

- $J = I/I_C =$  normalized bias current:  $J$  controls tilt of washboard;  $J > 1$  no wells
- Microwaves.
- Magnetic field ? critical current

$$H(p, \mathbf{g}) = 4E_C p^2 / \hbar^2 + U(\mathbf{g})$$

$$U(\mathbf{g}) = -E_J (\cos \mathbf{g}(t) + J \mathbf{g})$$



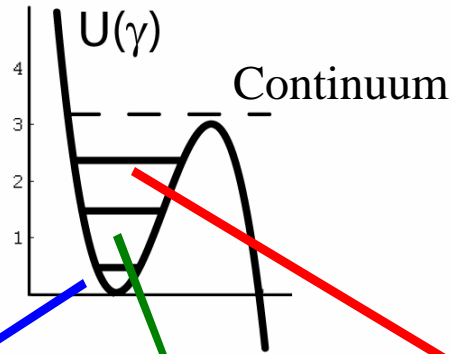
Canonical momentum:

$$p = \left( \frac{\Phi_0}{2\pi} \right)^2 C_J \frac{d\mathbf{g}}{dt} = \frac{\hbar Q}{2e}$$

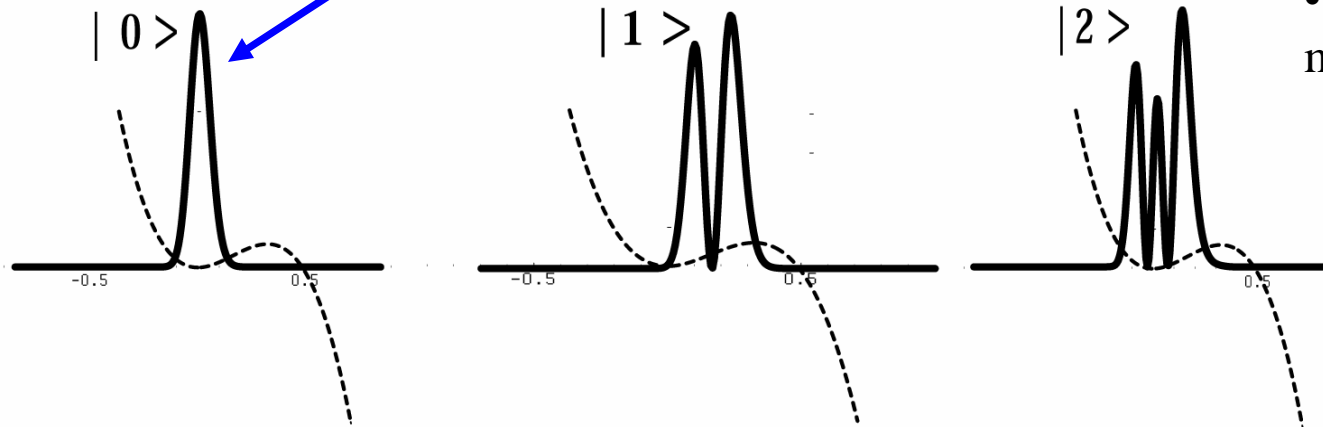
$\mu$  Number of pairs

# Single junction quantum states

Energy levels of  $N = 3$  metastable states



- Small  $N \rightarrow$  anharmonic  $\rightarrow$  selective level transitions.
- Small  $N \rightarrow$  strong nonlinearity  $\rightarrow$  significant tunneling.
- Tunneling limits fidelity of any gate; limits how small  $N$  can be.



• Tunneling used for state measurement/readout.

• Not a 2 level system; and other states aren't negligible.

**$N @ 4 - 5$  good compromise.**

# Readout / Measurement

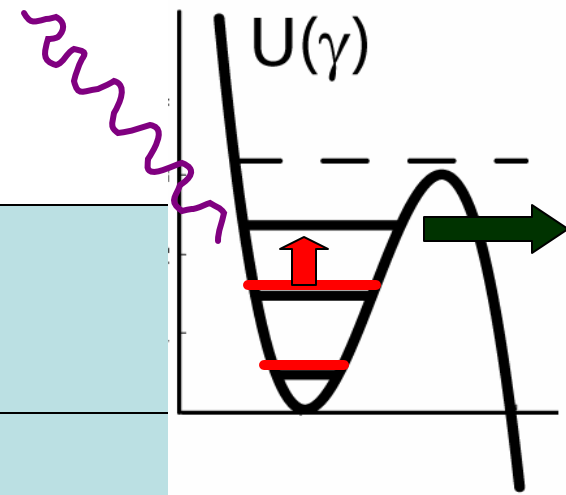
Qubit states are  $|0\rangle$  and  $|1\rangle$ .

Pump with microwaves at  $\omega_{21} = (E_2 - E_1) / \hbar$

If state is  $|0\rangle$ , no transition since  $(E_1 - E_0) \neq (E_2 - E_1)$ .

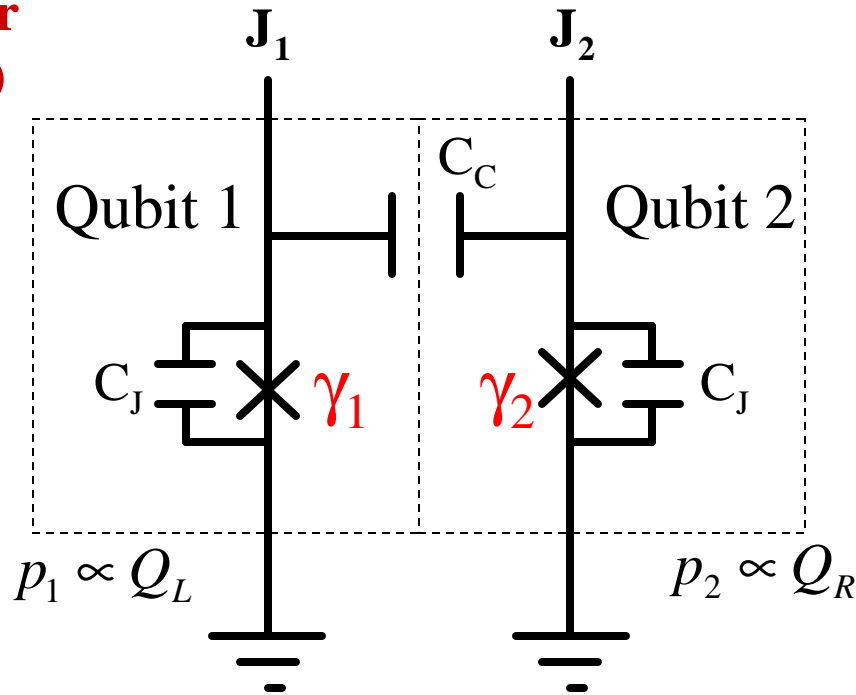
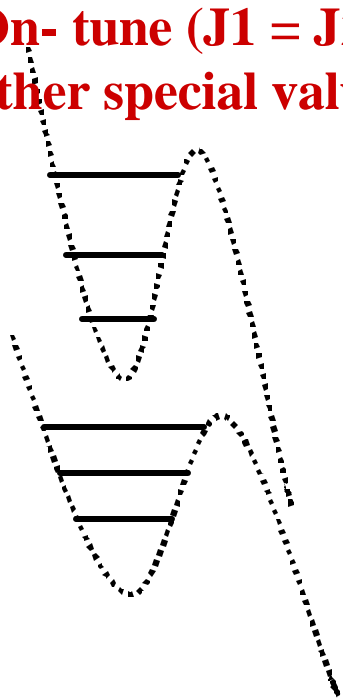
If state is  $|1\rangle$ , microwaves drive  $|1\rangle \rightarrow |2\rangle$

$|2\rangle$  quickly tunnels out of well (switches) giving measurable DC voltage across junction

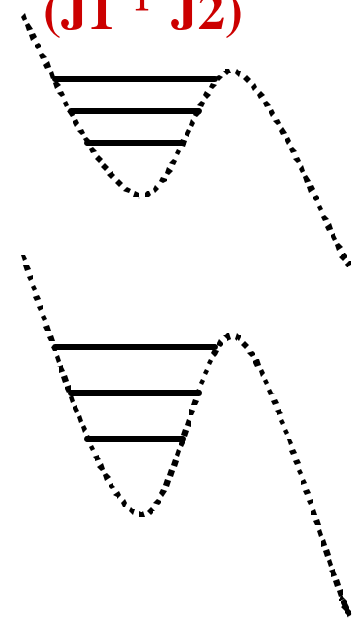


# Two capacitively coupled qubits

On-tune ( $J_1 = J_2$ , or other special values)



Off-tune ( $J_1 \neq J_2$ )



$$H = \underbrace{\frac{4E_C}{(1+z)\hbar^2} (p_1^2 + p_2^2 + 2z p_1 p_2)}_{\text{Kinetic energy}} - \underbrace{E_J (\cos \mathbf{g}_1 + J_1 \mathbf{g}_1 + \cos \mathbf{g}_2 + J_2 \mathbf{g}_2)}_{\text{Potential energy}}$$

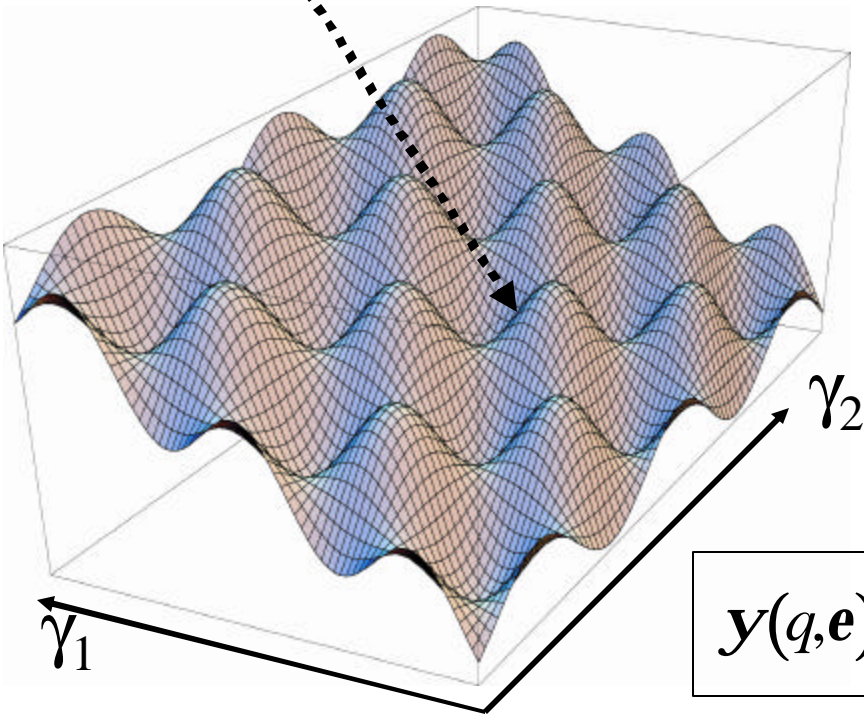
$$z = \frac{C_C}{C_C + C_J} = \text{Coupling parameter.}$$

*Momentum coupling:  
Electrostatic energy*



# Numerical solution of the 2D Schrödinger equation: split operator method.

Metastable states localized in well



Split-operator methods:

- Computes wave function on a lattice.
- Using imaginary time gives metastable stationary states.
- Using real time gives evolution including tunneling.
- Works for general nonlinear potentials.
- Highly accurate and proven method; unitary to machine precision.
- How it works:

$$\mathbf{y}(q, \mathbf{e}) = e^{-iV(q)\mathbf{e}/2} \underset{(p \rightarrow q)}{\text{FFT}} e^{-iT(p)\mathbf{e}} \underset{(q \rightarrow p)}{\text{FFT}} e^{-iV(q)\mathbf{e}/2} \mathbf{y}(q, 0)$$

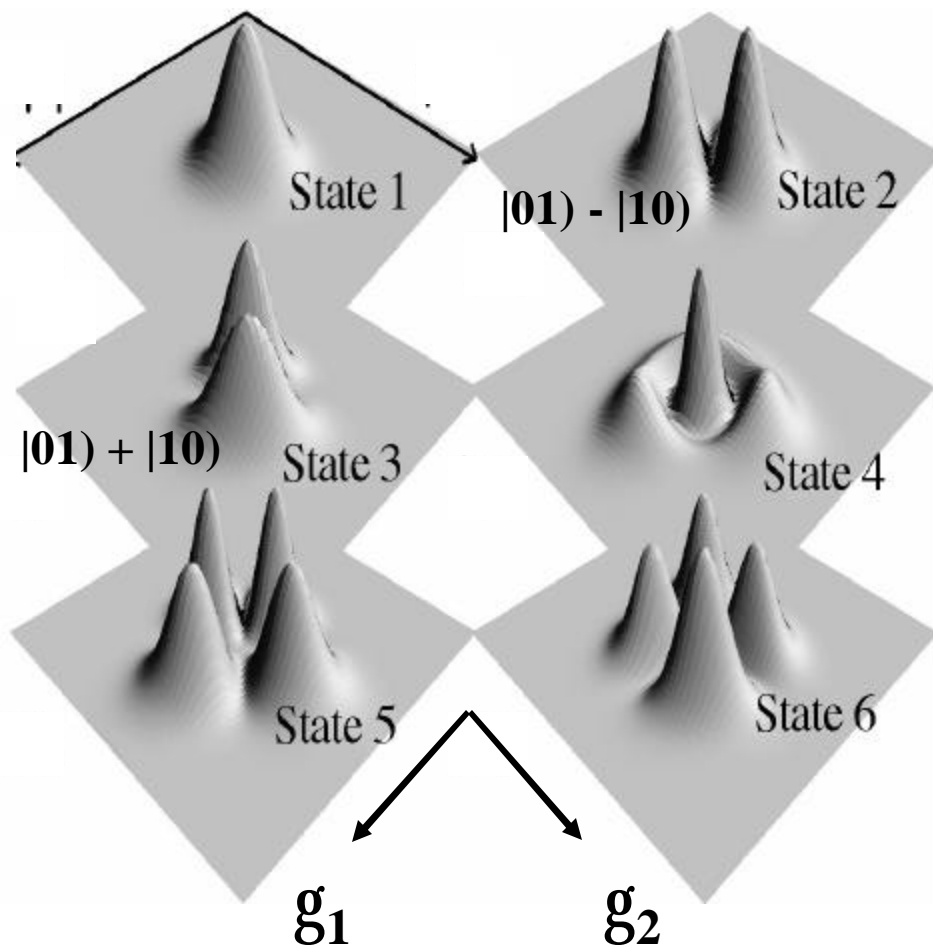
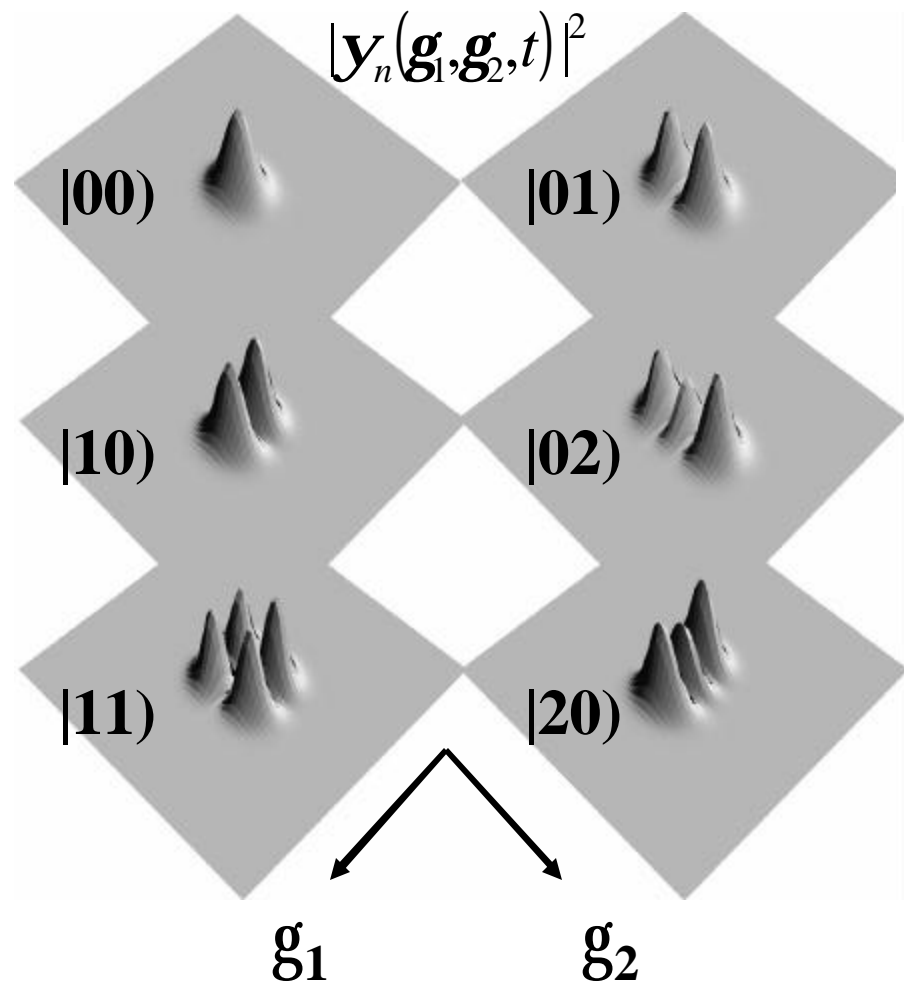
# Numerically computed quantum states:

(CJ = 4.3 pF, IC = 13.3  $\mu$ A, N = 3)

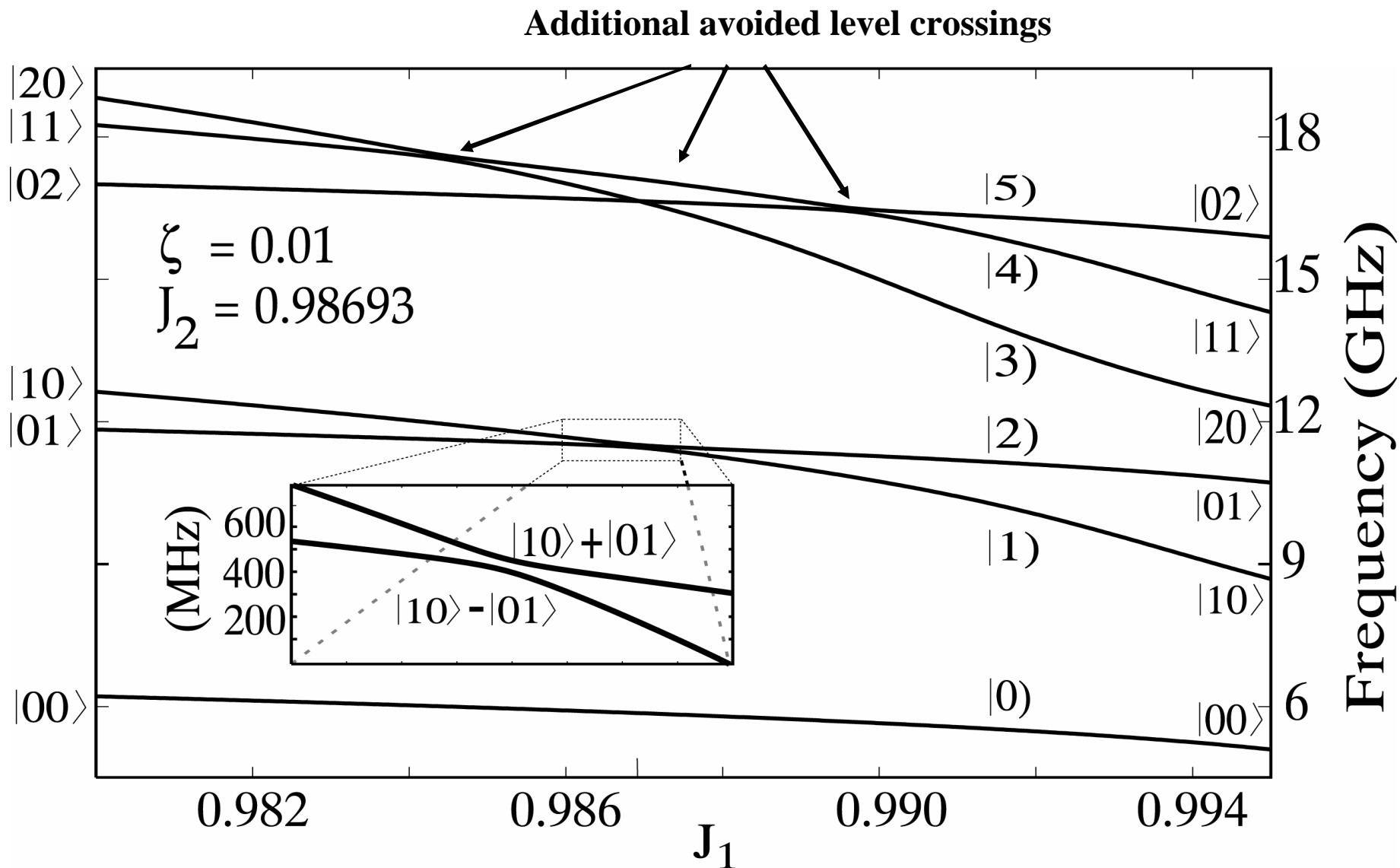
$$z = \frac{C_c}{C_c + C_J} = 0.01$$

**J1  $\neq$  J2 : Detuned**

**J1 = J2 : Tuned**



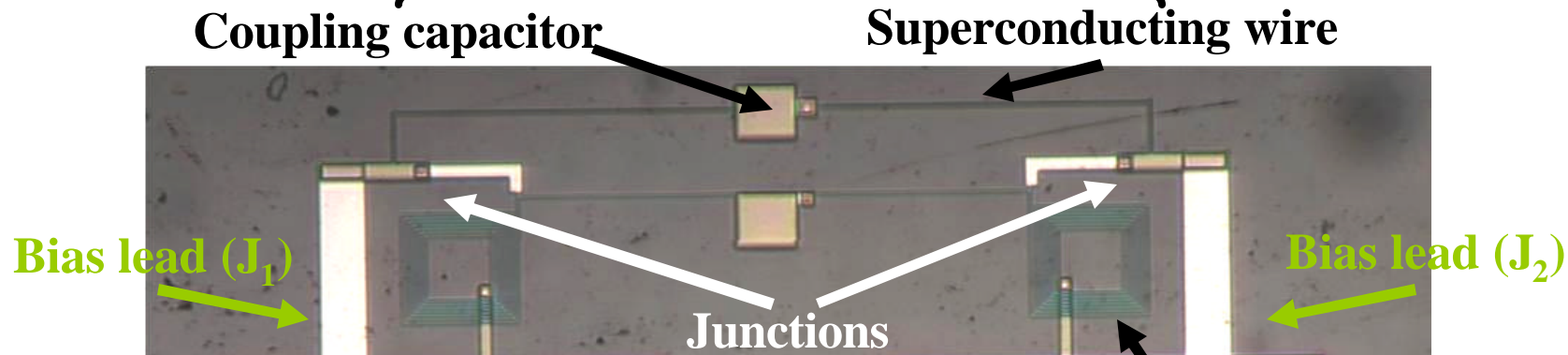
# Energy levels versus bias current: small coupling



Not 2-level system:  $|0,1,2,\dots\rangle \times |0,1,2,\dots\rangle = \{ |00\rangle, |01\rangle, |10\rangle, |20\rangle, |11\rangle, |02\rangle, \dots \}$

# Two qubit chip (Hypres): Large Nb-AlO<sub>x</sub>-Nb junction: 100 μm<sup>2</sup> area

**Approx. 1 mm separation**



$$I_C = 10\text{-}20 \mu\text{A}$$

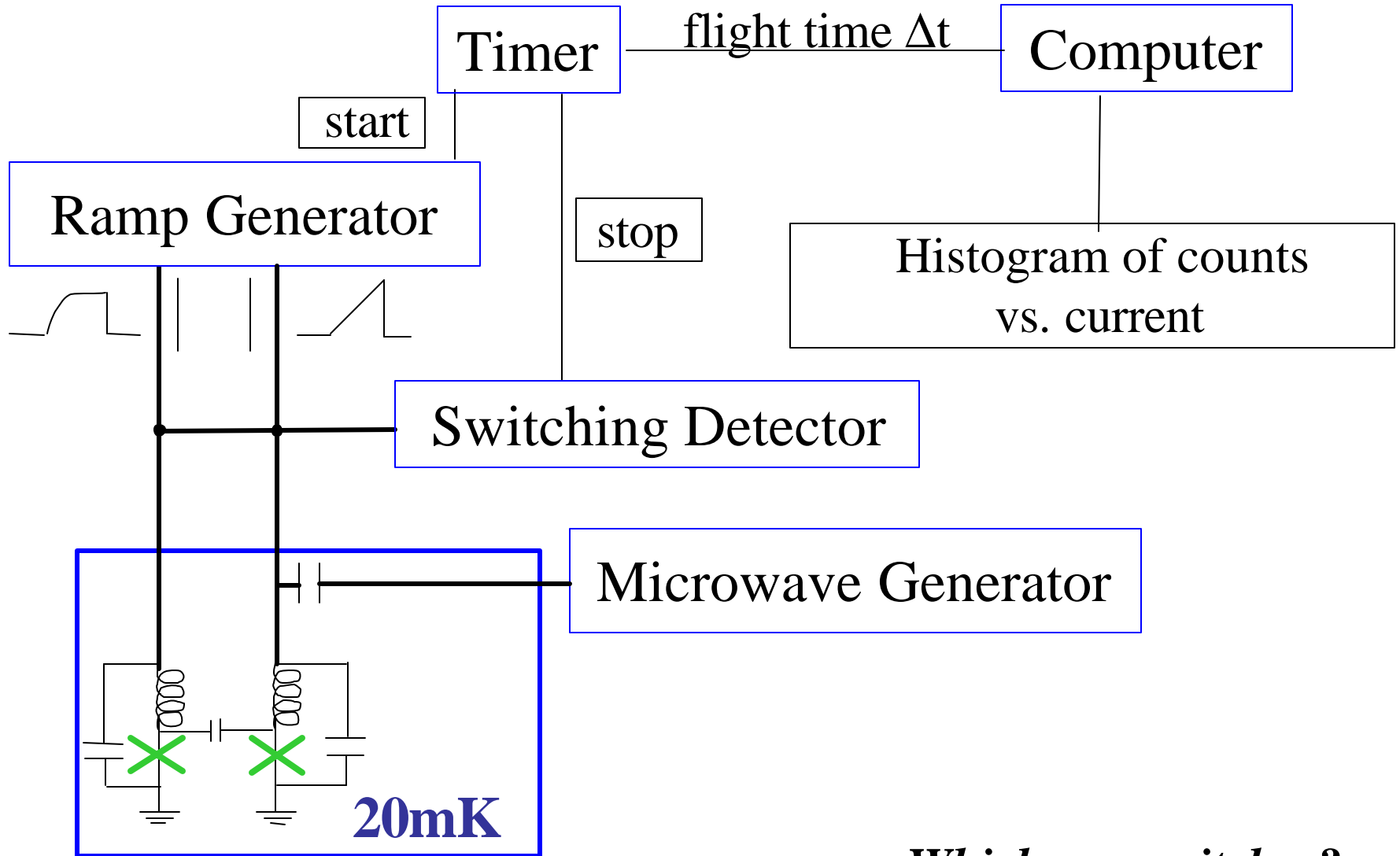
$$C_J = 4.8 \text{ pF}$$

$$C_C = 0.7 \text{ pF}$$

$$Z = 13\%$$

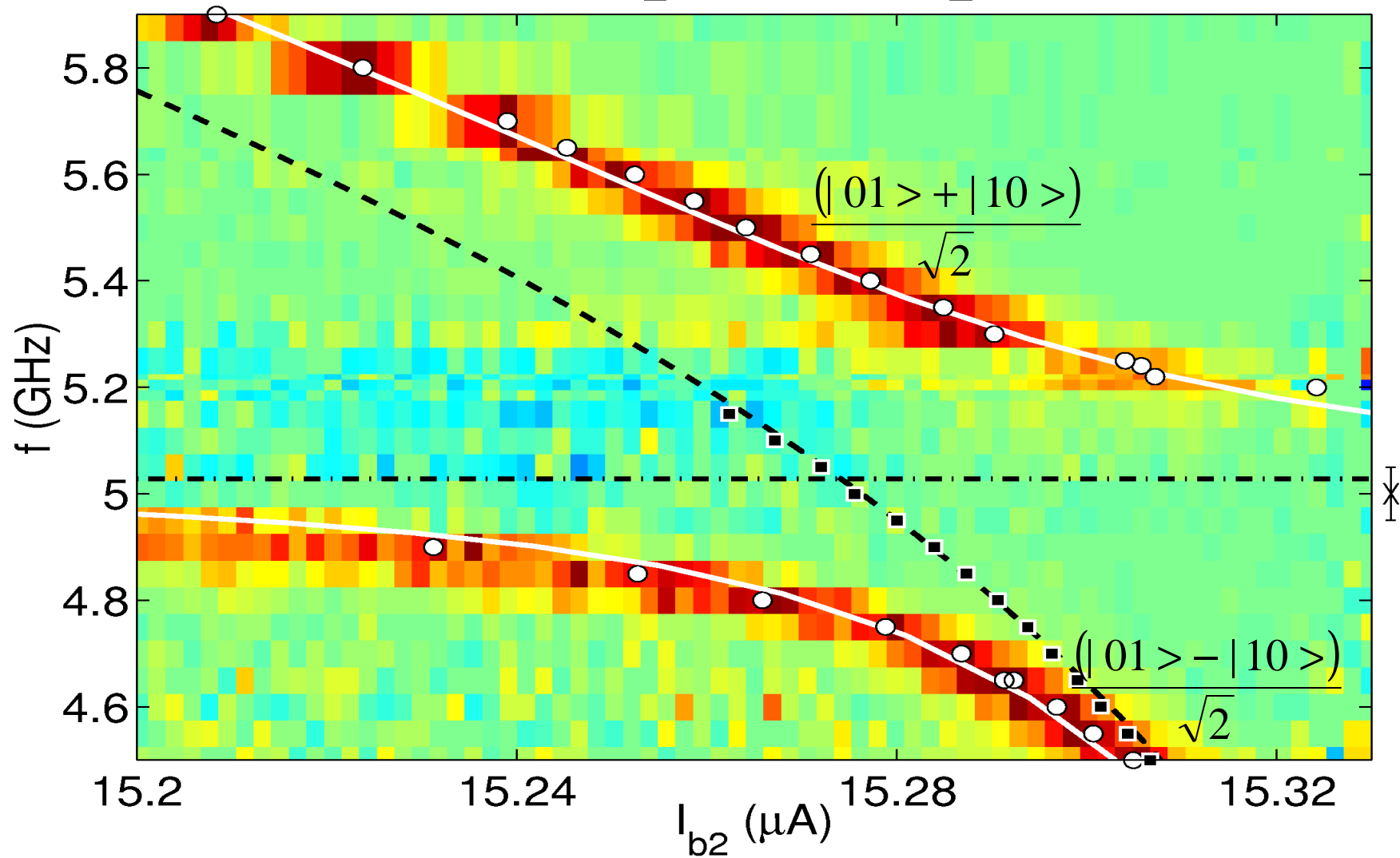
**Contact  
pads**

# 2-qubit spectroscopy method



*Which one switches?*

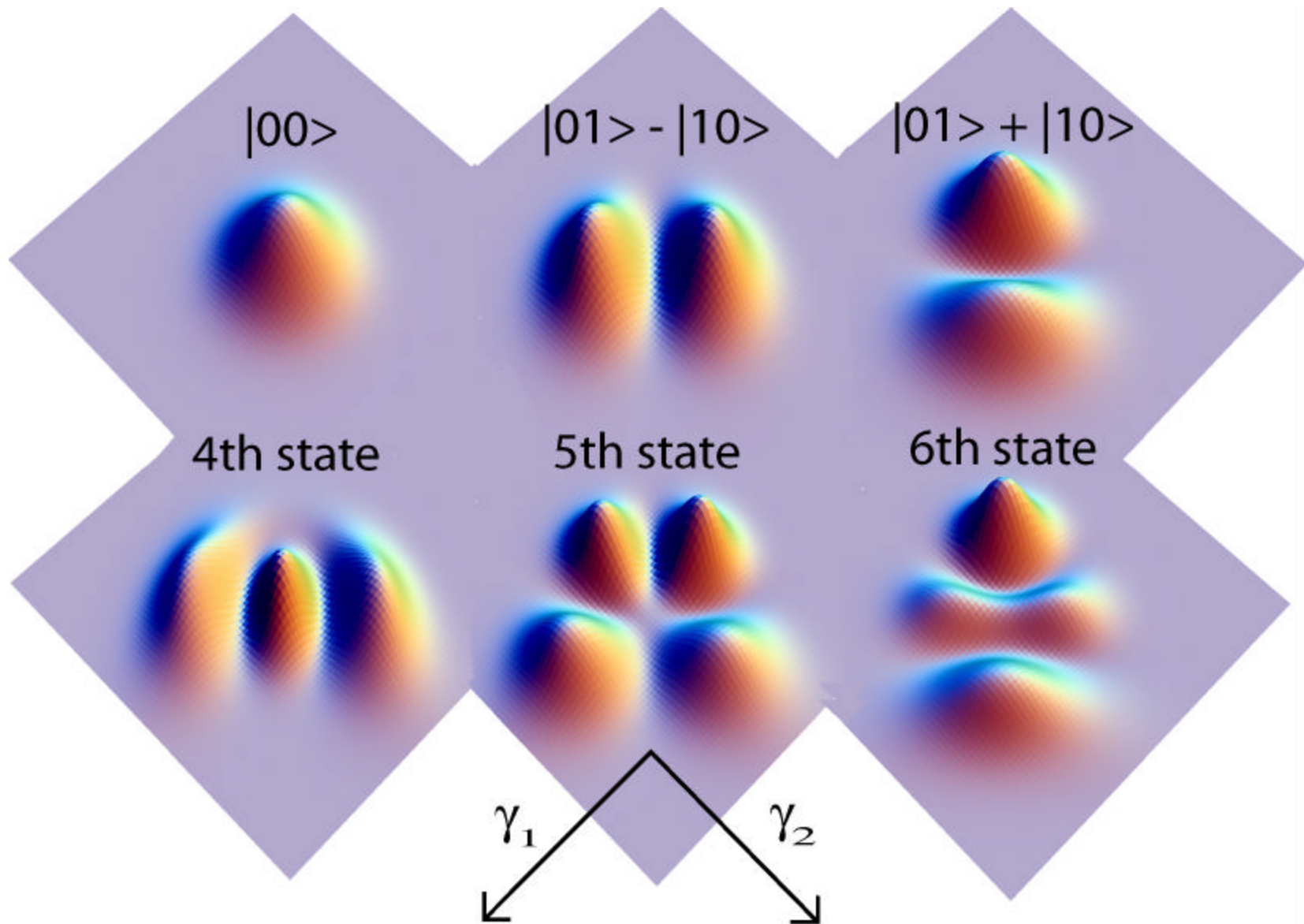
# Two qubit chip



Berkley et al., Science, May 15, 2003.



# First 6 states computed with experiment junction parameters



# Coherence and other qualitative features

- **Decoherence times may be estimated from widths of escape histograms: ranges from 2 - 6 ns.**
- **However, no excess decoherence from coupling.**
- **The 2 - 6 ns coherence time appears to be limited by low frequency current noise. Developing better isolation schemes should improve this (*how much?*).**
- **A little more coherence and many fundamental experiments are possible; QC will need a lot more coherence.**
- **Levels do not move with power, but new lines appear.**
- **If treated classical avoided levels appear but (because Hamiltonian is nonlinear) not in same place.**



# Conclusions

- **Capacitively coupled Josephson-junctions **phase qubits** are tunable qubits with effective coupling controlled by bias currents.**

*(Johnson et al. PRB 67 020509 (2003); cond-mat/0210278.)*

- **First test for coupled system passed: spectroscopy in excellent agreement with Schrödinger equation for nonlinear Hamiltonian, including quantum power dependence and MQT behavior.** *(Berkley et al. Science, May 15, 2003)*

- **Even more stringent experimental tests on the horizon. Increasing coherence is vital, there are a number of isolation schemes that should at least help in this regard. Coherent oscillations, ultimately correlations, must be measured.**

- **Next Fred Strauch is going to discuss quantum dynamics and show that good quantum gates should be possible; not immediately obvious with this system.**

*(Strauch et al., quant-ph/0303002)*