Universal Quantum Interfaces
or
The Power of One (Qubit)

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In collaboration with:
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Based on:
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Entering the Era of Quantum Control

Cavity QED

Liquid NMR

Quantum Dot

Lattice BEC

Ion Trap
Promises of Full Quantum Control

- Computation
- Teleportation
- Cryptography
- Communication
- Simulation
Missing: Roadmap!

How much control is enough?

1) Full control of one qubit.
2) Generic coupling to a closed quantum system.

“Quantum Interface”
Interfaces & Control Loops

Environment

Controller

Adaptive Measurement

System

Backaction

Observer

Computer
Interfaces & Control Loops

System

Interface

Computer
Universality

Kraus Representation Theorem:

Any linear, completely positive map between density matrices may be written as:

\[ \rho' = \sum_l M_{kl} \rho M_{kl}^\dagger \]

where

\[ \text{prob}(k) = \text{tr} \rho', \quad \sum_{kl} M_{kl}^\dagger M_{kl} = 1 \]

Church of the Larger Hilbert Space:

\[ \rho' = \text{tr}_B \langle k | U (\rho \otimes |0\rangle \langle 0|_A \otimes |0\rangle \langle 0|_B ) U^\dagger | k \rangle \]

\[ \text{prob}(k) = \text{tr} \rho', \quad U^\dagger U = 1 \]

*unnormalized
Outline of construction

Given:
- Fixed generic interaction \( \sigma_z \otimes A \)
- Closed-system Hamiltonian \( I \otimes H \)
- Projective spin measurement \( [(I \pm \sigma_z)/2] \otimes I \)
- Arbitrary qubit Hamiltonian \( \gamma \sigma \otimes I \)

Outline:
1) Fixed interaction \( \rightarrow \) Arbitrary interaction
2) Coherent system control
3) Any Hermitian two-outcome measurement
4) Any two-outcome measurement
5) Any generalized measurement
6) Any quantum operation
Fixed interaction $\rightarrow$ Any Interaction

Key formulas:

$$e^{i(A+B)} = \lim_{n \to \infty} (e^{iA/n}e^{iB/n})^n$$

$$e^{-[A,B]} = \lim_{n \to \infty} (e^{iA/n}e^{iB/n}e^{-iA/n}e^{-iB/n})^{n^2}$$

Generically $\{A, H\}$ generate full Lie Algebra on $\mathcal{H}_{sys}$

$$[H + \sigma_z \otimes A, H + \sigma_z \otimes A + \gamma \sigma_x] = i\sigma_y \otimes A$$

$$[H + \sigma_z \otimes A, H + \sigma_z \otimes A + \gamma \sigma_y] = -i\sigma_x \otimes A$$

$$[\sigma_x \otimes A, \sigma_y \otimes A] = i\sigma_z \otimes A$$

$$[H + \sigma_z \otimes A, \sigma_z \otimes A] = \sigma_z \otimes [H, A] = \sigma_z \otimes G$$
Coherent system control

Simple, given arbitrary interactions:

\[
[\sigma_z \otimes G_1, \sigma_z \otimes G_2] = I \otimes [G_1, G_2] = I \otimes G
\]
Any Hermitian two-outcome measurement

1) Prepare interface in state $|0\rangle$.
2) Evolve via Hamiltonian $\sigma_x \otimes G$.
3) Measure interface.

\[
\begin{align*}
\rho_0 &= \langle 0 | e^{-i \sigma_x G t} \rho e^{i \sigma_x G t} | 0 \rangle = M_0 \rho M_0^\dagger / p_0 \\
\rho_1 &= \langle 1 | e^{-i \sigma_x G t} \rho e^{i \sigma_x G t} | 1 \rangle = M_1 \rho M_1^\dagger / p_1 \\
M_0 &= \cos G t, \quad p_0 = \text{tr} M_0 \rho M_0^\dagger \\
M_1 &= \sin G t, \quad p_1 = \text{tr} M_1 \rho M_1^\dagger 
\end{align*}
\]
Any two-outcome measurement

Apply feedback:

\[
\begin{align*}
\tilde{M}_0 &= U_0 M_0 = e^{-iI \otimes G_0} M_0 \\
\tilde{M}_1 &= U_1 M_1 = e^{-iI \otimes G_1} M_1
\end{align*}
\]

Polar decomposition theorem:

Any operator can be written as

\[
M = U \sqrt{M^\dagger M}
\]

for some unitary \( U \).
Any generalized measurement

Build up iteratively:

\[ N_0^\dagger N_0 + N_1^\dagger N_1 + N_2^\dagger N_2 = 1 \]

\[ N_1' = \sqrt{N_1^\dagger N_1 + N_2^\dagger N_2} \]

If \( N_1' \) is obtained, measure

\[ M_0 = N_1 N_1'^{-1} \]
\[ M_1 = N_2 N_1'^{-1} \]

Etc.
Any quantum operation

Discard ("forget") measurement outcomes:

\[
\rho' = \sum_l p_l (M_{kl} \rho M_{kl}^\dagger / p_l)
\]

\[
= \sum_l M_{kl} \rho M_{kl}^\dagger
\]

The single qubit interface is universal.
Efficiency & Quantum Computation

Let $d = \dim \mathcal{H}_{sys}$

$O(d^2)$ basis of unitaries: $O(d^2)$ time to simulate any one

Efficient quantum algorithms: $d=2^n$, $O(\text{polylog } d)$ sized circuits

Need $O(n)$ Universal Quantum Interfaces (UQIs)!

Quantum Logic:
1) Connect qubits pairwise by UQIs
2) Measure UQIs to decouple qubits
3) $O(n)$ SWAPs to bring qubits adjacent
4) $O(1)$ time to effect a 2-qubit gate
5) 2-qubit gates are universal.

State Preparation & Measurement:
Decouple qubits and selectively measure, rotate qubits
### Candidate UQIs

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>Electromagnetic mode in optical cavity</td>
</tr>
<tr>
<td>2)</td>
<td>Ion in an ion trap</td>
</tr>
<tr>
<td>3)</td>
<td>Nuclear spin in a molecule</td>
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<tr>
<td>4)</td>
<td>Flux/charge qubit in a superconducting circuit</td>
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<tr>
<td>5)</td>
<td>Optically active site on a molecule</td>
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<tr>
<td>6)</td>
<td>…</td>
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A degree of freedom can be tailored to be a UQI!
Open Questions

Open quantum systems:

• Use of UQIs for decoupling open quantum systems from their environment? (DFS/QEC)

Efficiency:

• Quantify controllability/observability of open quantum systems by UQIs.

Networkability:

• $O(1)$ UQIs for quantum computation with additional controls?

Read more at: quant-ph/0303048