Quantum Logic Gates for Capacitively Coupled Josephson Junctions

Frederick W. Strauch

Department of Physics
University of Maryland

Quantum Logic Gates for Capacitively Coupled Josephson Junctions

Outline

• Motivation
• Energy Spectrum
• Logic Gates
• Experimental Prospects
• Conclusion
Theoretical Motivation:

• Current-biased Josephson junctions are seemingly *macroscopic* objects obeying quantum mechanics.

• Fabrication and interactions allow for directed design of *scalable* circuits.

• Electrical properties are easily measured.
Experimental Motivation:

• **Coherence**: Martinis *et al.*, Yu *et al.* (2002) demonstrated Rabi oscillations in single qubit dynamics.

• **Coupling**: Berkley *et al.* (2003) demonstrated tunable coupling through spectroscopy.

• Experimental **control** of coherent two-qubit dynamics of current-biased junctions should be possible—quantum logic gate design is necessary.
Current-Biased Josephson Junction

\[ H = 4E_C p_\gamma^2 - E_J (\cos \gamma + J \gamma) \]

\[ E_C = \frac{e^2}{2C_J}, \quad E_J = \frac{\hbar I_C}{2e}, \quad J = \frac{I}{I_C} \]

\[ \hbar \omega_p (J) = \left( 8E_C E_J \right)^{1/2} (1 - J^2)^{1/4} \]

\[ N_s (J) = \frac{2^{3/4}}{3} \left( \frac{E_J}{E_C} \right)^{1/2} (1 - J)^{5/4} \]
Capacitively Coupled Josephson Junctions

\[ H = 4E_C (1 + \zeta)^{-1} (p_1^2 + p_2^2 + 2\zeta p_1 p_2) - E_J (\cos \gamma_1 + J_1 \gamma_1 + \cos \gamma_2 + J_2 \gamma_2) \]

\[ E_C = \frac{e^2}{2C_J} , \quad E_J = \frac{\hbar I_c}{2e} \]

\[ J_1 = \frac{I_1}{I_c} , \quad J_2 = \frac{I_2}{I_c} \]

\[ \zeta = \frac{C}{C + C_J} \]
Energy Spectrum

\[ N_s = 4, \quad \zeta = 0.01 \]

\[
\sqrt{1 - J_{1,2}} = \sqrt{1 - J_0} (1 \pm \varepsilon)
\]

\[
\hbar \omega_0 = (8E_C E_J)^{1/2} (1 - J_0^2)^{1/4}
\]

- Energy states are unentangled away from avoided level crossings.
- Entanglement is maximized at the avoided level crossings.
Gate Design

- **Control**: Interactions controllable (tuned on and off) through bias currents for small coupling.
  
  (e.g. $\zeta = 0.01$)

- **Dynamical conditions**: Characteristic ramp time must satisfy
  
  \[
  \frac{2\pi}{\omega_0} < \tau_R < \frac{1}{\zeta} \frac{2\pi}{\omega_0} \approx 100 \times \frac{2\pi}{\omega_0}
  \]

- **Leakage**: Both tunneling and evolution into the auxiliary states $|02\rangle$ and $|20\rangle$ must be taken into account.
  
  \[N_S \geq 4\]
Gate Operation

- Start from detuned junctions.
- Ramp bias currents, in time $\tau_R$, from $\varepsilon_A$ to $\varepsilon_B$.
- Wait for time $\tau_I$.
- Detune the junctions.
Phase Gate Operation

This avoided level crossing is isolated, so the other two-qubit states $|00\rangle$, $|01\rangle$ and $|10\rangle$ are unaffected.
Phase Gate Simulation

\[ |11\rangle, |02\rangle, |11\rangle \]

\[ \gamma_2 \]

\[ \omega_0 t = 10 \]
\[ \omega_0 t = 340 \]
\[ \omega_0 t = 710 \]

\[ \gamma_1 \]

Probability

\[ 0 \rightarrow 1 \rightarrow 0 \]

Time \((1/\omega_0)\)
Swap-Like Operation

\[ N_s \approx 5.16, \quad \zeta = 0.01 \]
\[ \theta_1 = \pi / 2, \quad \theta_2 \approx \pi / 4 \]

\[ U_2 = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & \cos \theta_1 & -i \sin \theta_1 & 0 \\
0 & -i \sin \theta_1 & \cos \theta_1 & 0 \\
0 & 0 & 0 & e^{-i\theta_2}
\end{pmatrix} \]

\[ |01\rangle \rightarrow \cos \theta_1 |01\rangle - i \sin \theta_1 |10\rangle \]
\[ |10\rangle \rightarrow \cos \theta_1 |10\rangle - i \sin \theta_1 |01\rangle \]
Energy Spectrum

$N_s = 4, \ zeta = 0.01$

![Energy Spectrum Diagram]

$\frac{|02\rangle + |11\rangle}{\sqrt{2}}$  
$\frac{|20\rangle + |11\rangle}{\sqrt{2}}$

$a |11\rangle + b \frac{|02\rangle + |20\rangle}{\sqrt{2}} = b |11\rangle - a \frac{|02\rangle + |20\rangle}{\sqrt{2}}$
Swap Gate Simulation

\[ |10\rangle \]
\[ \gamma_2 \]
\[ \omega_0 t = 10 \]

\[ \frac{|10\rangle - i|01\rangle}{\sqrt{2}} \]
\[ \gamma_1 \]
\[ \omega_0 t = 260 \]

\[ |01\rangle \]
\[ \omega_0 t = 560 \]

\[ \text{Probability} \]

\[ \text{Time (} 1/\omega_0 \text{)} \]

\[ 0 \to 10 \]
\[ |10\rangle \to \frac{|10\rangle - i|01\rangle}{\sqrt{2}} \to |01\rangle \]
Fidelity and Leakage

Fidelity is the probability that the gate operation is successful.

Leakage is the probability that the gate drives transitions out of the two-qubit basis.

<table>
<thead>
<tr>
<th></th>
<th>Fidelity</th>
<th>Leakage</th>
<th>Time(ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase</td>
<td>0.996</td>
<td>0.003</td>
<td>14.85</td>
</tr>
<tr>
<td>Swap</td>
<td>0.972</td>
<td>0.006</td>
<td>10.7</td>
</tr>
</tbody>
</table>

$\omega_0/2\pi = 6$ GHz
Experimental Demonstration

- The nanosecond pulse times can be generated with conventional electronics.
- Energy levels can be determined spectroscopically.
- Coherence requires characteristic (“RC”) dissipation times $\geq 1 \mu s$:
  - $C=6\text{pF} \Rightarrow R \geq 160 \text{k}\Omega$ at GHz frequencies.
  - BUT, bias lines typically have $R \approx 50 \text{\Omega}$!
Achieving Coherence: Impedance Transformers

- Resistor (Gubrud 2001)
- LC isolator (Berkley 2002)
- Junction (inductor) (Martinis 2002)
- Inductor (tunable $T_1$) (Martinis 2003)
- Flux Detector
Properties and Optimizations

• Fixed coupling requires undesirably large detuning of junctions.
  • Non-identical qubit energy levels.
  • Possibly large tunneling rates.
• Swap gate uses delicate energy level structure.
  • Sensitive to bias current noise.
  • Difficult with non-identical junctions.
• Gate times are close to time scales for high-fidelity single-qubit operations.
Three-Junction Scheme

- Junction B is used to entangle junctions $A_1$ and $A_2$.
- Before and after the operation, junction B is in its ground state.

Conclusion

• Designed and numerically simulated two fundamental quantum logic operations, each with fidelity $F>0.97$.

• Explored multiple junction schemes for controlled coupling.

• Experimental demonstration of these logic gates is possible.

Strauch et al., quant-ph/0303002