Spin-Based Quantum Information Processing in Nanostructures

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Overview

- electron spin as qubit in quantum dots

- exchange and transport through double dots

- decoherence of spins in GaAs dots:
  1. how to measure spin: ESR and electrical current
  2. dominant source of decoherence: nuclear spin (hyperfine interaction) ➔ non-Markovian behavior (power laws)

Spin qubits in solids

Loss & D. DiVincenzo, 1997

Key Idea: spin-to-charge conversion, i.e. control of spin via electrical gates:

1. single qubit: via Zeeman, magnets, QHE edge states, magnetic semicond., g-factor, ESR,…

2. XOR gate: via double quantum dot & exchange control
=> deterministic entanglement

3. Read-out: - spin filter and charge detection (SET)
- spin-polarized charge current

advantage of spin over charge:

long decoherence times

$$\tau_{\phi}^{spin} \gg \tau_{\phi}^{charge}$$

> 0.1-100 μs

1 ns

➔ natural choice for qubit: spin ½ of electron

spin qubits in solid state:

Loss & DiVincenzo,	1997
Privman et al.,	1998
Kane/Clarke	1998
Awschalom	1999
Imamoglu et al.	1999
Barnes et al.,	2000
Yablonovich et al.,	2000
Das Sarma, Hu & Koiller,	2000/2
Yamamoto et al.,	2000/2
Levy,	2001
Whaley, Lidar et al.	2000
Kouwenhoven, Tarucha,	2001
Marcus, Westervelt	2001
Friesen et al.	2002
Ensslin, Salis	2002/3
Abstreiter, Kotthaus, Blick,..	2000/3

Quantum XOR via Heisenberg exchange

$$U(t) = T\exp\left\{-\frac{i}{\hbar}\int_0^t H(t')\,dt'\right\},\ H \neq 0 \text{ during } \tau_s$$

•**Heisenberg exchange** $H = J\mathbf{S}_1 \cdot \mathbf{S}_2$ for U_{sw} and $U^{1/2}_{sw}$:

$$\Rightarrow U(t) = e^{i\pi/4}U_{sw} = e^{i\pi/4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

basis:
$\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$

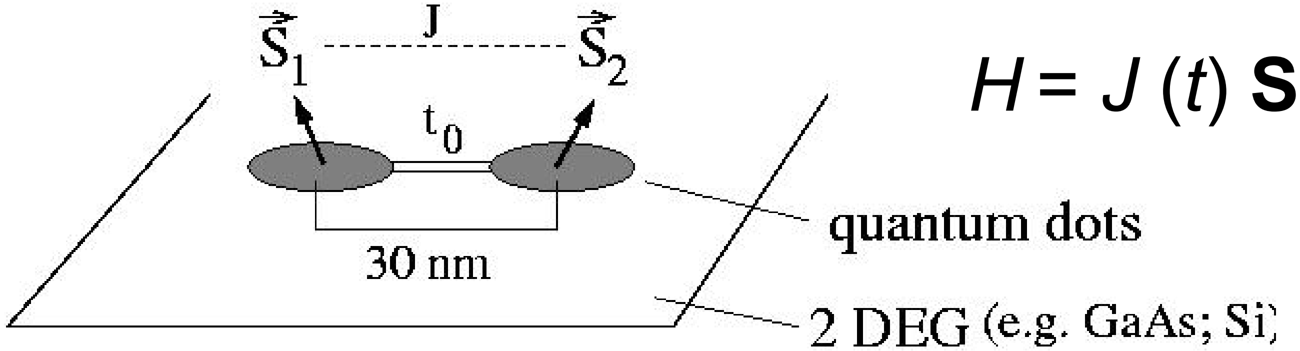
i.e. swap gate: qubit 1 ⟷ qubit 2, for $\int^t J(t)/\hbar \approx J_0\tau_s/\hbar = \pi\,(\mathrm{mod}\,2\pi)$

•**Zeeman** $H_B = \mathbf{B}_1 \cdot \mathbf{S}_1 + \mathbf{B}_2 \cdot \mathbf{S}_2$ for single-qubit operations

$$➔\quad U_{\mathrm{XOR}} = e^{i\frac{\pi}{2}S_1^z}\ e^{-i\frac{\pi}{2}S_2^z}\ U_{\mathrm{sw}}^{\frac{1}{2}}\ e^{i\pi S_1^z}\ U_{\mathrm{sw}}^{\frac{1}{2}}$$

Loss+DiVincenzo, PRA **57** (120), 1998

- **quantum gate = two coupled dots**



$$H = J\,(t)\; \mathbf{S}_1 \cdot \mathbf{S}_2$$

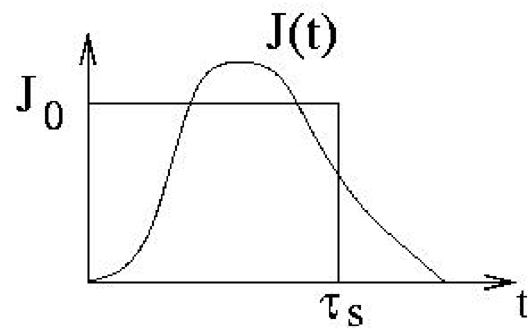
- idea: Hubbard physics: $J(t) \approx 4\; t_0(t)^2/U$

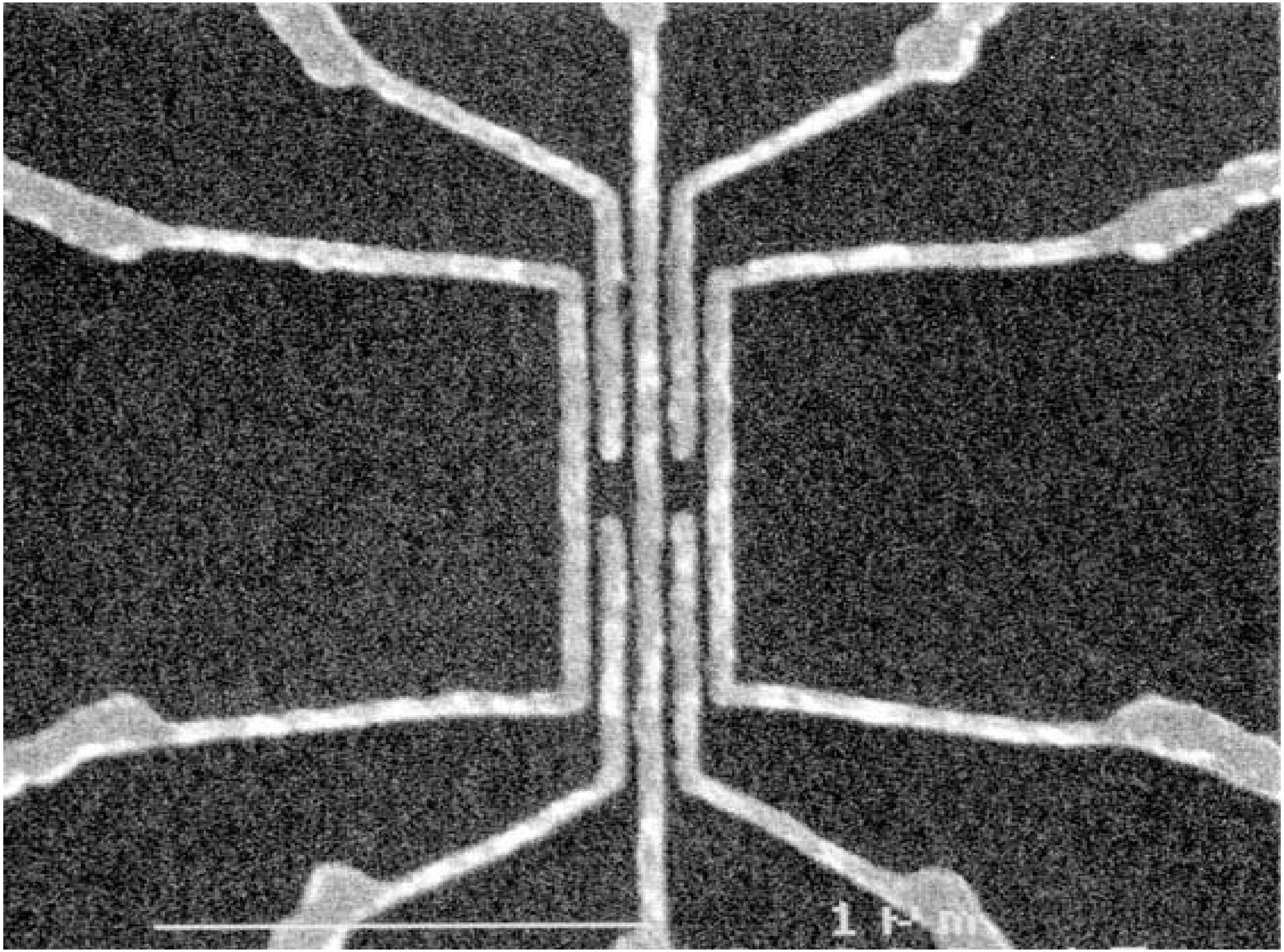
  $t_0 = t_0(t)$: tunable tunneling barrier

- e.g. square root of swap $U_{SW}^{1/2}$:

$$\int_0^t J(t')dt'/\hbar \approx J_0 \tau_S / \hbar = \pi / 2 (\mathrm{mod}\, \pi)$$

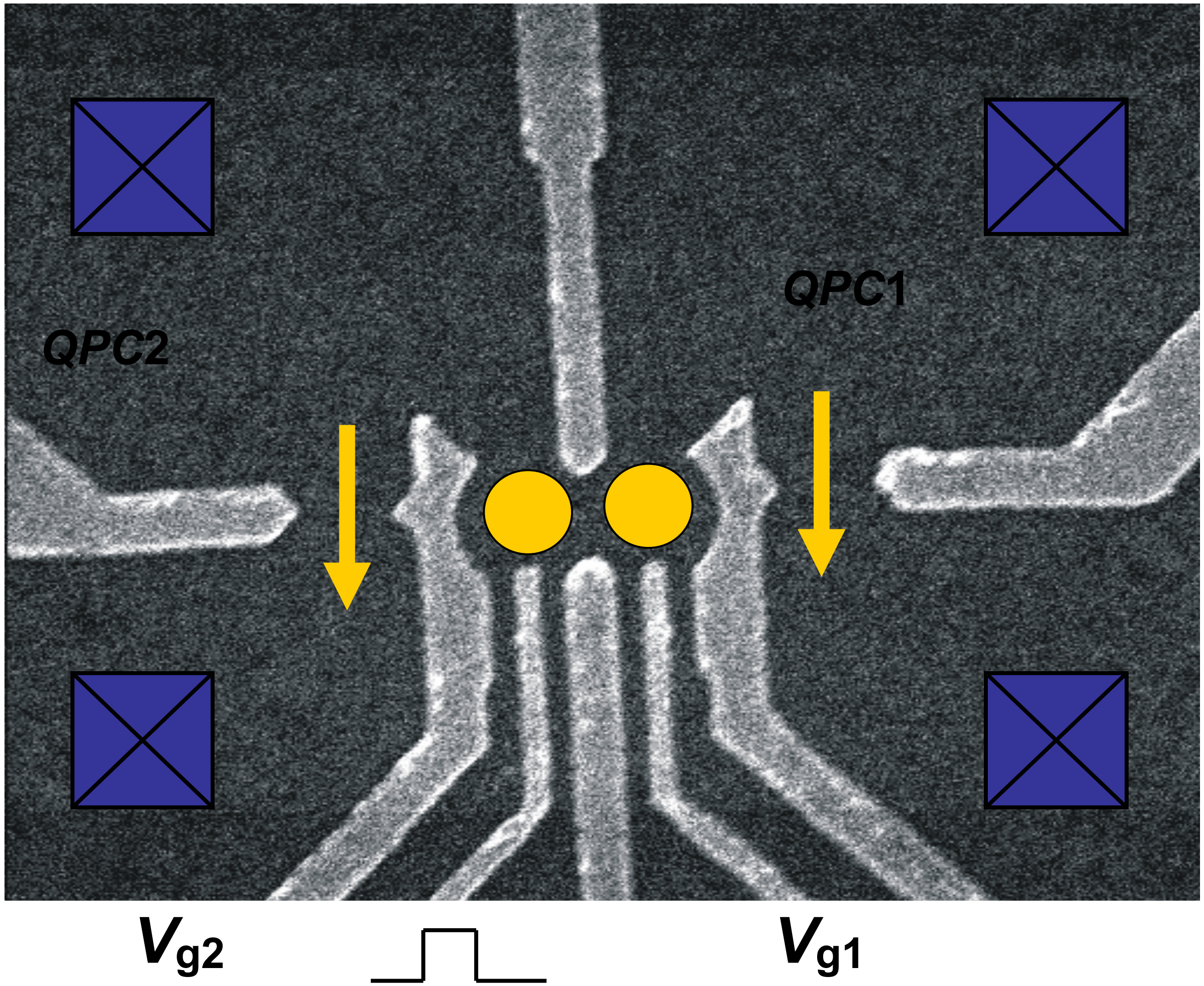
note: τ_s = 50 ps << T_2 = 150 ns (GaAs)





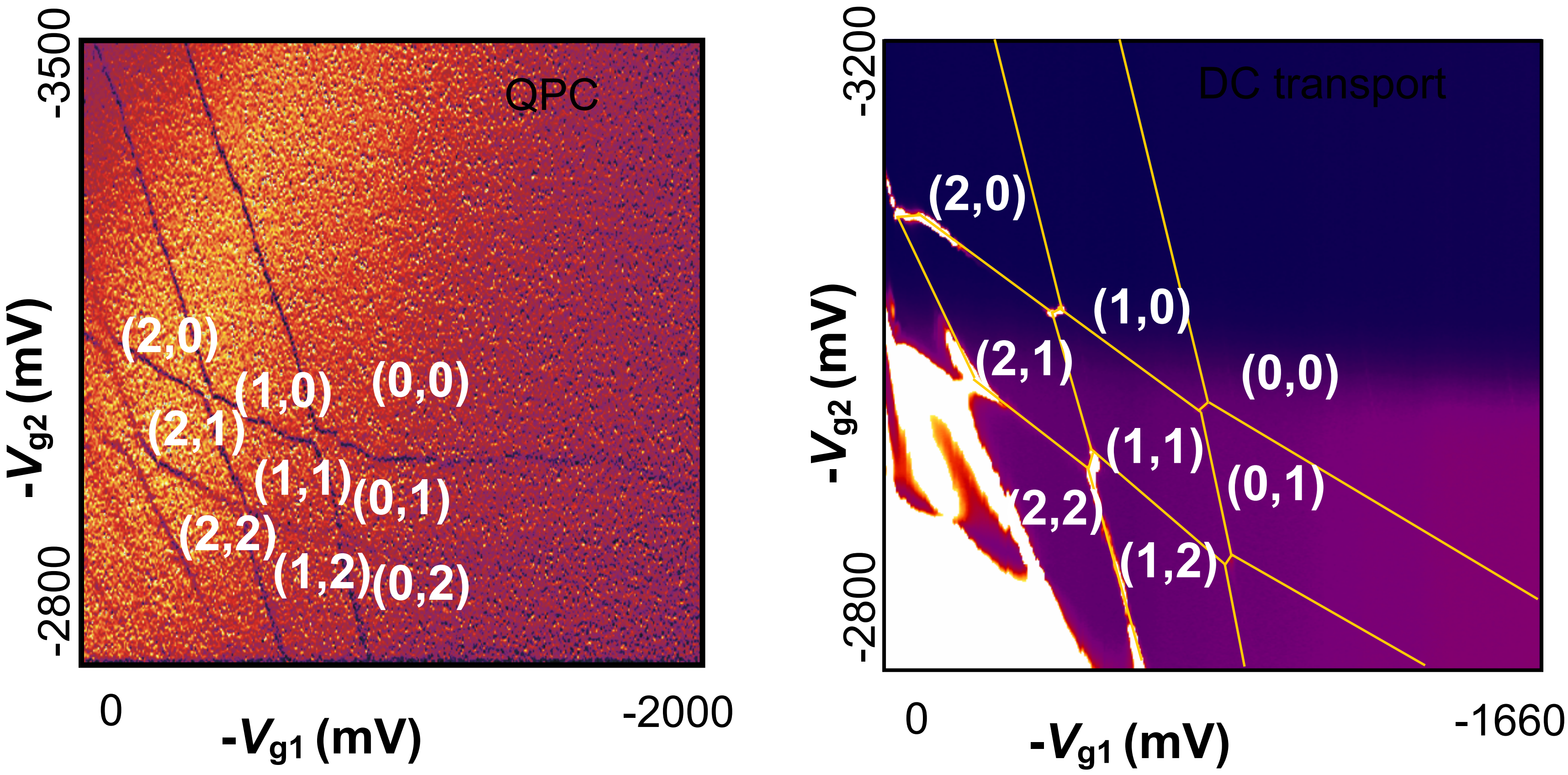
Westervelt et al., 2002

Double dots, GaAs, density = $2.9 \times 10^{15}\ m^{-2}$



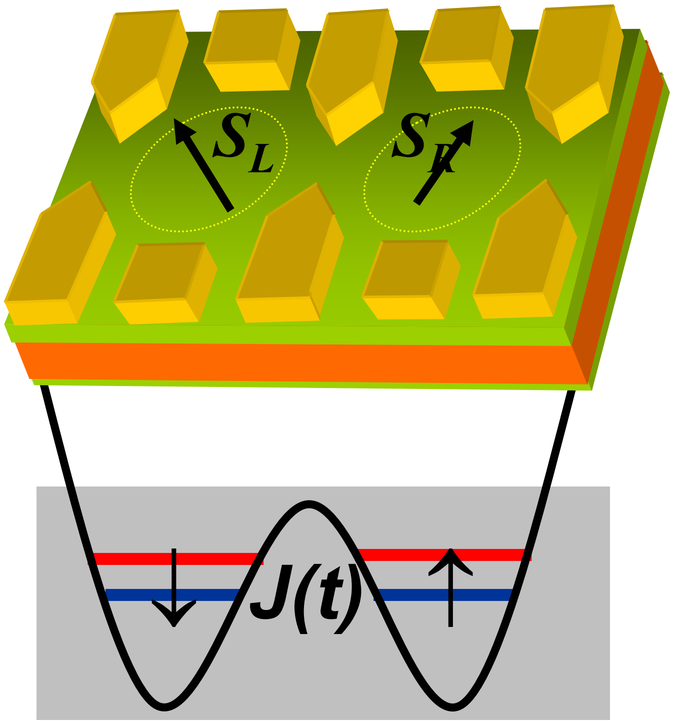
Kouwenhoven & Tarucha et al., 2002
cond-mat/0212489 (PRB03)

Kouwenhoven et al., 2002



Exchange coupling J(t) in double dot:

$$H_s(t) = J(t)\, \mathbf{S_L} \cdot \mathbf{S_R}$$



``deterministic entanglement``

1. theory for artificial atoms and molecules ➔ exchange J
2. theory for electrical current through system (➔ measurements)

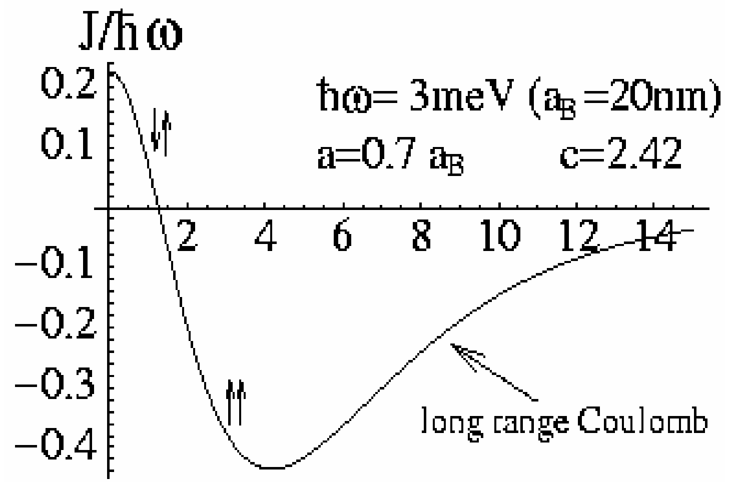
Heitler-London

- single-dot problem in a magnetic field has exact solution (Fock '28,Darwin '30) $\rightarrow \quad \varphi(\vec{r})$
  two-particle trial wavefunction (Heitler-London)

$$\begin{aligned} \Psi_\pm &= N\left[\varphi_{-a}(\vec{r}_1)\varphi_{+a}(\vec{r}_2) \pm \varphi_{-a}(\vec{r}_2)\varphi_{+a}(\vec{r}_1)\right] \\ J &= \langle \Psi_- | H_{el} | \Psi_- \rangle - \langle \Psi_+ | H_{el} | \Psi_+ \rangle \end{aligned}$$

- results: $d = a/a_B,\ b^2 = 1 + \omega_L^2/\omega_0^2,\ c \sim (e^2/\epsilon a_B)/\hbar\omega_0,\ \omega_L = eB/2m$
  (Burkard,Loss,DiVincenzo '99)

$$\begin{aligned} J \;=\; & \frac{\hbar\omega_0}{\sinh\left(2d^2(2b-1/b)\right)}\left[c\sqrt{b}\left(e^{-bd^2}\mathrm{I}_0\left(bd^2\right)\right.\right. \\ & \left.\left. -\, e^{d^2(b-1/b)}\mathrm{I}_0(d^2(b-1/b))\right) + \frac{3}{4b}\left(1+bd^2\right)\right] \end{aligned}$$



- Theorem: $J > 0$ for 2 electrons and $B = 0$.

  (see also numerics by X. Hu et al., PRB '00, include higher orbitals)

Transport through double dots in Coulomb Blockade

Loss & Sukhorkov, Phys. Rev. Lett. **84**, 1035 (2000); V. Golovach & D. L., '01, '03

via current?

two spins interact via exchange interaction: $H_{\mathrm{dot}} = K\,\mathbf{S}_L \cdot \mathbf{S}_R$

$$K = E_t - E_S$$

The relevant states are:

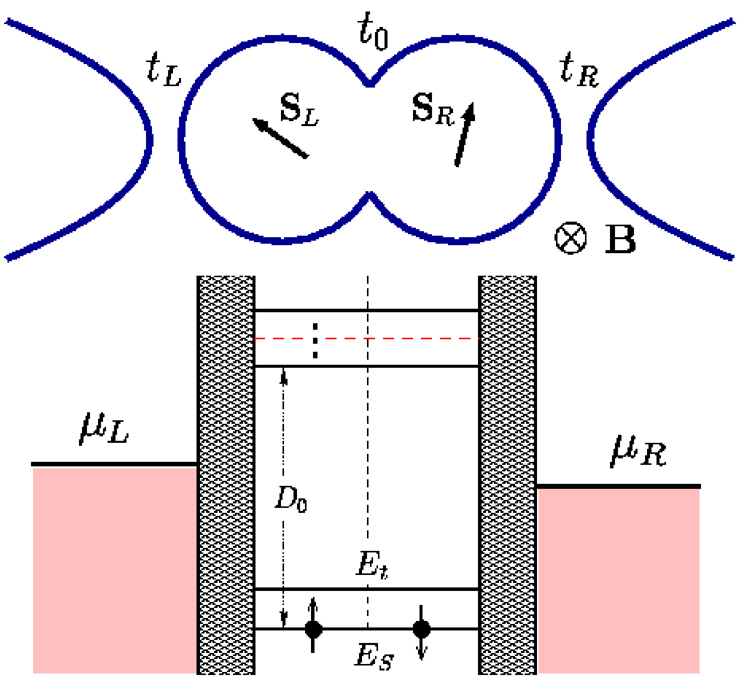
$$|00\rangle = \frac{1}{\sqrt{1+\phi^2}}(d^\dagger_{+\uparrow}d^\dagger_{+\downarrow} - \phi d^\dagger_{-\uparrow}d^\dagger_{-\downarrow})|0\rangle\ ,$$

$$|11\rangle = d^\dagger_{-\uparrow}d^\dagger_{+\uparrow}|0\rangle\ , \quad |1-1\rangle = d^\dagger_{-\downarrow}d^\dagger_{+\downarrow}|0\rangle\ ,$$

$$|10\rangle = \frac{1}{\sqrt{2}}(d^\dagger_{-\uparrow}d^\dagger_{+\downarrow} + d^\dagger_{-\downarrow}d^\dagger_{+\uparrow})|0\rangle\ ,$$

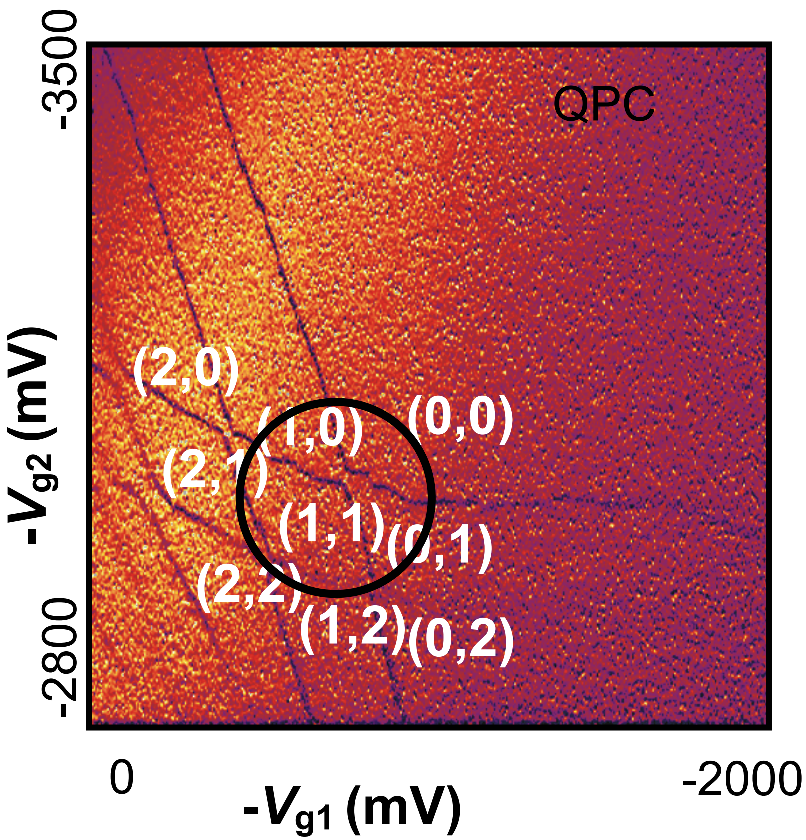
$$\phi = \sqrt{1+\left(\frac{4t_H}{U_H}\right)^2} - \frac{4t_H}{U_H}$$

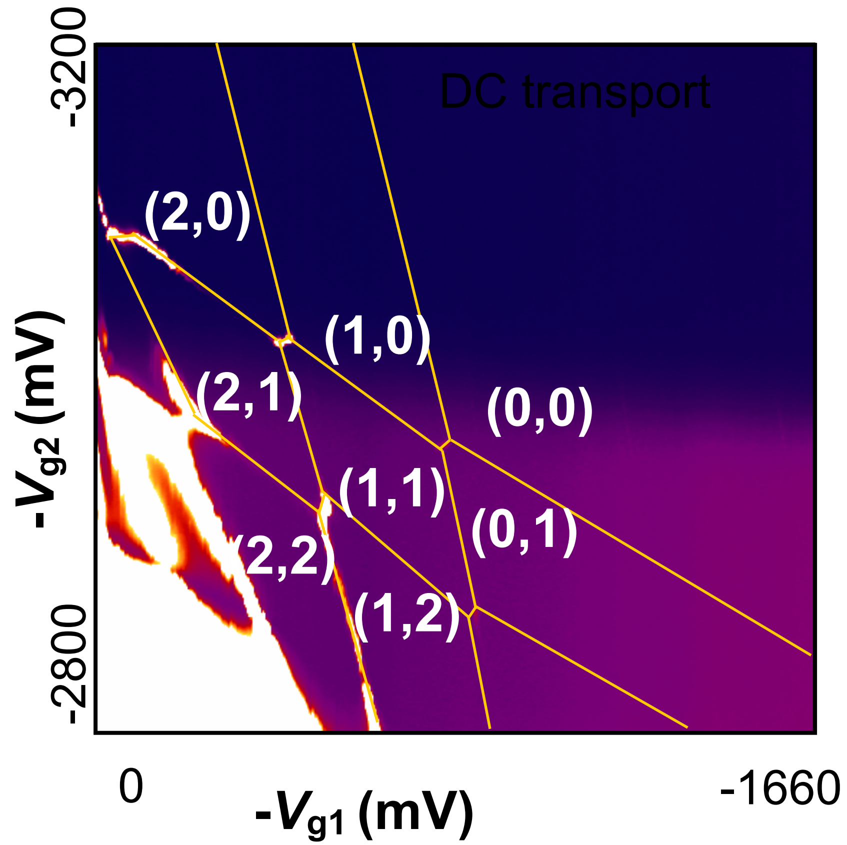
tunnel coupling: $t_H = t_0 + t_C$



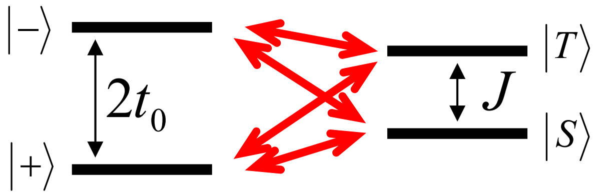
e.g. sequential or cotunneling regime

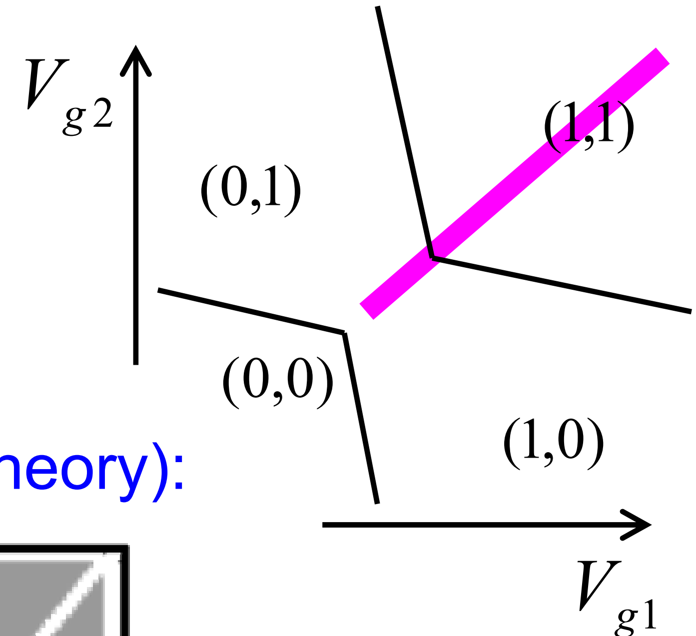
Kouwenhoven et al., 2002



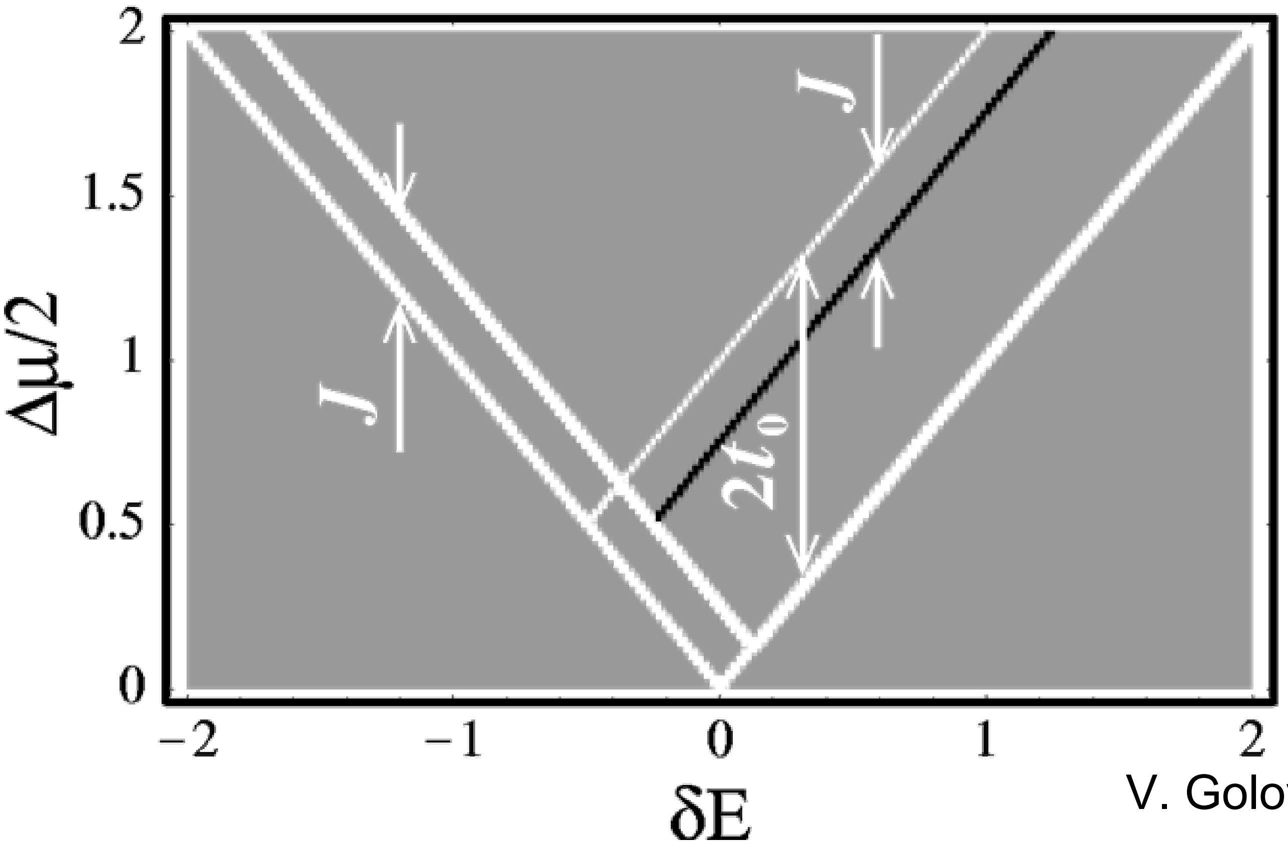


Calculate I vs V ➔ peaks (dips) in $\mathrm{d}I/\mathrm{d}V$ (heating effects)





result for sequential tunneling regime (theory):

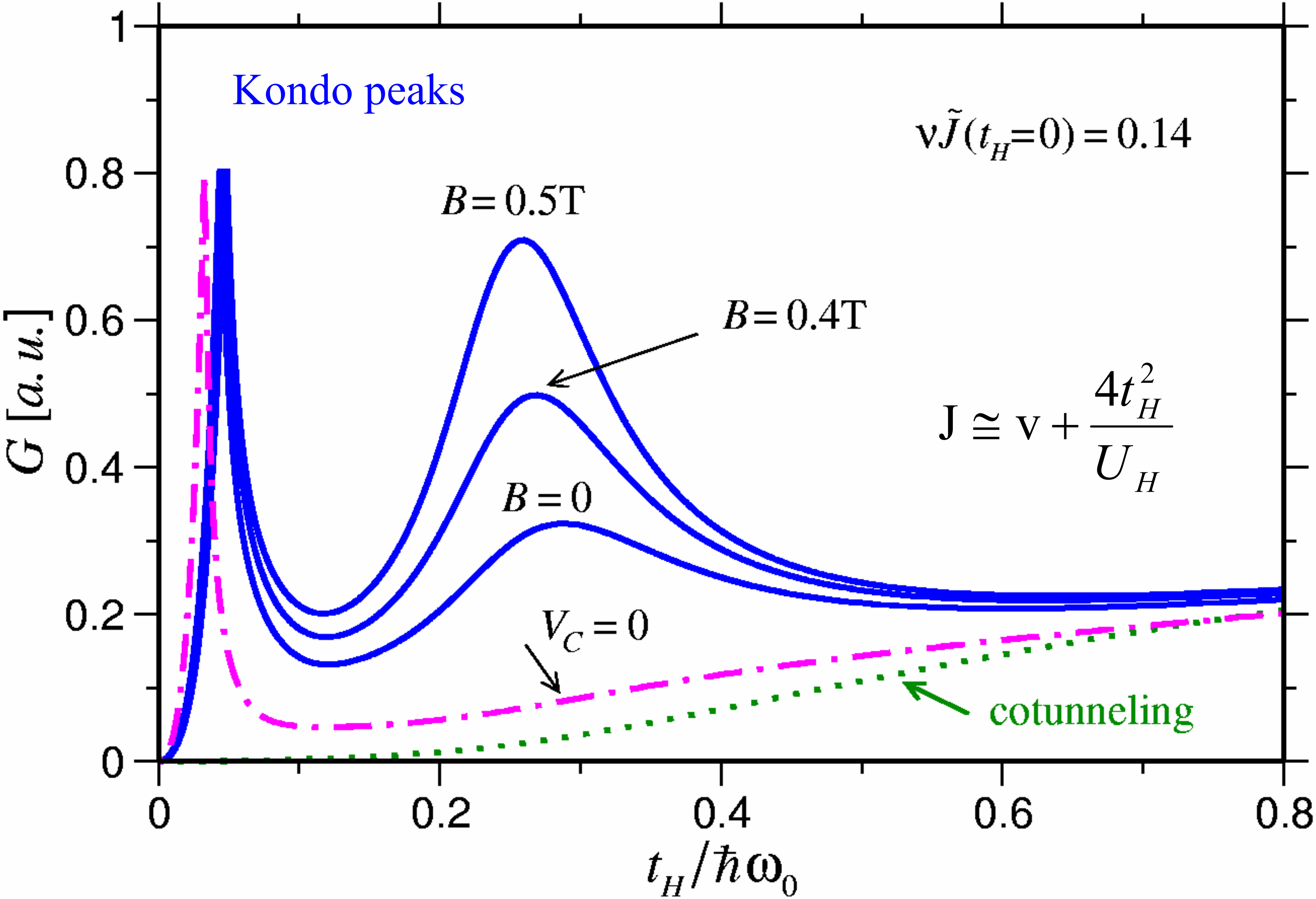


$$\delta E = E_{|+\rangle} - E_{|S\rangle} + \mu$$

$$\mu_L = \mu + \Delta\mu/2$$

$$\mu_R = \mu - \Delta\mu/2$$

V. Golovach & D. Loss, '03



V.N. Golovach and D. Loss, Europhys. Lett. **62**, 83 (2003)

Swichting Rate

Determine $N_{Op} \approx \tau_\phi / \tau_s$ for GaAs

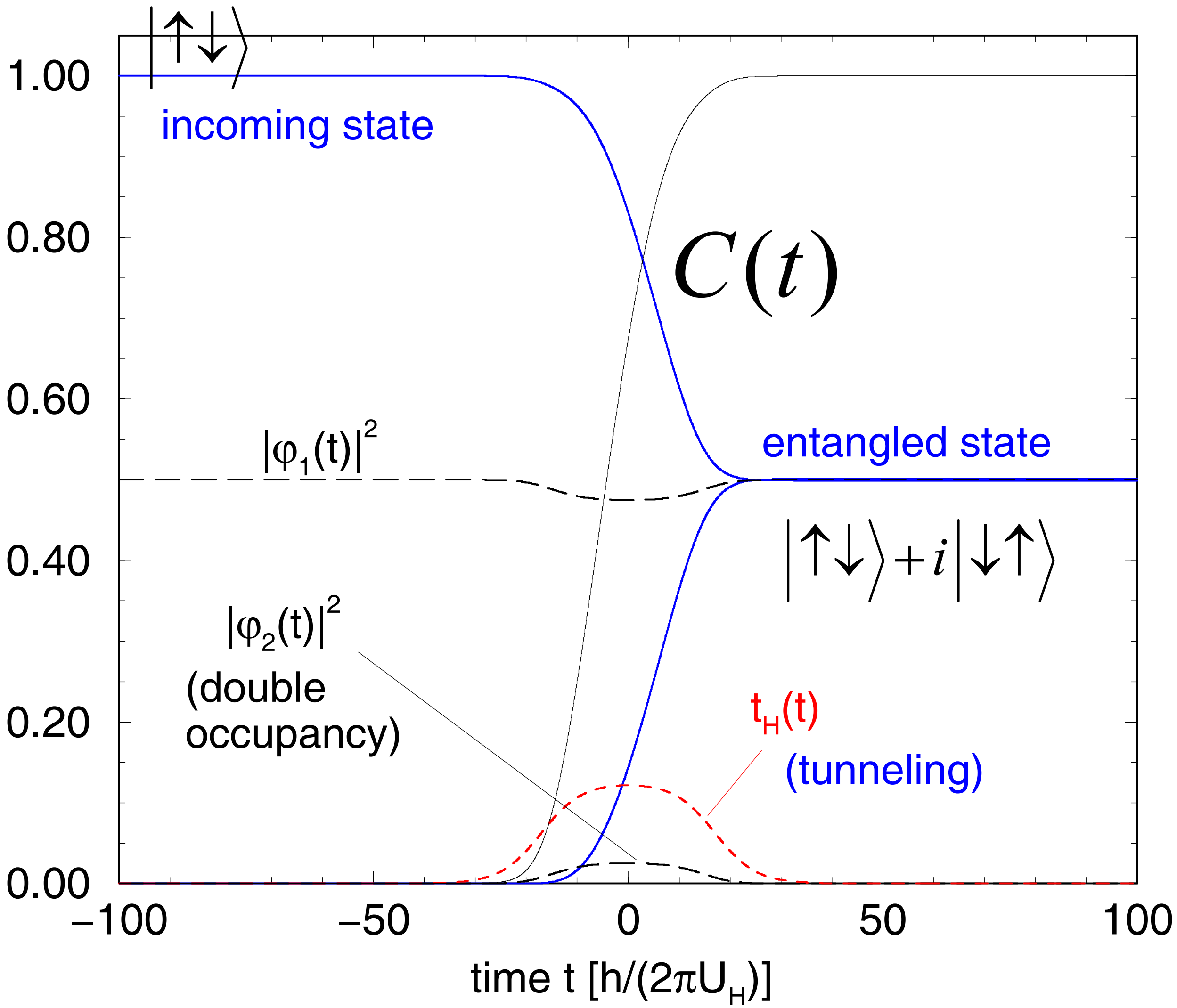
- calculate *J*(*v*) **statically** and then take *J*(*t*) = *J*(*v*(*t*)) for time-dependent *v*(*t*) (where *v* = *V*, *B*, *a*, or *E* = control parameter)
- sufficient criterion for this to work [$\bar{J} = (1/\tau_S) \int_0^{\tau_S} dt\, J(t)$]

$$1/\tau_S \approx |\dot{v}/v| \ll \bar{J}/\hbar$$ **adiabaticity condition**

- compatible with $J\tau_S = n\pi$, $n = 1, 3, 5, \ldots$ (needed for XOR)
- self-consistency of calculation of *J*: $J \ll \Delta\epsilon$
- thus: $1/\tau_S \ll \bar{J}/\hbar \ll \Delta\epsilon/\hbar$ $, \pi U^2 / 8t_0$ (no double occupancy)
- numbers: $J \approx 0.2\,\text{meV} \rightarrow$ $\tau_S \gtrsim 50\,\text{ps}$
- decoherence of spin ca. 100 ns (Awschalom & Kikkawa, `97)

➡ $$N_{Op} \approx \tau_\phi / \tau_s \approx 10^4$$ sufficient for upscaling

Dynamics of Entanglement for the square-root-of-swap



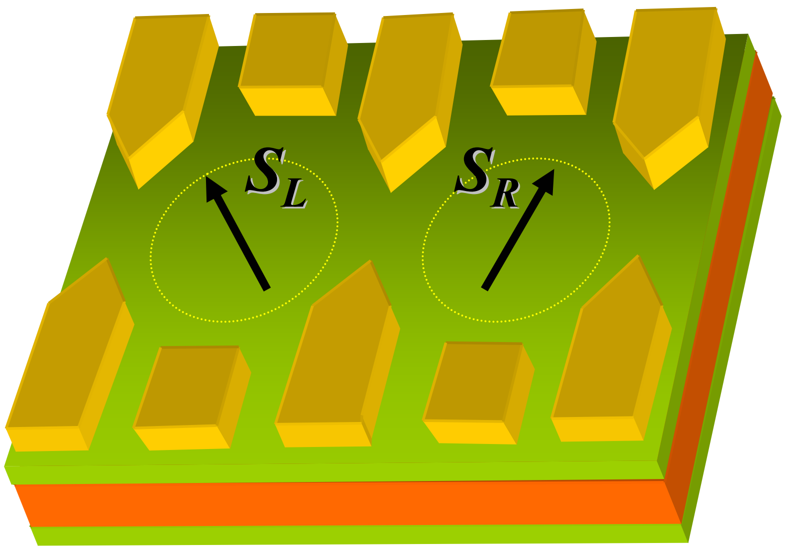
The square-root of a swap is obtained by halfing the duration of the tunneling pulse. The result is a fully entangled two-qubit state having only a vanishingly small amplitude for double-occupancies of one of the dots.

As before, during the process the indistinguishable character of the electrons and their fermionic statistics are essential.

J. Schliemann, D. Loss, and A. H. MacDonald, Phys. Rev. B **63**, 085311 (2001)

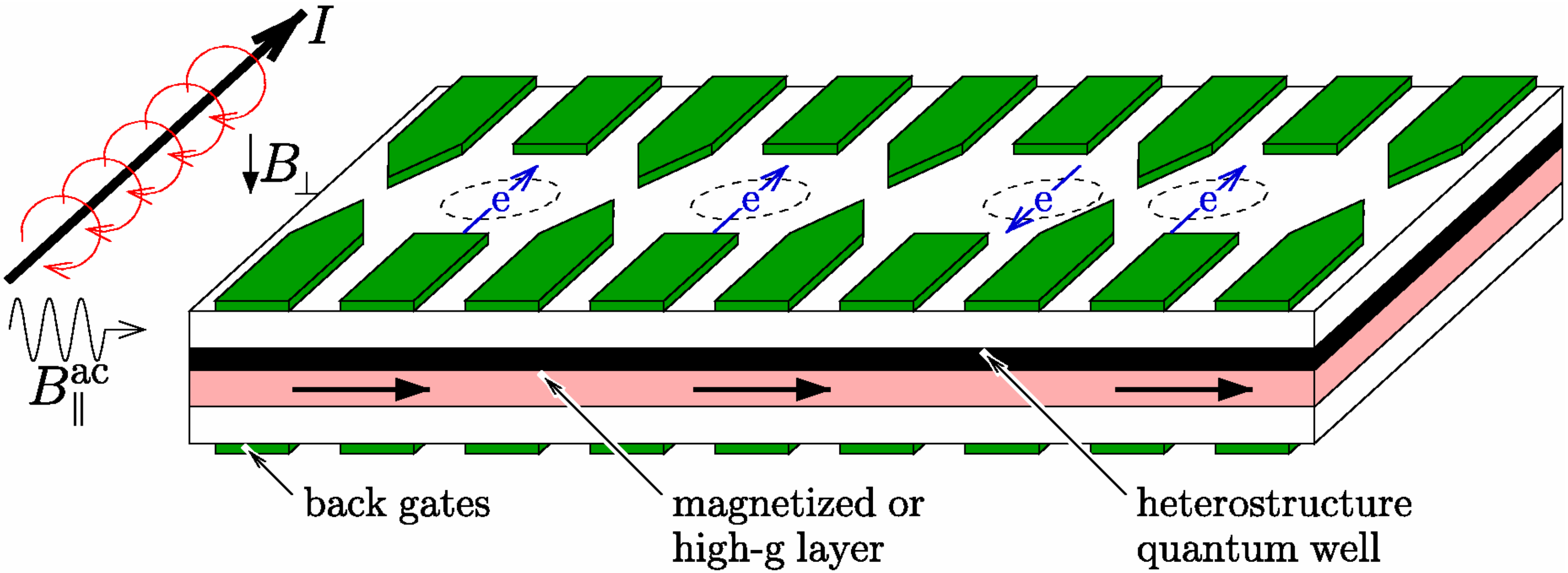
‘Quantum transistor’: double dot with gate control over exchange splitting $J(t)$

$$H_s(t) = J(t)\, \mathbf{S}_L \cdot \mathbf{S}_R$$



up scaling: connect N quantum transistors ➔

Scalable system: quantum dot array



$$H = \sum_{\langle ij \rangle} J_{ij}(t)\mathbf{S}_i \cdot \mathbf{S}_j + \sum_i (g_i \mu_B \mathbf{B}_i)(t) \cdot \mathbf{S}_i$$

n.n. exchange — local Zeeman

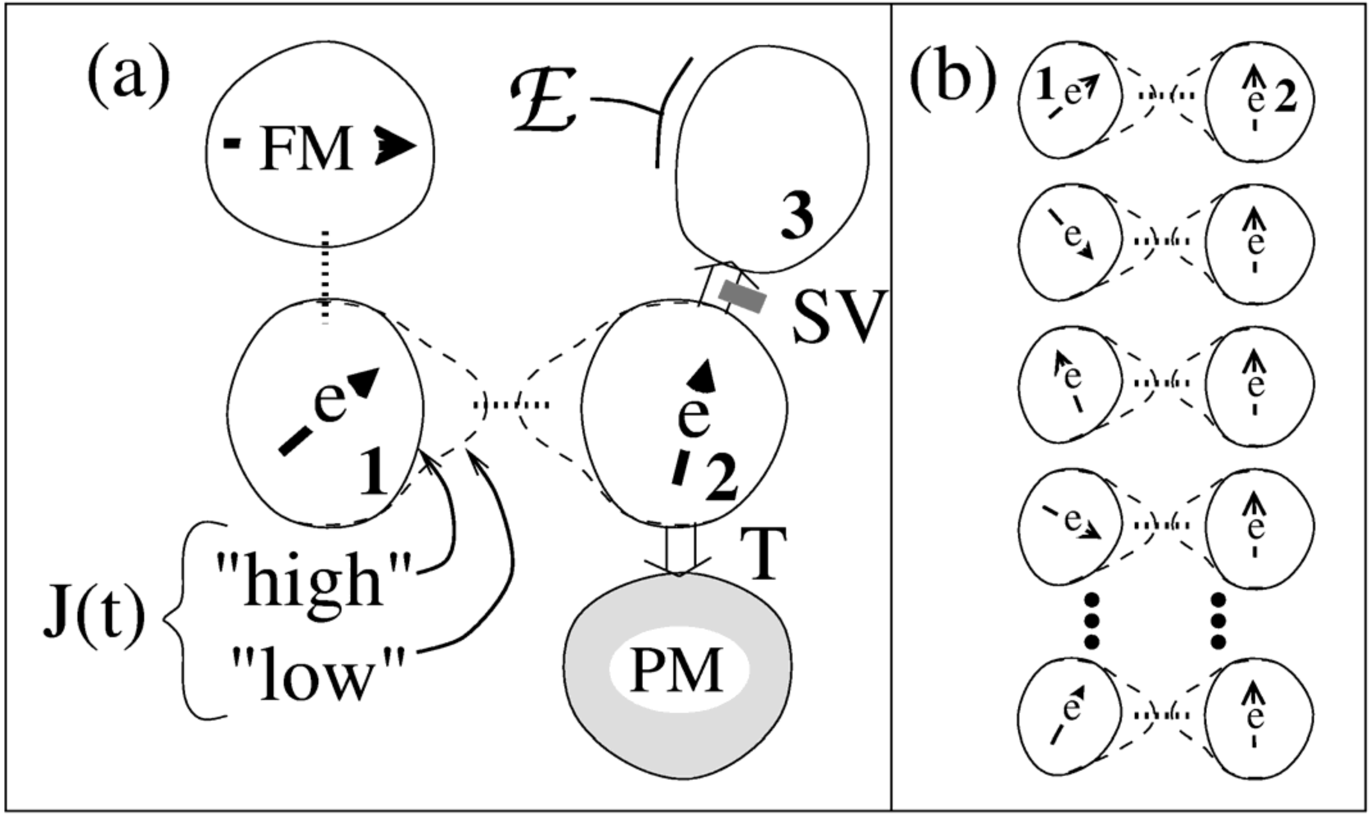
D. Loss & D. DiVincenzo, PRA **57** (1998) 120; cond-mat/9701055

All-electrical control of spin is possible:

1. single qubit: via Zeeman, magnets, QHE edge states, magnetic semicond., g-factor, ESR,…

2. XOR gate: via double quantum dot & exchange control
➔ deterministic entanglement

3. Read-out: - spin filter and charge detection (SET)
- spin-polarized charge current

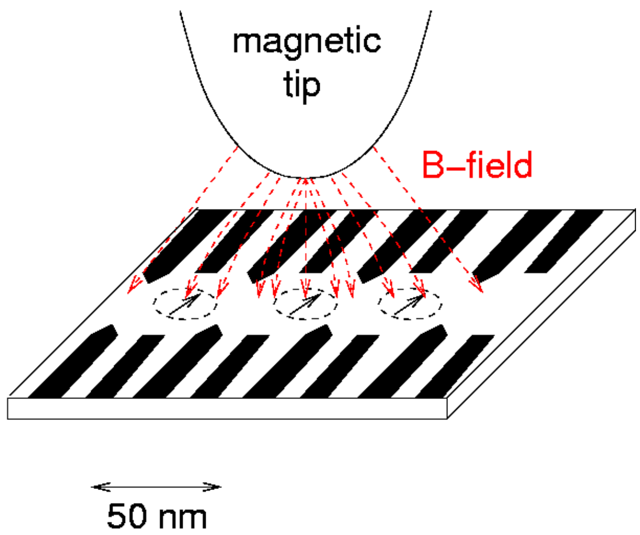


Loss&DiVincenzo, '97
PRA 57 (1998) 120

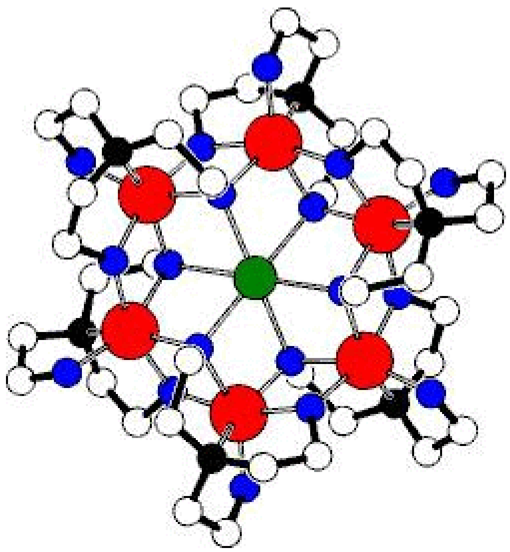
spin-to-charge conversion

FIG. 1. a) Schematic top view of two coupled quantum dots labeled 1 and 2, each containing one single excess electron (e) with spin 1/2. The tunnel barrier between the dots can be raised or lowered by setting a gate voltage "high" (solid equipotential contour) or "low" (dashed equipotential contour). In the low state virtual tunneling (dotted line) produces a time-dependent Heisenberg exchange $J(t)$. Hopping to an auxiliary ferromagnetic dot (FM) provides one method of performing single-qubit operations. Tunneling (T) to the paramagnetic dot (PM) can be used as a POV read out with 75% reliability; spin-dependent tunneling (through "spin valve" SV) into dot 3 can lead to spin measurement via an electrometer $\mathcal{E}$. b) Proposed experimental setup for initial test of swap-gate operation in an array of many non-interacting quantum-dot pairs. Left column of dots is initially unpolarized while right one is polarized; this state can be reversed by a swap operation (see Eq. (31)).

When local control difficult ➔ make your qubit large(r)



Local control of QDs?

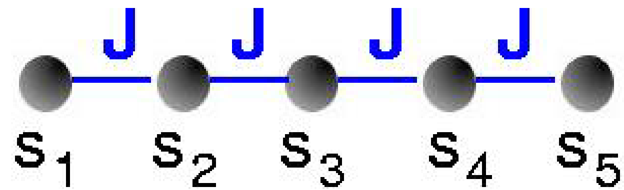


Is it necessary to control single ion spins for QC?

Collective qubit: spin clusters

F. Meier, J. Levy & D. Loss, Phys. Rev. Lett. **90**, 047901 (2003)

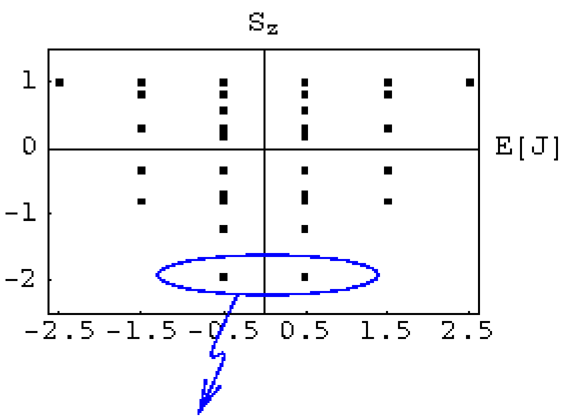
e.g. isotropic spin chain with n_c sites:
(e.g., neighboring QDs or P atoms)



$$\hat{H} = J \sum_{i=1}^{n_c-1} \hat{s}_i \cdot \hat{s}_{i+1}$$ with *J>0* (antiferromagnetic)

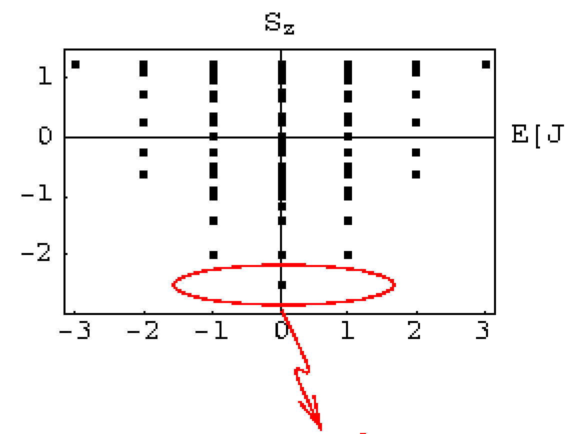
Spectrum?

$n_c = 5$



ground state doublet ➔ qubit

$n_c = 6$



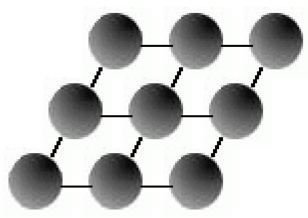
ground state singlet

Dimension d>1

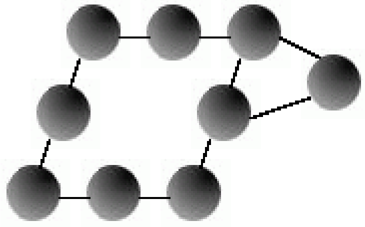
2d and 3d clusters

QC with spin clusters relies on existence of S=1/2 ground state
⇒ scheme extends to

- any bipartite lattice



- even to lattices with partial geometrical frustration



note: dipole interaction between cluster qubits reduced

Central issue for quantum computing: decoherence of qubit (spin,…)

• decoherence is unavoidable in realistic systems under realistic conditions

→ 1. How to measure decoherence for single spin?

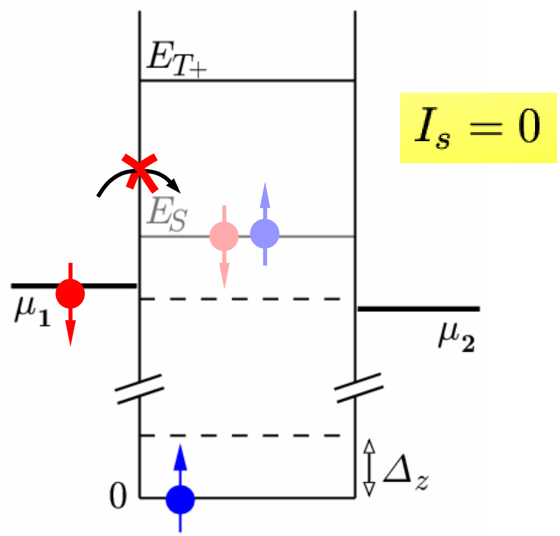
→ 2. Quantitative theories of spin decoherence

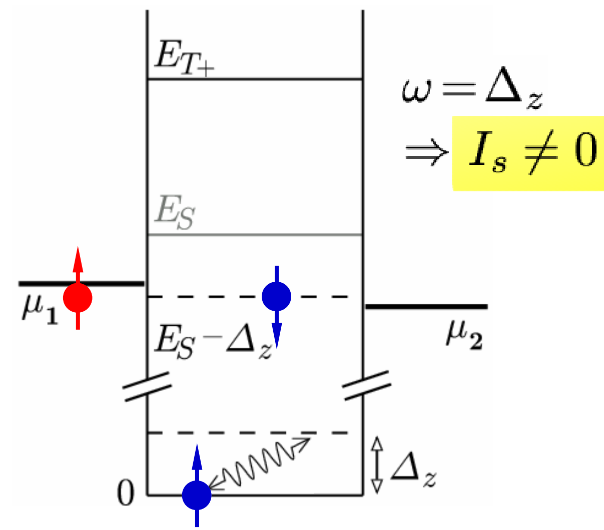
• hierarchy of decoherence times → eventually need to identify shortest one!

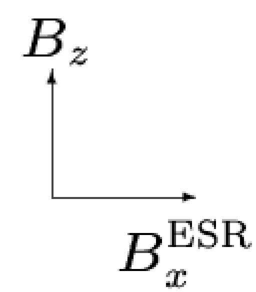
Spin decoherence T_2 via charge current

H.-A. Engel and D. Loss, Phys. Rev. Lett. **86**, 4648 (2001); Phys. Rev. B **65** 195321 (2002)

Quantum dot in sequential tunneling regime



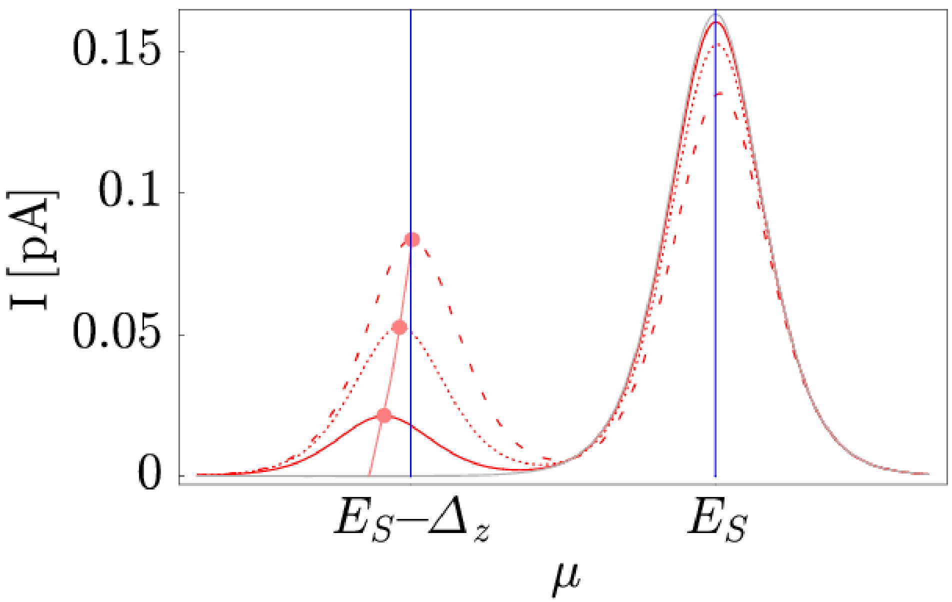




- Coulomb blockade regime, $E_S - E_\uparrow > \mu_1 > E_S - E_\downarrow > \mu_2$
  $[E_\uparrow = 0\!: \; E_S > \mu_1 > E_S - \Delta_z > \mu_2]$
- dot: Zeeman splitting $\Delta_z = g\mu_B B_z > k_{\mathrm{B}} T$
  leads: $\Delta_z^{\mathrm{leads}} \not\approx \Delta_z$ and $\Delta_z^{\mathrm{leads}} \ll \varepsilon_{\mathrm{F}}$
- ESR field $H_{\mathrm{ESR}} = \frac{1}{2}\Delta_x \cos(\omega t)\sigma_x$ of frequency $\omega \approx \Delta_z$
  $\Rightarrow$ Rabi flips are produced and current flows through the dot, involving state $|\downarrow\rangle$.

Spin satellite peak in ST current

- Stationary current $I(V_{\text{Gate}}) = I(\mu)$, $\mu = (\mu_1+\mu_2)/2$
  $I(\mu)$ peaked at $\mu \approx E_S$ and $\mu \approx E_S - \Delta_z$:



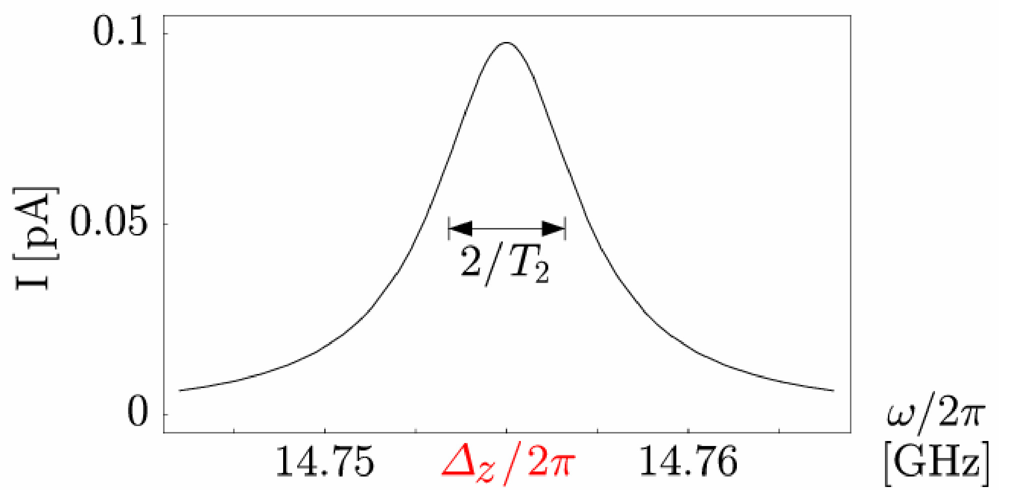
$B_z = 0.5$ T,
$g = 2$,
$T = 70$ mK,
$\Delta\mu = 6\ \mu$V,
$T_1 = 1\ \mu$s,
$T_2 = 100$ ns,
$\gamma_1 = 5 \times 10^6\ \text{s}^{-1}$, and
$\gamma_2 = 5\gamma_1$.

gray:	$W_\omega = 0$,	$B_x^0 = 0$,	
solid:	$W_\omega = \gamma_1/5$,	$B_x^0 = 0.75$ G,	(at $\omega = \Delta_z$)
dotted:	$W_\omega = \gamma_1$,	$B_x^0 = 1.7$ G,	(at $\omega = \Delta_z$)
dashed:	$W_\omega = 9\gamma_1$,	$B_x^0 = 5$ G,	(at $\omega = \Delta_z$)

- Spin satellite peak due to ESR field
- Peak height changes as function of ω and/or B_x^0, since $I=I(W_\omega)$ and $W_\omega = \frac{(g\mu_B B_x^0)^2/8T_2}{(\omega-\Delta_z)^2+1/T_2^2}$
- Satellite peak increases, main peak decreases for increasing W_ω

Spin T_2 via linewidth of current

- Stationary current $I(\omega) \propto W_\omega$, for $W_\omega^{\max} < \max\{W_{\uparrow\downarrow}, \gamma_1\}$.
  $I(\omega)$: Lorentzian in ω, peaked at $\omega = \Delta_z$.



$k_B T < \Delta\mu$,
$B_z = 0.5$ T,
$B_x^0 = 0.45$ G,
$g = 2$,
$T_1 = 1\ \mu$s,
$T_2 = 100$ ns,
$\gamma_1 = 5 \times 10^6\ \mathrm{s}^{-1}$, and
$\gamma_2 = 5\gamma_1$, i.e.
$W_\omega^{\max} \lesssim \gamma_1 \lesssim 1/T_2$.

- Linewidth $2V_{\downarrow\uparrow}$ gives lower bound for intrinsic spin decoherence time T_2.

- $V_{\downarrow\uparrow} = 1/T_2$ for $W_\omega^{\max} < \gamma_1 < 1/T_2$
  [e.g. $B_x^{\,0} = 0.08\ G$, $\gamma_1 = 5\times10^5\ s^{-1}$, thus $I(\Delta_z) \approx 1.5\ fA$]

Measurement of charge current $I(\omega)$ yields lower bound for single-spin decoherence time T_2 on dot.

Sources of spin decoherence in GaAs quantum dots:

- spin-orbit interaction (relativistic band structure effects):
  couples lattice vibrations with spin ➔ spin-phonon interaction, but
  weak in quantum dots due to 1. low momentum, 2. no 1st order s-o terms
  due to symmetry (Khaetskii&Nazarov, '00)

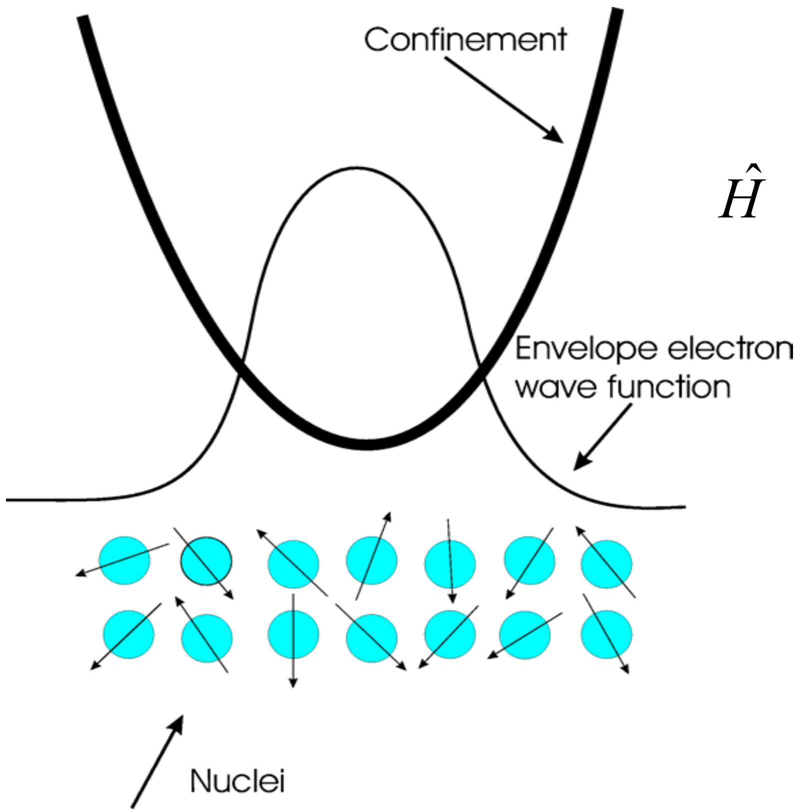
- note: gate errors (XOR) due to s-o can be minimized
  (Bonesteel et al., Burkard et al. '02, '03)

- dipole-dipole interaction: weak

- hyperfine interaction with nuclear spins: dominant decoherence source
  (Burkard, DL, DiVincenzo, PRB '99)

Electron spin decoherence in quantum dot due to nuclei

Khaetskii, Loss, Glazman, Phys. Rev. Lett. 88, 186802 (2002); cond-mat/0211678 (PRB)
Schliemann, Khaetskii, Loss, Phys. Rev. B66, 245303 (2002)



$$\hat{H} = g\mu_B \hat{\mathbf{S}}\cdot\mathbf{B} + \hat{\mathbf{S}}\cdot\sum_i A_i \hat{\mathbf{I}}_i + \hat{H}_{d-d}$$

hyperfine interaction

$$A_i \propto A\cdot|\Psi(\mathbf{r}_i)|^2$$

i.e. non-uniform hyperfine coupling: $A_i = A(r_i)$ varies with position r_i ,

➔ power-law decay of spin coherence $\langle S_z(t)\rangle \propto 1/t^{3/2}$

➔ decoherence suppressed when nuclei become polarized

Neglect dipole interaction ➔ total spin **J** conserved:

$$\mathbf{J} = \mathbf{S} + \Sigma_i \mathbf{I}_i = \text{const.}$$

But: each flip-flop process (due to hyperfine interaction) creates a different nuclear configuration ➔ different hyperfine field $\mathbf{H}_N$ seen by the electron spin in time due to spatial variation of the hyperfine constants A_i ➔ average over different electron spin precession frequencies ω_N ➔ electron spin decays !

Result (below): decay is non-exponential and is characterized by time $(A/N)^{-1} \sim 1\ \mu s$
➔ consistency check: $T_{n2} \sim 100\ \mu s >> (A/N)^{-1} \sim 1\ \mu s$ ➔ no averaging over nuclear configurations is needed ➔ dipolar interactions can be neglected for $t < T_{n2}$!

1) I=3/2, and 2 different hyperfine constants A_i in GaAs
   ➔ simplify (non-essential): I=1/2 and only one value for A_i
2) consider first a particular and unpolarized nuclear configuration $|\{I^i_z\}>$, with $I^i_z = \pm ½$, i.e. tensor product state.
   ➔ typical nuclear magnetic field is $H_N \sim A / (\sqrt{N}\, g\, \mu_B) << A/ g\, \mu_B$.

Perturbative evaluation of spin correlator C_n :

Consider decay of the electron spin from its initial (t=0) $\hat{S}_z$-eigenstate $|\Uparrow\rangle$
➔ evaluate spin correlator (time scale of decay = decoherence time):

$$C_n(t) = \langle n | \delta \hat{S}_z(t) \hat{S}_z | n \rangle$$

where $\delta \hat{S}_z(t) = \hat{S}_z(t) - \hat{S}_z$, $\hat{S}_z(t) = e^{it\hat{H}} \hat{S}_z e^{-it\hat{H}}$, *with* $\hat{H} = \hat{H}_0 + \hat{V}$

Here $\hat{H}_0 = \hat{S}_z \hat{h}_{Nz}$ is the free part, with eigenenergy ε_n, and the 'perturbation'

$$\hat{V} = (1/2)(\hat{S}_+ \hat{h}_{N-} + \hat{S}_- \hat{h}_{N+})$$

describes the flip-flop processes, i.e. $\Uparrow;..,\downarrow_k,.... \mapsto \Downarrow;...,\uparrow_k,.... \quad \forall\, k = 1,...,N$

In leading order in V, we obtain for the spin correlator

$$C_n(t) = \sum_k \frac{|V_{nk}|^2}{\omega_{nk}^2} (\cos(\omega_{nk} t) - 1) \,,$$

Define $\boldsymbol{\tau = At/2\pi N}$ [$N = a_z a^2 / v_0 >> 1$ nuclei inside dot, and a, a_z lateral/transverse dot lengths]
Then, asymptotically for $\boldsymbol{\tau >> 1}$ spin correlator becomes (pertub. theory):

$$C_n(t) \cong -\alpha + \frac{\beta}{\tau^{3/2}} \sin(\tilde{h}_n t - \phi_0),$$
$$\tilde{h}_n = \varepsilon_z + (h_z)_n + A_0 / 2$$

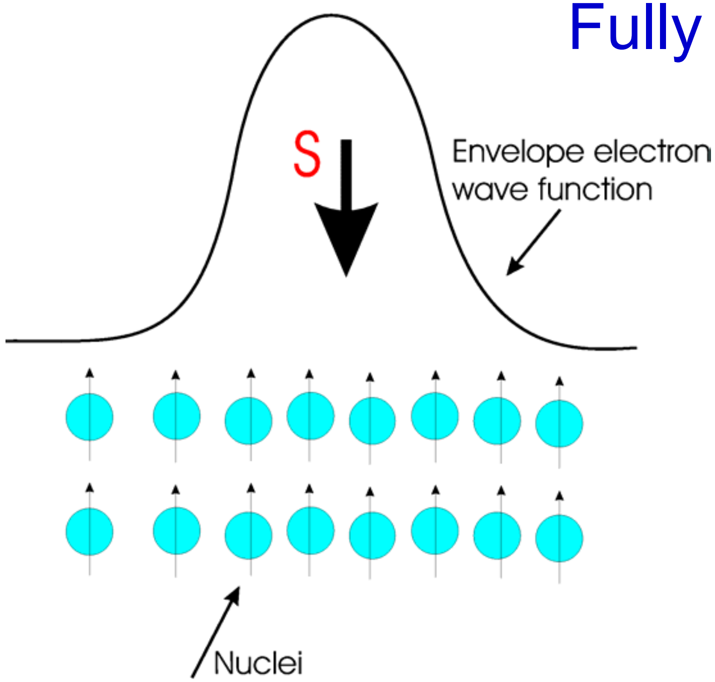
power law decay
(quite unexpected)

Note: for weak Zeeman field, i.e. $\varepsilon_z = g\,\mu_B B < \omega_N$, we obtain $\alpha \sim \beta \sim 1/2$,
but if $g\,\mu_B B >> \omega_N$ ➔ $\alpha \sim \beta \sim (\omega_N / g\,\mu_B B)^2 << 1$.

Thus: spin decay follows **power law** for times $t >> (A/N)^{-1}$ (~ 1 μs for GaAs).
The power law is universal, and amplitude of precession at end of decay is > 0
note: can suppress decay amplitude by magnetic field!

But: higher order terms diverge due to memory effects

Fully polarized nuclei: exactly solvable case



The initial nuclear spin configuration is fully polarized. With the initial wave function Ψ_0 we construct the exact wave function of the system for t > 0 :

$$\Psi_0 = |\Downarrow;\uparrow,\uparrow,\uparrow\ \ldots\ >, \Psi(t) = \alpha(t)\Psi_0 + \sum_k \beta_k(t)|\Uparrow;\uparrow,\uparrow,\downarrow_k\ \ldots\ >,$$

"magnon"
→ entangled

Normalization condition is: $|\alpha(t)|^2 + \sum_k |\beta_k(t)|^2 = 1$, and we assume that

$\alpha(t = 0^+) = 1, \alpha(t < 0) = 0.$ From the Schroedinger equation we obtain:

$$i\frac{d\alpha(t)}{dt} = -\frac{1}{4}A\alpha(t) + \sum_k \frac{A_k}{2}\beta_k(t) - \epsilon_z\alpha(t)/2,$$
$$i\frac{d\beta_l(t)}{dt} = (\frac{A}{4} - \frac{A_l}{2})\beta_l(t) + \frac{A_l}{2}\alpha(t) + \epsilon_z\beta_l(t)/2, \quad (1)$$

where $A = \Sigma_k A_k$; $l=1,\ldots, N'$ → set of N'+1 coupled differential eqs. (N'>>N)

Correlation function: $C_0(t) = -\langle\psi_0|\delta\hat{S}_z(t)\hat{S}_z|\psi_0\rangle = (1-|\alpha(t)|^2)/2$

Laplace transform of (1) gives:

$$\alpha(t) = \frac{\exp(-iA't/4)}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} d\omega \frac{i\exp(\omega t)}{[i\omega + \epsilon_z + \pi N i\omega \int dz \ln(1 - \frac{iA\chi_0^2(z)}{2\pi N\omega})]},$$

here $A' = A + 2\epsilon_z$.

note: sums Σ_k replaced by integrals over r_k^3 (valid for τ < N), with x,y (Gaussians) integrated out → non-analyticity

$$
\begin{aligned}
i\frac{d\alpha(t)}{dt} &= -\frac{1}{4}A\alpha(t) + \sum_k \frac{A_k}{2}\beta_k(t) - \epsilon_z\alpha(t)/2, \\
i\frac{d\beta_l(t)}{dt} &= (\frac{A}{4} - \frac{A_l}{2})\beta_l(t) + \frac{A_l}{2}\alpha(t) + \epsilon_z\beta_l(t)/2,
\end{aligned} \qquad (1)
$$

Laplace transform of (1) gives:

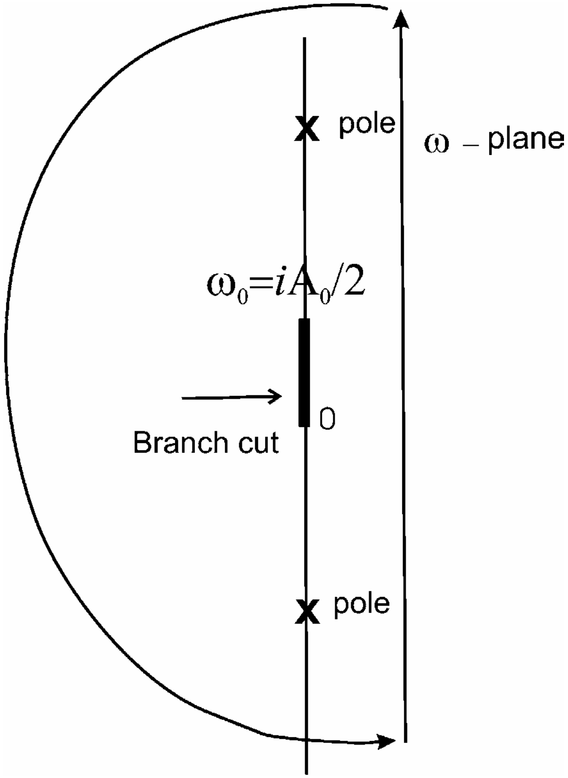
$$\alpha(u) = i\frac{\alpha(t=0)}{D(u)} + \frac{i}{2D(u)}\sum_k \frac{\mathrm{A_k}\beta_k(t=0)}{iu-(\mathrm{A}+2\varepsilon_z)/4+\mathrm{A_k}/2},$$

$$D(u) = iu + \frac{(\mathrm{A}+2\varepsilon_z)}{4} - \frac{1}{4}\sum_k \frac{\mathrm{A_k}^2}{iu-(\mathrm{A}+2\varepsilon_z)/4+\mathrm{A_k}/2}$$

← self-energy

Introducing $iu = i\omega + (\mathrm{A}+2\varepsilon_z)/4$, using α(t=0)=1, β_k(t=0)=0,
and replacing the sum over k by an integral

$$\sum_k \frac{\mathrm{A_k}^2}{i\omega+\mathrm{A_k}/2} = 2\,[\,\mathrm{A} - 2i\pi\,\mathrm{N}\omega\int dz\,\ln(1-\frac{i\mathrm{A}\chi_0^2(z)}{2\pi\,N\omega})\,]$$



integration contour γ and singularities

The singularities are: two branch points ($\omega=0$, $\omega_0 = i\, A\chi_0^2(0)/ 2\pi N$), and first order poles which lie on the imaginary axis (one pole for $\varepsilon_z > 0$, two poles for $\varepsilon_z < 0$). For the contribution from the branch cut (decaying part) we obtain:

$$\tilde{\alpha}(t) = \frac{\exp(-iA't/4)}{\pi N}\int_0^1 \frac{d\kappa 2 z_0 \kappa \exp(i\tau'\kappa)}{[\kappa \int dz \ln|-1+\chi_0^2(z)/\chi_0^2(0)\kappa| + (\kappa/\pi N) - (2\epsilon_z/A\chi_0^2(0))]^2 + (2\pi z_0)^2\kappa^2},$$

here $\tau' = \tau\chi_0^2(0)$ and $z_0 = z_0(\kappa)$, $\chi_0^2(z_0) = \chi_0^2(0)\kappa$.

1) Large Zeeman field $|\varepsilon_z| >> A$.

The asymptotic behavior ($\tau >>1$) is determined by $\kappa =1$ (dot center), and we find

$$\tilde{\alpha}(\tau \gg 1) = \frac{\exp(-iA't/4)\exp(i\tau')}{\pi N} \frac{\chi_0^2(0)}{\sqrt{(\chi_0^2)''}} \frac{A^2}{\epsilon_z^2} \frac{(1-i)\sqrt{\pi}}{4i\tau^{3/2}}$$

~ 1/ N

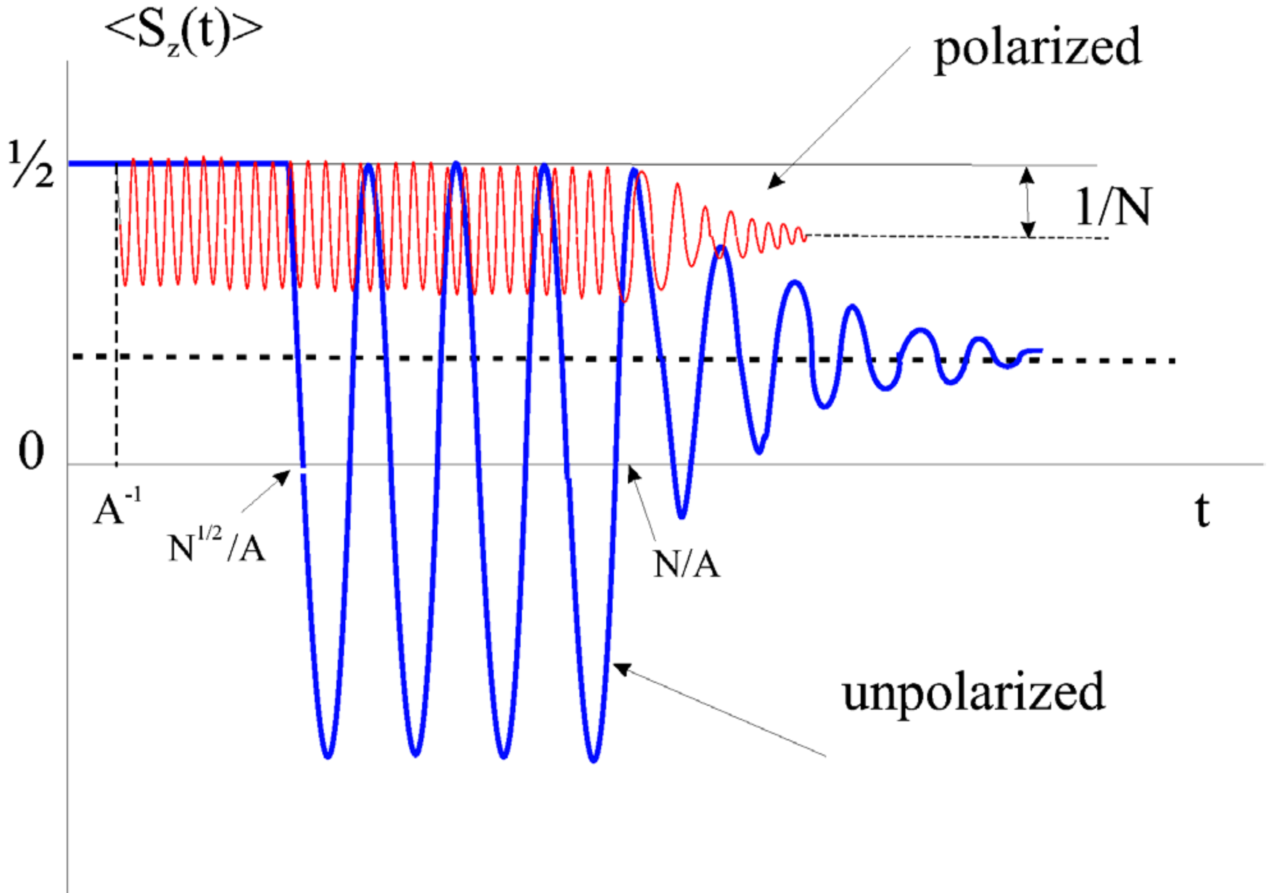
and the correlator $C_0(t)$ agrees with the perturbative result for the fully polarized state, i.e. $C_0(t)- C_0(\infty) \sim 1 / t^{3/2}$, i.e. **power law** (in d-dimensions: $\sim 1 / t^{d/2}$)

Thus, the decay law depends on the magnetic field strength . However, the characteristic time scale for the onset of the non-exponential decay is the same for all cases and given by $(A/N)^{-1}$ (microseconds in GaAs dot).

Spin Decoherence: unpolarized vs polarized

typical decay law for electron spin: $\langle S_z(t) \rangle \propto 1/t^{3/2}$ $(1/\ln^{3/2}\tau \;, for\;\; B = 0)$

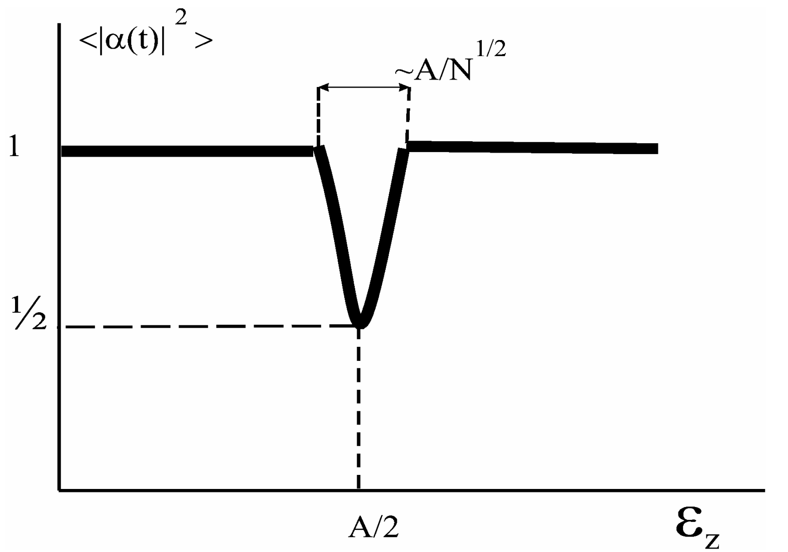
valid for **t > N/A** for unpolarized and polarized nuclear spins (N/A~ 1 μs for GaAs dots).
But note: The decaying part ~ 1/N for polarized nuclei, in contrast to the unpolarized case where decaying part ~ O(1) :



Some interesting features of the fully polarized state

1) Resonance regime for negative external Zeeman field $|\varepsilon_z| = A/2$.

Effective gap seen by the electron spin nearly zero (residual gap is of order $\omega_N \sim A/\sqrt{N}$).
Near this field, $|\alpha(t)|^2 = \cos^2(\omega_+ t)$, $\omega_+ \sim \omega_N$ ➔ $< |\alpha(t)|^2 > = ½$ (time-average) (see Fig.), i.e. up and down spin of electron strongly coupled. But away from resonance: $|\alpha(t)|^2 \approx 1$.
Note: width of the resonance $\sim A/\sqrt{N} \ll A$.



➔ detect polarized nuclei via abrupt change of oscillation amplitude of $<\hat{S}_z(t)>$!

not related to decoherence (the latter is O(1/N) even near the resonance)

The weight of upper pole also drops abruptly from a value ~1 to a value <<1 in the same narrow interval of Zeeman field ➔ experimental check by Fourier analysis.

2) $\varepsilon_z = \varepsilon_z^* = b\,A/2$, where $b = \chi_0^2(0) \int dz \ln|1 - \chi_0^2(z)/\chi_0^2(0)| < -1$ is a non-universal number which depends on the dot shape.

This Zeeman field value ε_z^* corresponds to the case when the upper pole is close to the upper edge of the branch cut. When approaching the critical Zeeman field ε_z^* there is a slow down of the asymptotics, i.e.

$$\frac{1}{\tau^{3/2}} \rightarrow \frac{1}{\tau^{1/2}}$$

This slow down is related to a strong modulation of the density of states of one-magnon excitations near the edge of the continuum band (branch cut).

The DOS is $\upsilon(u) = \mathrm{Im}\,[G_0(u) + d/du \ln D(u)]$,

$u=i\omega$, $G_0(u)= \Sigma_k\, 1/(u + A_k/2)$ is the "unperturbed Green's function" , and $\alpha(u) = i/D(u)$. When $\varepsilon_z \rightarrow \varepsilon_z^*$, we get $\upsilon(u) \propto 1/\sqrt{(\omega_0 - u)}$, i.e. edge singularity of the continuum

Decay of electron spin due to entanglement with nuclear spins:

Schliemann, Khaetskii, Loss, Phys. Rev. B**66**, 245303 (2002)

measure of entanglement: von Neumann entropy E of reduced density matrix (see C.H. Bennett et al., Phys. Rev. A53 2046 (1996))

i.e. trace out pure-state density matrix $|\Psi(t)\rangle\langle\Psi(t)|$ of electron-nuclear spin system over nuclei ➔

$$\rho_{el}(t) = Tr_{nuclei}\,|\Psi(t)\rangle\langle\Psi(t)| = \begin{pmatrix} 1/2+\langle S_z(t)\rangle & \langle S_+(t)\rangle \\ \langle S_z(t)\rangle & 1/2-\langle S_z(t)\rangle \end{pmatrix}$$

$$\textit{with eigenvalues}\;\; \lambda_\pm = 1/2 \pm \left|\langle \vec{S}(t)\rangle\right| \quad \Rightarrow \quad E(|\Psi(t)\rangle) = -\lambda_+ \log\lambda_+ - \lambda_- \log\lambda_-$$

i.e. entanglement reaches max. (E_{max} = log 2) for completely decayed spin of electron, i.e. for $\left|\langle \vec{S}(t)\rangle\right| = 0$

Possible choices of initial nuclear spin states:

1a) product-states in I_z -basis for all nuclei:

$$|i\rangle = |\uparrow\rangle_1 \otimes |\downarrow\rangle_2 \otimes |\downarrow\rangle_3 \otimes \ldots \otimes |\uparrow\rangle_N$$

1b) product states in random basis:

$$|\tilde{i}\rangle = |\uparrow + a_1 \downarrow\rangle_1 \otimes |\downarrow + a_2 \uparrow\rangle_2 \otimes |\downarrow + a_3 \uparrow\rangle_3 \otimes \ldots \otimes |\uparrow + a_N \downarrow\rangle_N$$

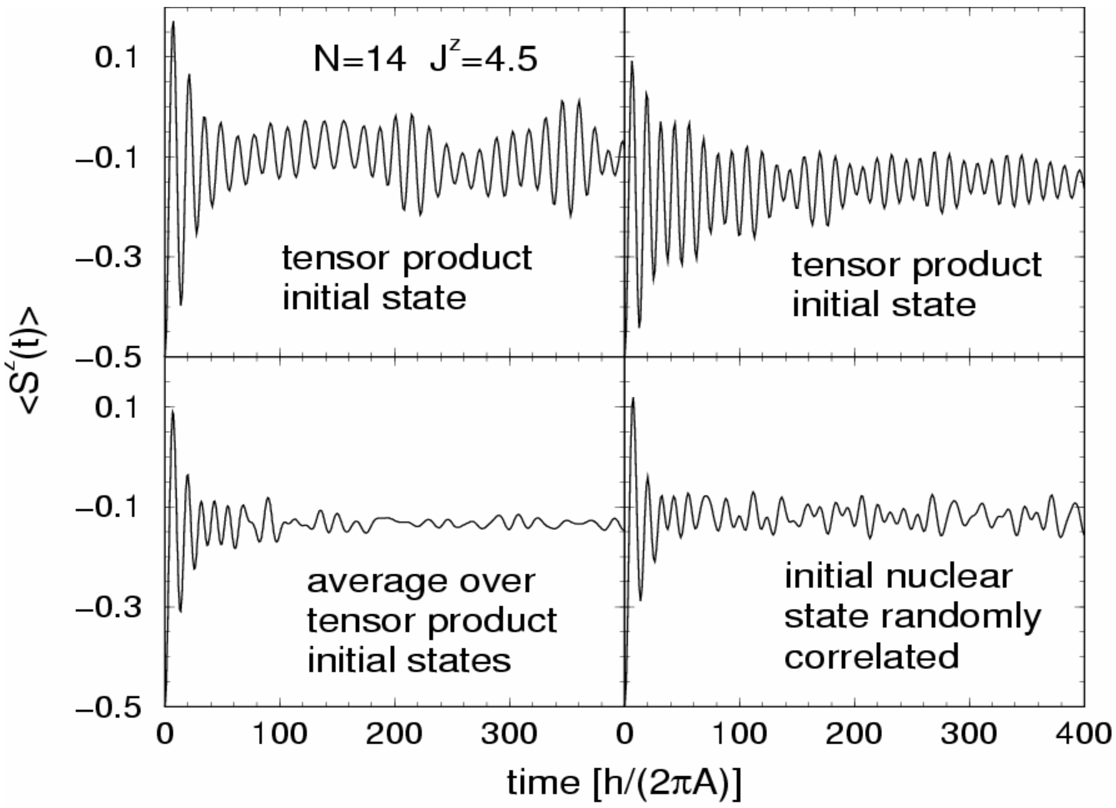
2) correlated basis (entangled):

$$|\psi_c\rangle = \sum_i a_i |i\rangle, \quad where\ |i\rangle = |\uparrow\rangle_1 \otimes |\downarrow\rangle_2 \otimes |\downarrow\rangle_3 \otimes \ldots \otimes |\uparrow\rangle_N$$

➔ possible nuclear spin averages for observable **S**:

1. $\langle i|\vec{S}|i\rangle$ single product state
2. $\sum_i a_i \langle i|\vec{S}|i\rangle$ average over product states <=> ensemble average
3. $\langle \psi_c|\vec{S}|\psi_c\rangle$ single correlated state, e.g. with random phases

Numerics:
Schliemann et al., Phys. Rev. B**66**, 245303 (2002)



- Upper panels: time evolution of the electron spin $\langle S^z(t)\rangle$ for a system with 14 nuclear spins being initially in an uncorrelated tensor product state in the subspace with total angular momentum $J^z=9/2$.
- Lower left panel: data of the same type as above but averaged overall possible uncorrelated initial states with $J^z=9/2$.
- Lower right panel: $\langle S^z(t)\rangle$ for the same system being initially in a randomly chosen correlated state (pure state!)

Averaging over nuclear configurations → dephasing

Since $\omega_N^{-1} = \sqrt{N}\ /A \ll N/A$, the electron spin undergoes many precessions in a given nuclear field configuration before decoherence sets in due to the non-uniform hyperfine couplings A_k .
This behavior changes dramatically when we average over nuclear configurations.
We demonstrate this for the case when the nuclear field is treated classically, i.e. as a c-number.
The exact calculation of the correlator gives:

$$C_n(t) = -\frac{h_{N\perp}^2}{4h_N^2}(1 - \cos h_N t),$$

here $h_N = \sqrt{h_{Nz}^2 + h_{N\perp}^2}$ is the nuclear field , $h_{N\perp}^2 = h_{Nx}^2 + h_{Ny}^2.$

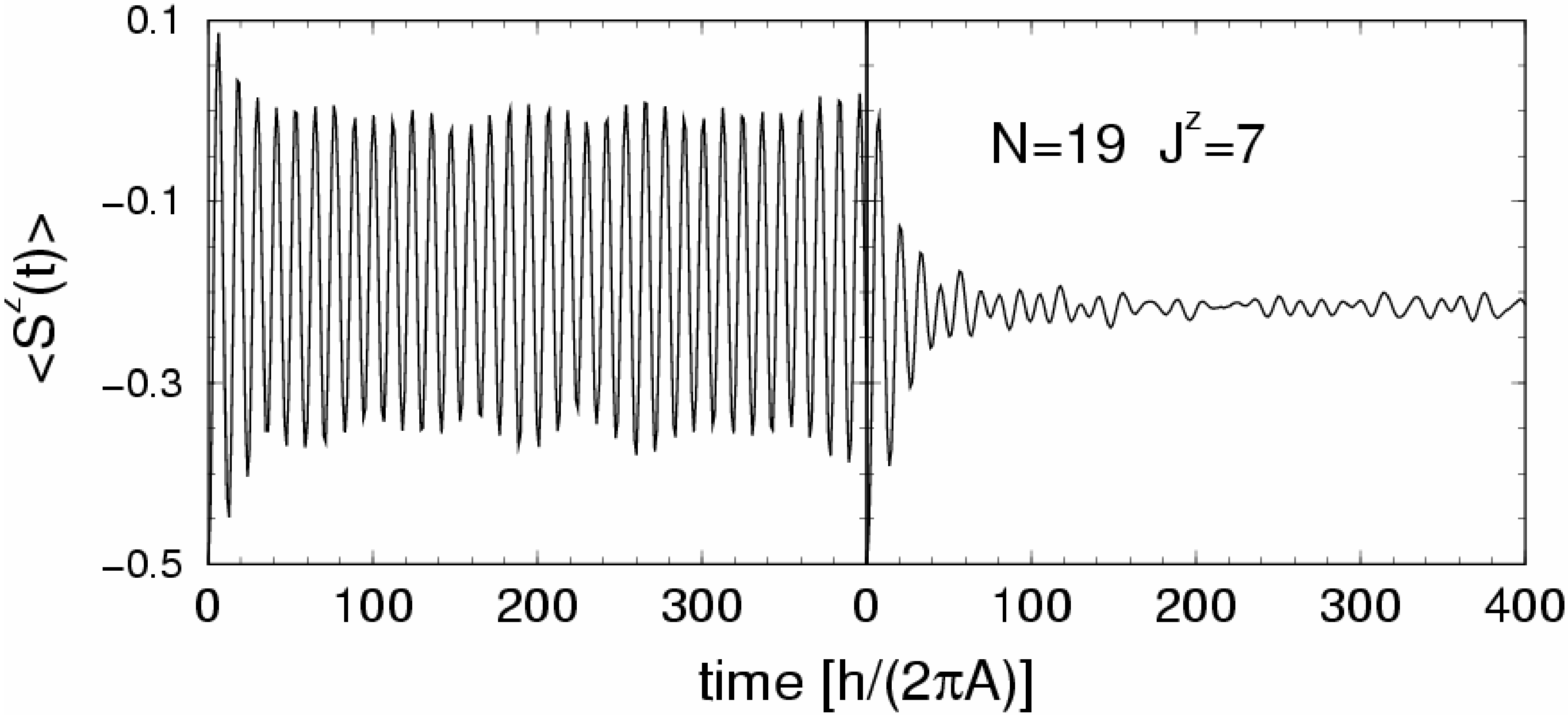
We average this correlator over a Gaussian distribution for h_N, i.e. over

$$P(h_N) \propto \exp(-3h_N^2/2\omega_N^2).$$

With the definition $C_{cl}(t) = \int dh_N P(h_N) C_n(t)$, we obtain

$$C_{cl}(t) = -\frac{1}{6}[1 + (\frac{\omega_N^2 t^2}{3} - 1)\, e^{-\omega_N^2 t^2/6}].$$

Thus, we get rapid decay of the correlator for $t \gg \omega_N^{-1}$, giving the dephasing time $\sim \sqrt{N}\ /A = \omega_N^{-1}$ ($\ll$ decoherence time $\sim N/A$)



Time evolution of $\langle S^z(t) \rangle$ for two types of initially random correlated nuclear spin states. In the left panel the amplitudes α_T are restricted to have non-negative real and imaginary part, while in the right panel they have all the same modulus but completely random phases.

Conclusions

- spin-based quantum computing scheme in quantum dots
- all-electrical spin control via gates acting on electron charge:
  1. single qubit: via magnetic semicond./g-factor & ESR
  2. XOR gate = double quantum dot & exchange control → deterministic entanglement
- $N_{Op} \approx \tau_{\phi} / \tau_{s} \approx 10^{4}$ & parallel switching (QEC) → scale up
- detection of single spin decoherence: via transport current and ESR in quantum dot
- spin decoherence of electron due to nuclear spins is non-exponential (power law).
  But: amplitude of decayed part ~ 1/N for N polarized nuclear spins

See review in *Semiconductor Spintronics and Quantum Computation*, eds. D. D. Awschalom, D. Loss, and N. Samarth, Springer, Berlin, 2002.