Universal Quantum Interfaces
or
The Power of One (Qubit)

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Based on:
quant-ph/0303048

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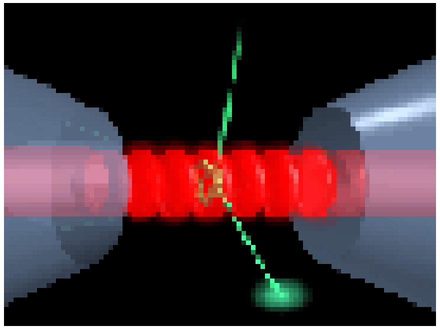
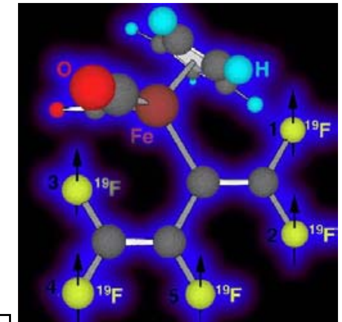
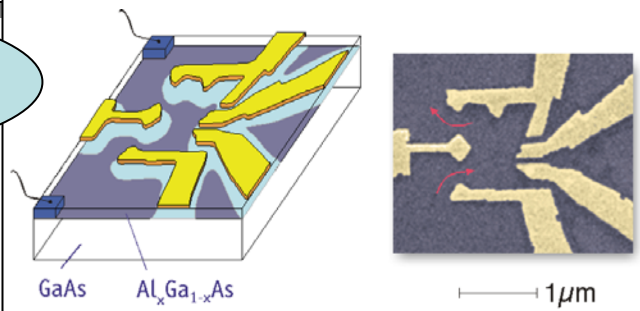
Entering the Era of Quantum Control



Cavity QED

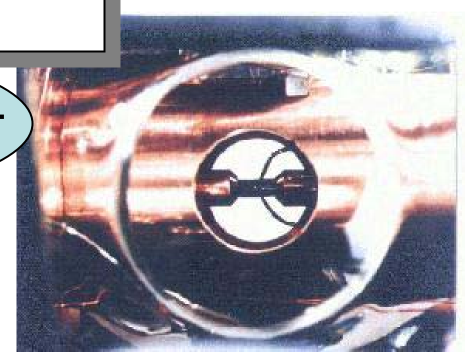


Liquid NMR



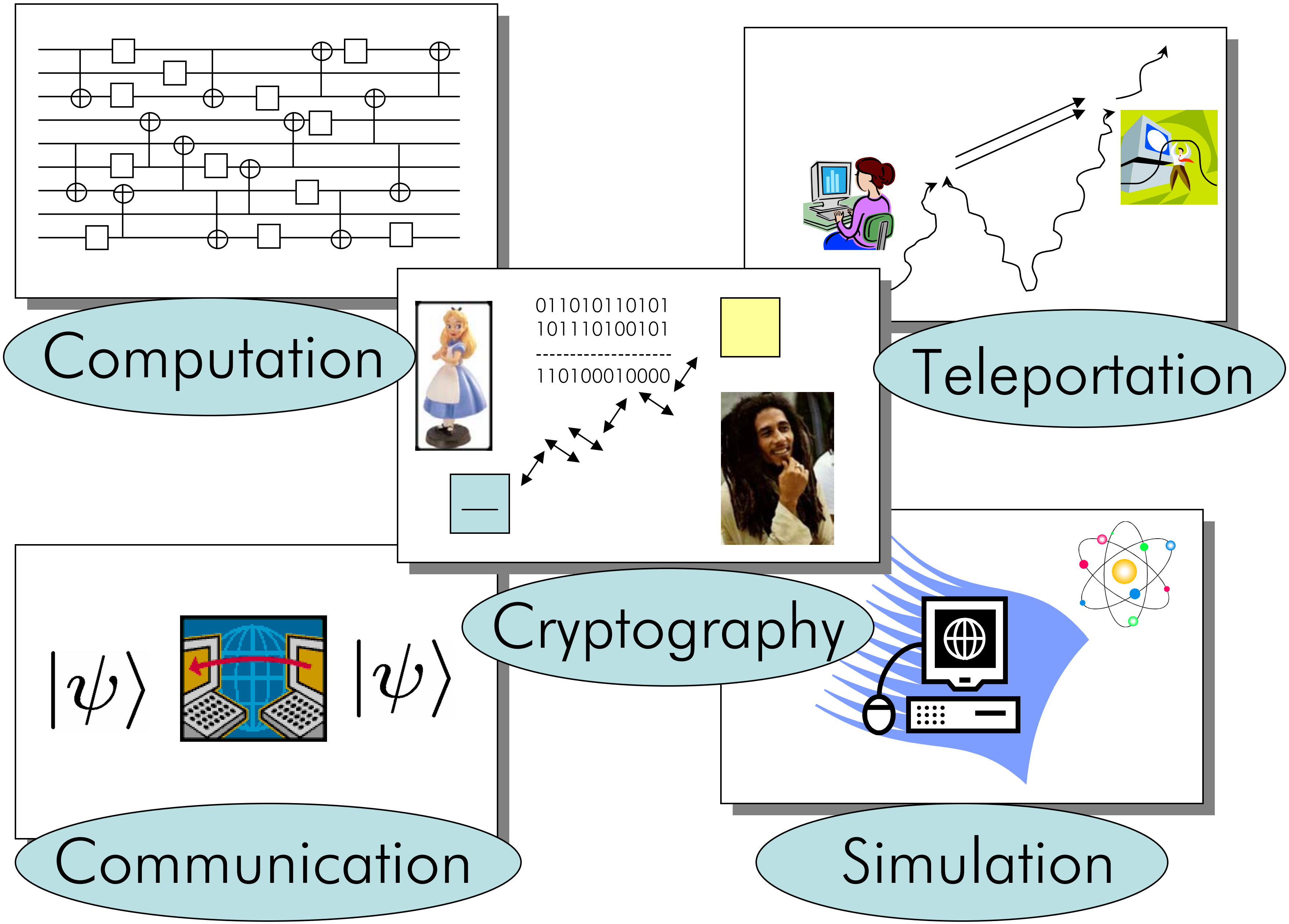
Quantum Dot

Lattice BEC

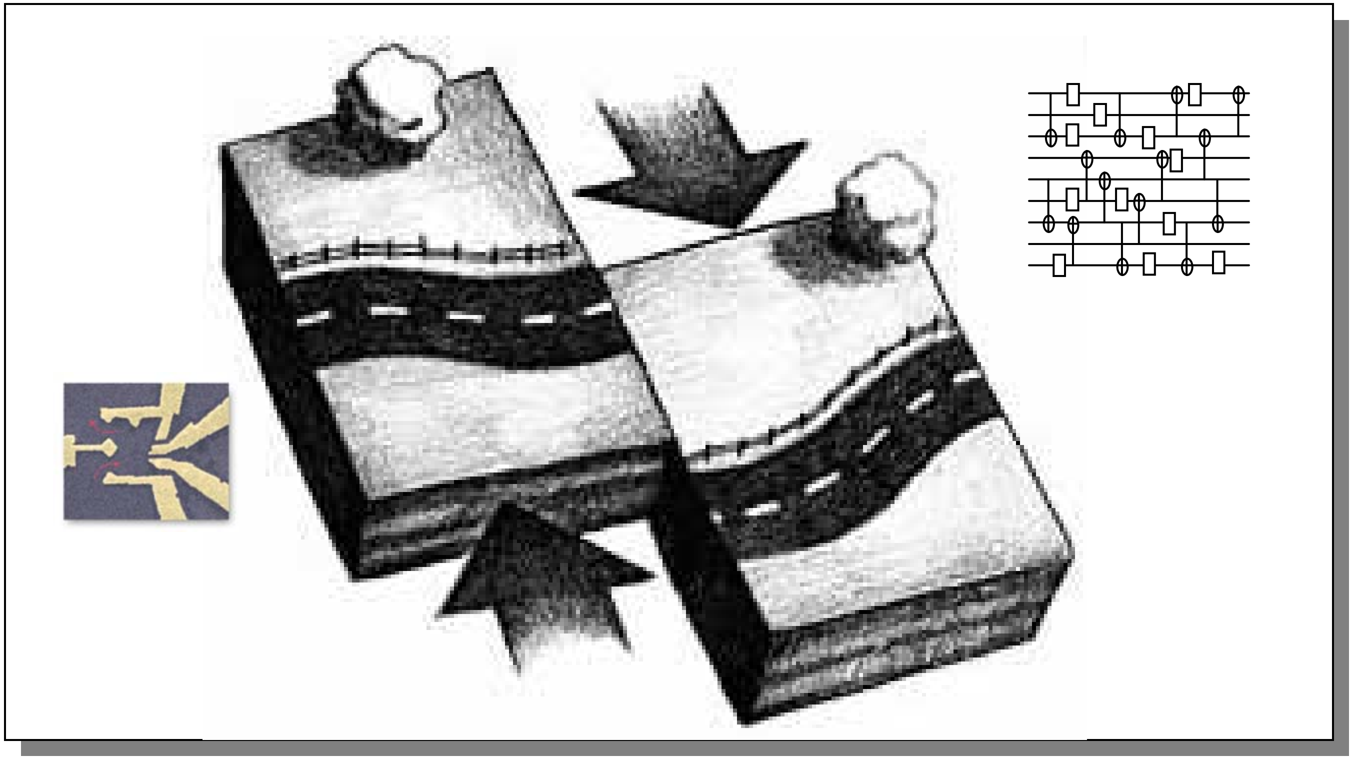


Ion Trap

Promises of Full Quantum Control



Missing: Roadmap!

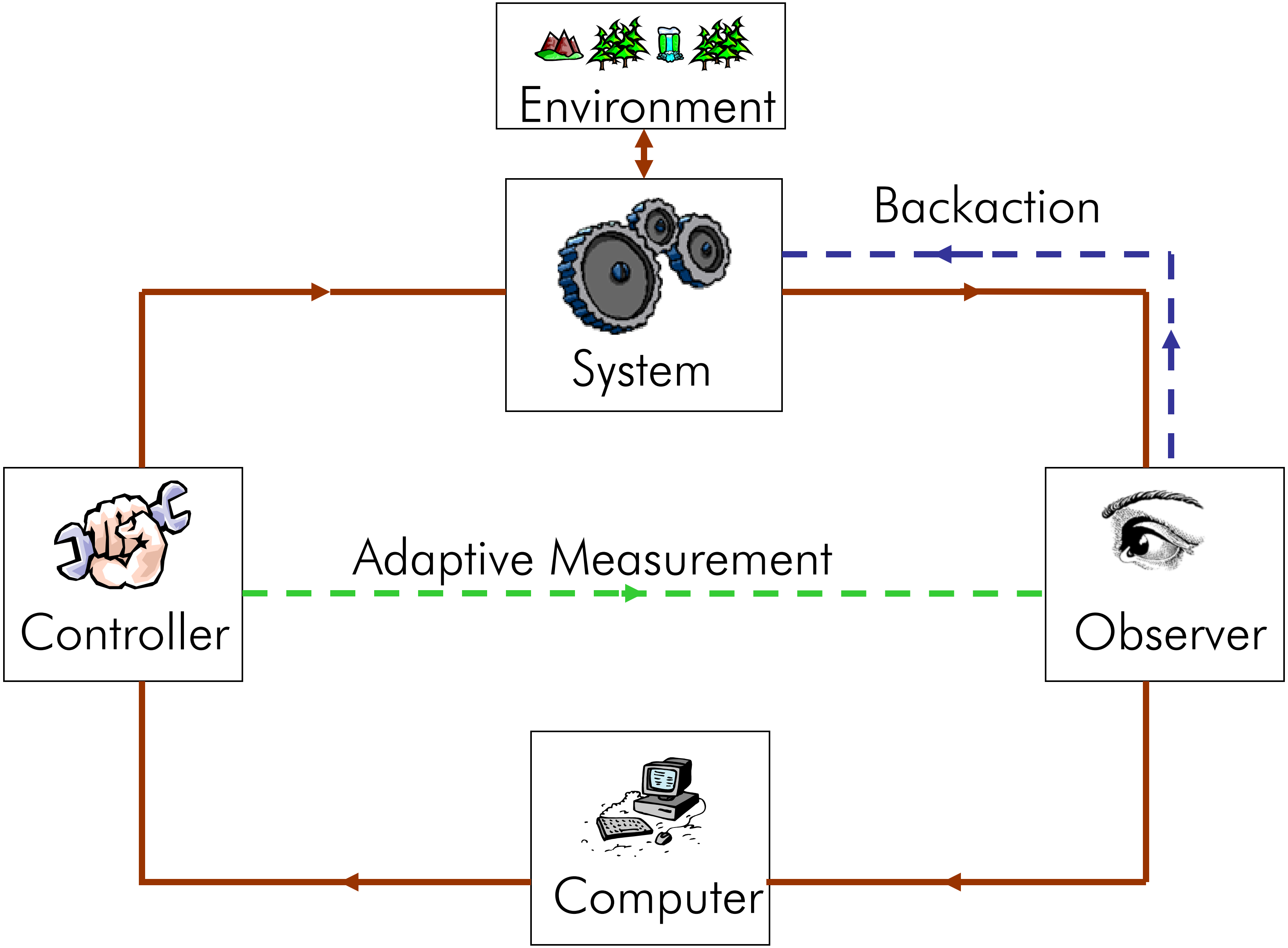


How much control is enough?

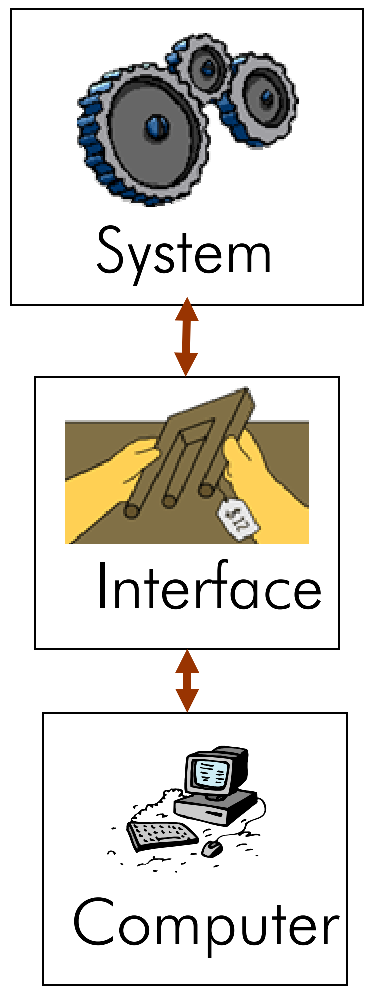
1) Full control of one qubit.
2) Generic coupling to a closed quantum system.

"Quantum Interface"

Interfaces & Control Loops



Interfaces & Control Loops



Universality

Kraus Representation Theorem:

Any linear, completely positive map between density matrices may be written as*

$$\rho' = \sum_l M_{kl} \rho M_{kl}^\dagger$$

"Quantum Operation"

where

$$\text{prob}(k) = \text{tr}\rho', \qquad \sum_{kl} M_{kl}^\dagger M_{kl} = 1$$

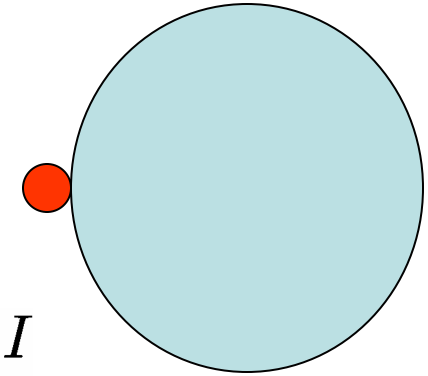
Church of the Larger Hilbert Space:

$$\rho' = \text{tr}_B \langle k | U (\rho \otimes |0\rangle\langle 0|_A \otimes |0\rangle\langle 0|_B) U^\dagger | k \rangle$$

$$\text{prob}(k) = \text{tr}\rho', \qquad U^\dagger U = 1$$

*unnormalized

Outline of construction



<u>Given:</u>

- Fixed generic interaction $\sigma_z \otimes A$
- Closed-system Hamiltonian $I \otimes H$
- Projective spin measurement $[(I \pm \sigma_z)/2] \otimes I$
- Arbitrary qubit Hamiltonian $\gamma\sigma \otimes I$

<u>Outline:</u>

1) Fixed interaction → Arbitrary interaction
2) Coherent system control
3) Any Hermitian two-outcome measurement
4) Any two-outcome measurement
5) Any generalized measurement
6) Any quantum operation

Fixed interaction ⟶ Any Interaction

Key formulas:

$$e^{i(A+B)} = \lim_{n\to\infty} (e^{iA/n} e^{iB/n})^n$$

$$n = 1/\Delta t$$

$$e^{-[A,B]} = \lim_{n\to\infty} (e^{iA/n} e^{iB/n} e^{-iA/n} e^{-iB/n})^{n^2}$$

Generically $\{A, H\}$ generate full Lie Algebra on $\mathcal{H}_{sys}$

$$[H + \sigma_z \otimes A, H + \sigma_z \otimes A + \gamma\sigma_x] = i\sigma_y \otimes A$$

$$[H + \sigma_z \otimes A, H + \sigma_z \otimes A + \gamma\sigma_y] = -i\sigma_x \otimes A$$

$$[\sigma_x \otimes A, \sigma_y \otimes A] = i\sigma_z \otimes A$$

$$\begin{aligned}[H + \sigma_z \otimes A, \sigma_z \otimes A] &= \sigma_z \otimes [H, A] \\ &= \sigma_z \otimes G\end{aligned}$$

Coherent system control

Simple, given arbitrary interactions:

$$
\begin{aligned}
[\sigma_z \otimes G_1, \sigma_z \otimes G_2] &= I \otimes [G_1, G_2] \\
&= I \otimes G
\end{aligned}
$$

Any Hermitian two-outcome measurement

1) Prepare interface in state $|0\rangle$.
2) Evolve via Hamiltonian $\sigma_x \otimes G$.
3) Measure interface.

$$\rho_0 = \langle 0|e^{-i\sigma_x G t}\rho e^{i\sigma_x G t}|0\rangle = M_0 \rho M_0^\dagger / p_0$$
$$\rho_1 = \langle 1|e^{-i\sigma_x G t}\rho e^{i\sigma_x G t}|1\rangle = M_1 \rho M_1^\dagger / p_1$$

$$M_0 = \cos Gt, \qquad p_0 = \mathrm{tr}\, M_0 \rho M_0^\dagger$$
$$M_1 = \sin Gt, \qquad p_1 = \mathrm{tr}\, M_1 \rho M_1^\dagger$$

Any two-outcome measurement

Apply feedback:

$$\tilde{M}_0 = U_0 M_0 = e^{-iI\otimes G_0} M_0$$
$$\tilde{M}_1 = U_1 M_1 = e^{-iI\otimes G_1} M_1$$

Polar decomposition theorem:

Any operator can be written as $M = U\sqrt{M^\dagger M}$ for some unitary U.

Any generalized measurement

Build up iteratively:

$$N_0^\dagger N_0 + N_1^\dagger N_1 + N_2^\dagger N_2 = 1$$

$$N_1' = \sqrt{N_1^\dagger N_1 + N_2^\dagger N_2}$$

If N_1' is obtained, measure

$$\begin{aligned} M_0 &= N_1 N_1'^{-1} \\ M_1 &= N_2 N_1'^{-1} \end{aligned}$$

Etc.

Any quantum operation

Discard ("forget") measurement outcomes:

$$\begin{aligned}\rho' &= \sum_l p_l (M_{kl}\rho M_{kl}^\dagger / p_l) \\ &= \sum_l M_{kl}\rho M_{kl}^\dagger\end{aligned}$$

The single qubit interface is *universal*. ■

Efficiency & Quantum Computation

Let d $= \dim \mathcal{H}_{sys}$

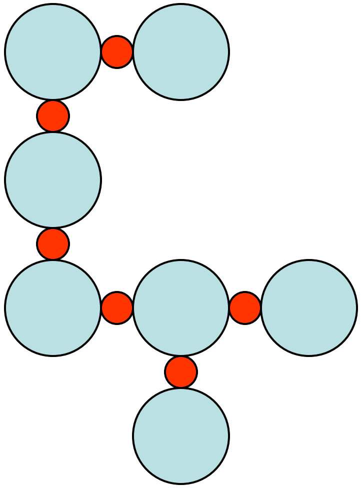
$O(d^2)$ basis of unitaries: $O(d^2)$ time to simulate any one

Efficient quantum algorithms: $d=2^n$, O(polylog d) sized circuits

Need O(n) Universal Quantum Interfaces (UQIs)!

Quantum Logic:

1) Connect qubits pairwise by UQIs
2) Measure UQIs to decouple qubits
3) O(n) SWAPs to bring qubits adjacent
4) O(1) time to effect a 2-qubit gate
5) 2-qubit gates are universal.



State Preparation & Measurement:

Decouple qubits and selectively measure, rotate qubits

Candidate UQIs

1) 1 Electromagnetic mode in optical cavity
2) 1 Ion in an ion trap
3) 1 Nuclear spin in a molecule
4) 1 Flux/charge qubit in a superconducting circuit
5) 1 Optically active site on a molecule
6) …

A degree of freedom can be tailored to be a UQI!

Open Questions

Open quantum systems:

- Use of UQIs for decoupling open quantum systems from their environment? (DFS/QEC)

Efficiency:

- Quantify controllability/observability of open quantum systems by UQIs.

Networkability:

- O(1) UQIs for quantum computation with additional controls?

Read more at:
quant-ph/0303048