Quantum Logic Gates for Capacitively Coupled Josephson Junctions

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Quantum Logic Gates for Capacitively Coupled Josephson Junctions

Outline

- Motivation
- Energy Spectrum
- Logic Gates
- Experimental Prospects
- Conclusion

Theoretical Motivation:

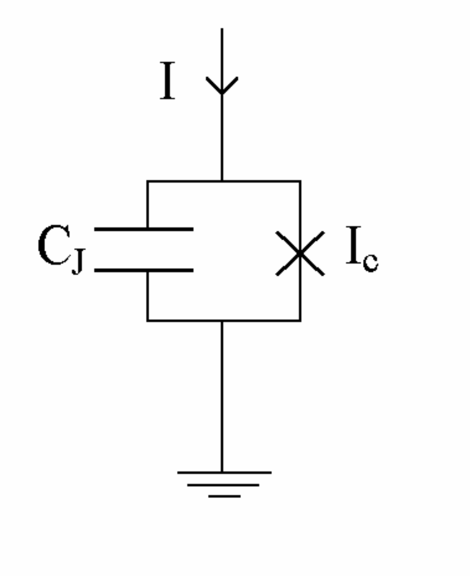
- Current-biased Josephson junctions are seemingly **macroscopic** objects obeying quantum mechanics.
- Fabrication and interactions allow for directed design of **scalable** circuits.
- Electrical properties are easily **measured**.

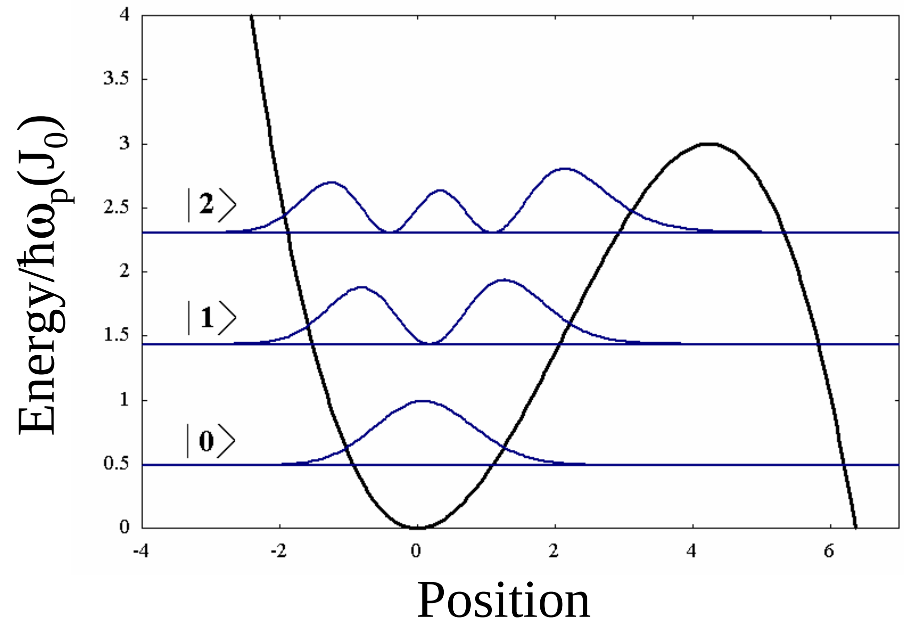
Experimental Motivation:

- **Coherence**: Martinis *et al.*, Yu *et al.* (2002) demonstrated Rabi oscillations in single qubit dynamics.
- **Coupling**: Berkley *et al.* (2003) demonstrated tunable coupling through spectroscopy.

- Experimental **control** of coherent two-qubit dynamics of current-biased junctions should be possible—quantum logic gate design is necessary.

Current-Biased Josephson Junction





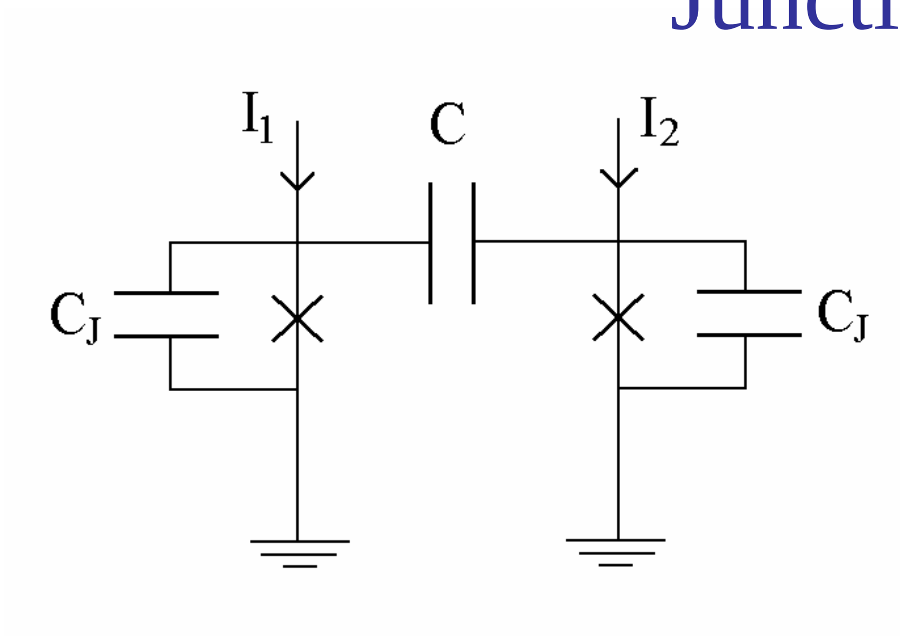
$$H = 4E_C p_\gamma^{\ 2} - E_J(\cos\gamma + J\gamma)$$

$$\hbar\omega_p(J) = \left(8E_C E_J\right)^{1/2}(1 - J^2)^{1/4}$$

$$E_C = \frac{e^2}{2C_J},\ \ E_J = \frac{\hbar I_C}{2e},\ \ J = \frac{I}{I_C}$$

$$N_s(J) = \frac{2^{3/4}}{3}\left(\frac{E_J}{E_C}\right)^{1/2}(1 - J)^{5/4}$$

Capacitively Coupled Josephson Junctions



$$E_C = \frac{e^2}{2C_J}\ , \ E_J = \frac{\hbar I_C}{2e}$$

$$J_1 = I_1 / I_C\ ,\ \ J_2 = I_2 / I_C$$

$$\boxed{\zeta = C/(C + C_J)}$$

$$H = 4E_C(1+\zeta)^{-1}(p_1^{\,2} + p_2^{\,2} + 2\zeta p_1 p_2)$$
$$- E_J(\cos\gamma_1 + J_1\gamma_1 + \cos\gamma_2 + J_2\gamma_2)$$

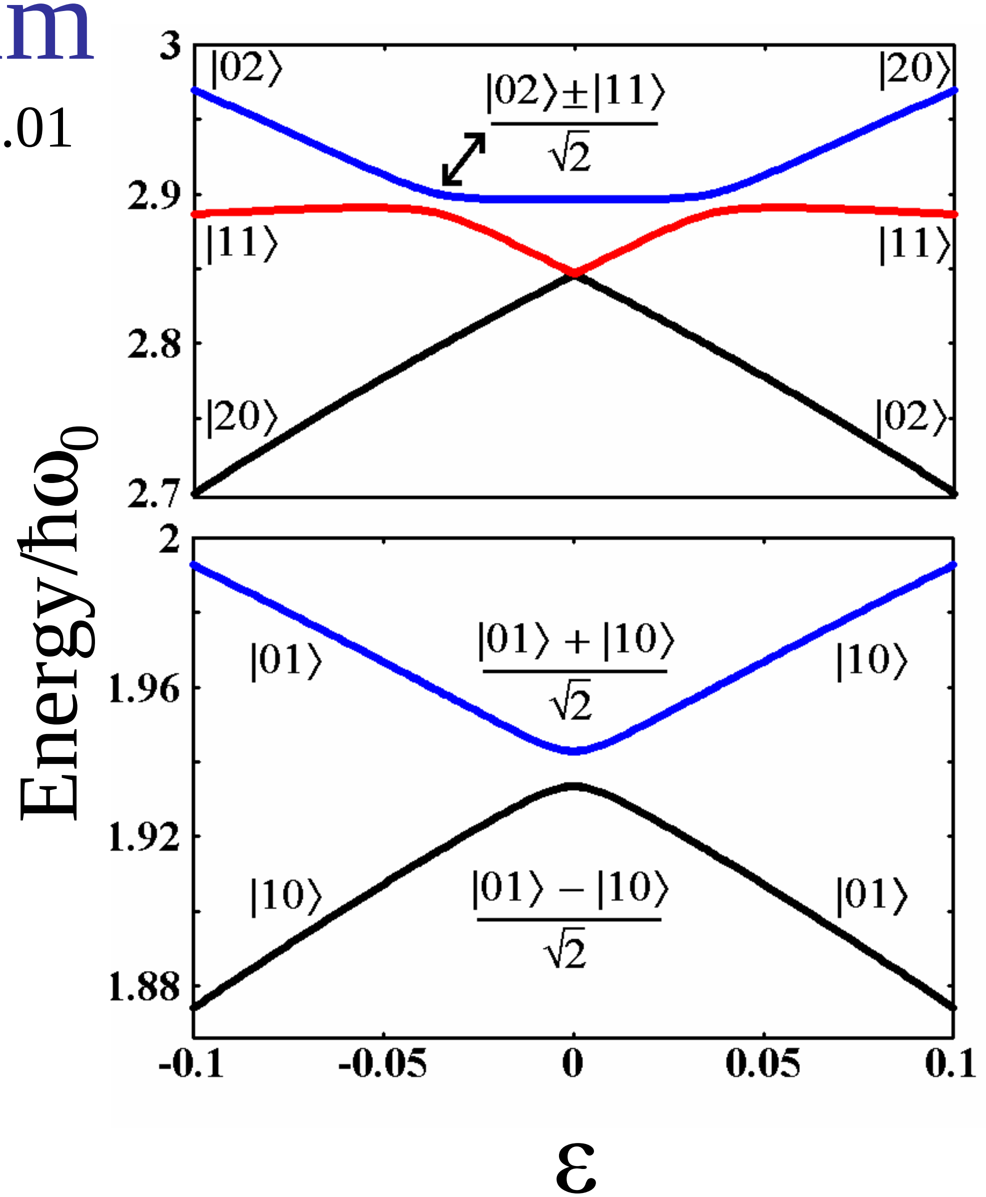
Energy Spectrum

$$N_s = 4,\ \zeta = 0.01$$

$$\sqrt{1-J_{1,2}} = \sqrt{1-J_0}\,(1 \pm \varepsilon)$$

$$\hbar\omega_0 = \left(8E_C E_J\right)^{1/2}\left(1-J_0^2\right)^{1/4}$$

- Energy states are unentangled away from avoided level crossings.
- Entanglement is maximized at the avoided level crossings.



Gate Design

- **Control**: Interactions controllable (tuned on and off) through bias currents for small coupling.

  (e.g. $\zeta = 0.01$)

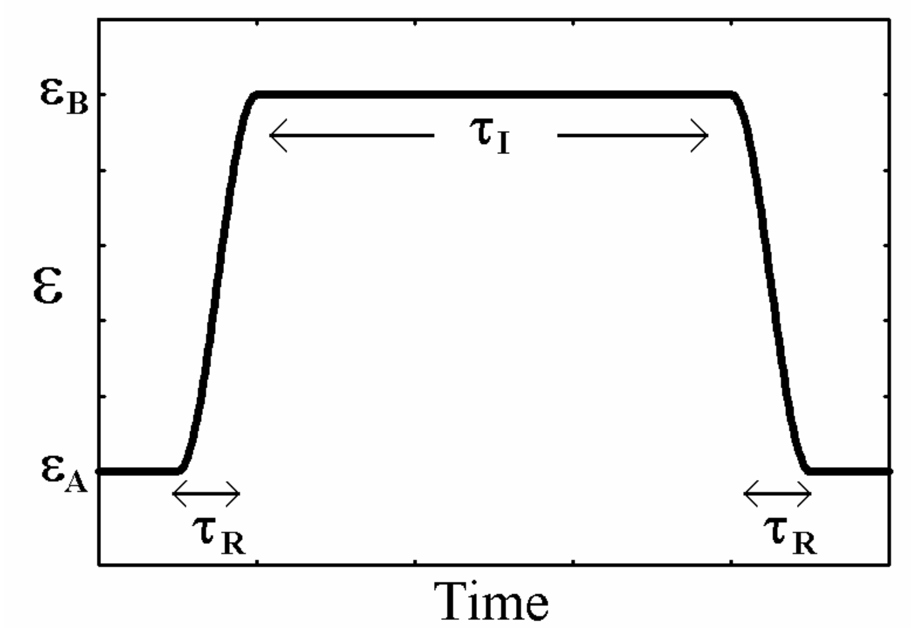
- **Dynamical conditions**: Characteristic ramp time must satisfy $$\frac{2\pi}{\omega_0} < \tau_R < \frac{1}{\zeta}\frac{2\pi}{\omega_0} \approx 100 \times \frac{2\pi}{\omega_0}$$

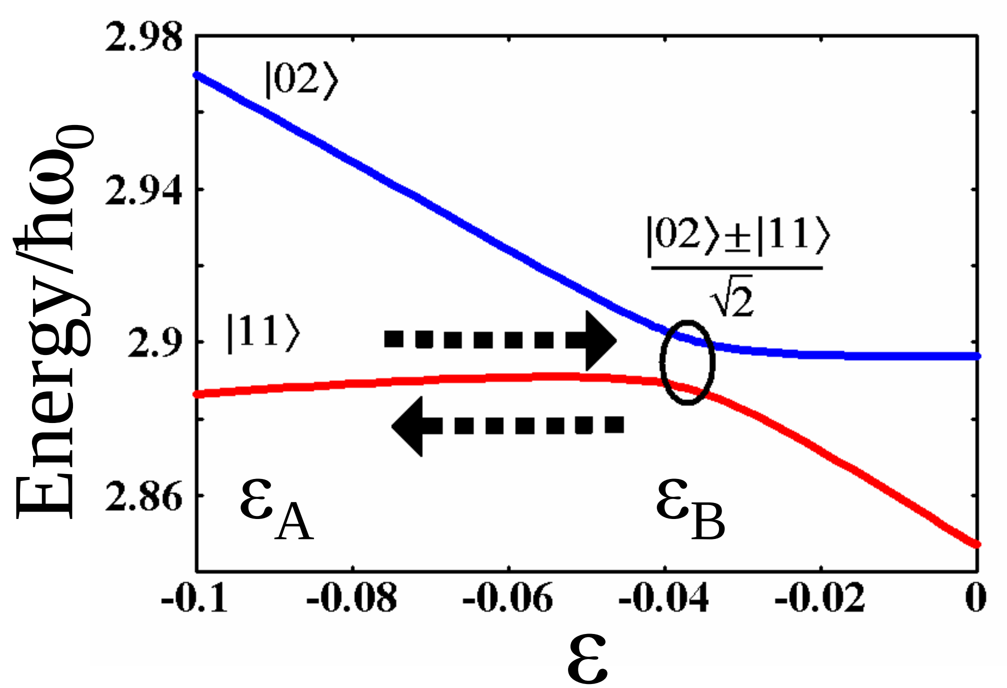
- **Leakage**: Both tunneling and evolution into the auxiliary states $|02\rangle$ and $|20\rangle$ must be taken into account.

$$N_S \geq 4$$

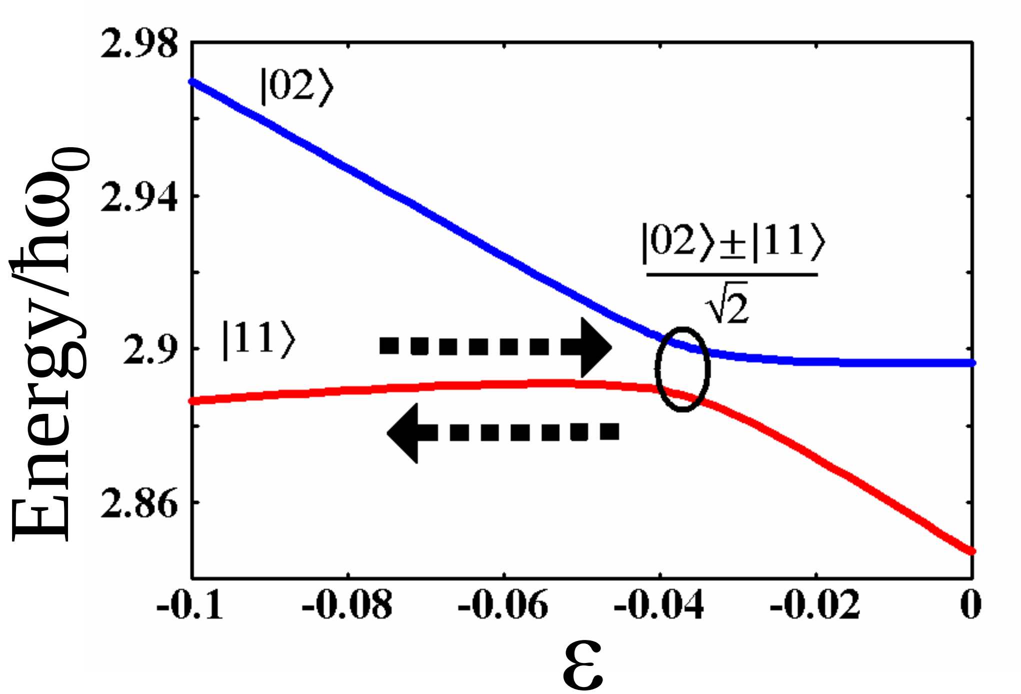
Gate Operation

- Start from detuned junctions.
- Ramp bias currents, in time τ_R, from ε_A to ε_B.
- Wait for time τ_I.
- Detune the junctions.





Phase Gate Operation

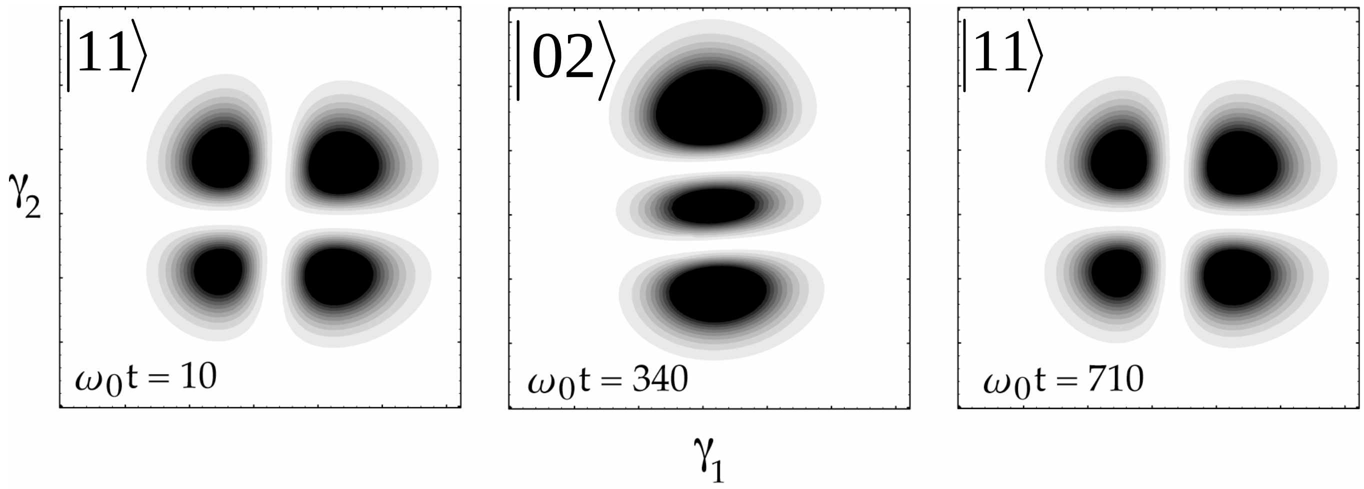


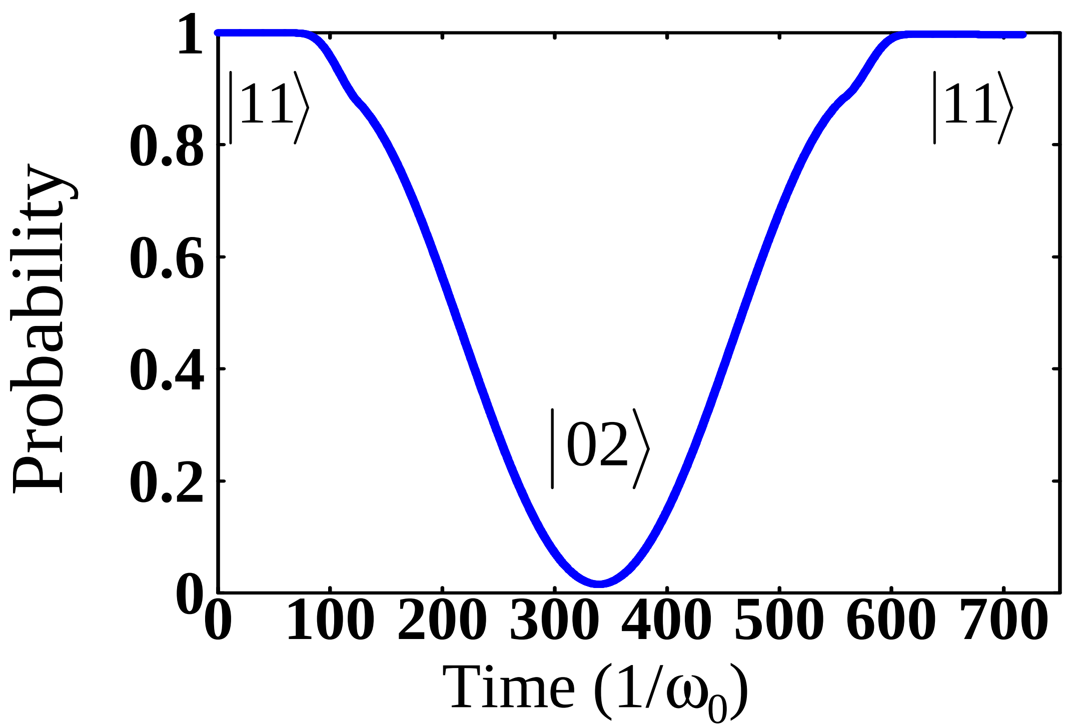
$$|11\rangle \rightarrow |02\rangle \rightarrow -|11\rangle$$

$$U_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

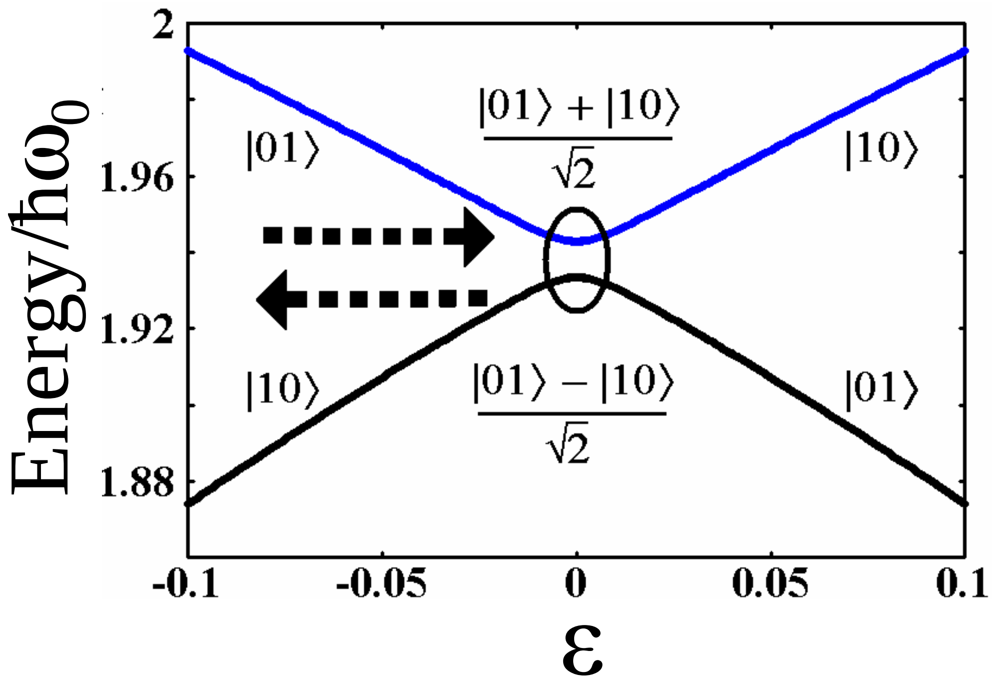
This avoided level crossing is isolated, so the other two-qubit states $|00\rangle$, $|01\rangle$ and $|10\rangle$ are unaffected.

Phase Gate Simulation





Swap-Like Operation



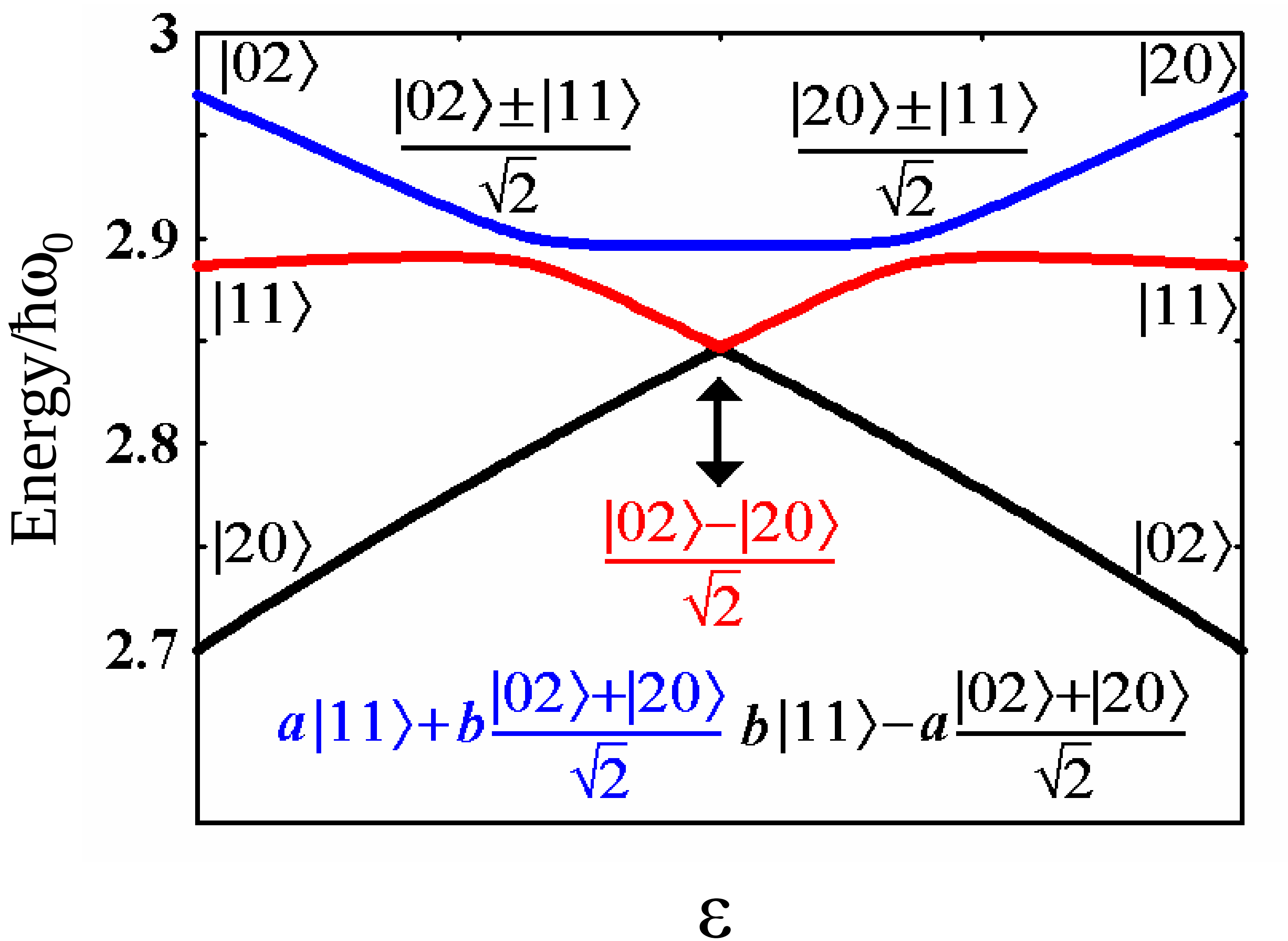
$$|01\rangle \rightarrow \cos\theta_1|01\rangle - i\sin\theta_1|10\rangle$$
$$|10\rangle \rightarrow \cos\theta_1|10\rangle - i\sin\theta_1|01\rangle$$

$$N_s \cong 5.16, \;\; \zeta = 0.01$$
$$\theta_1 = \pi/2 \quad \theta_2 \cong \pi/4$$

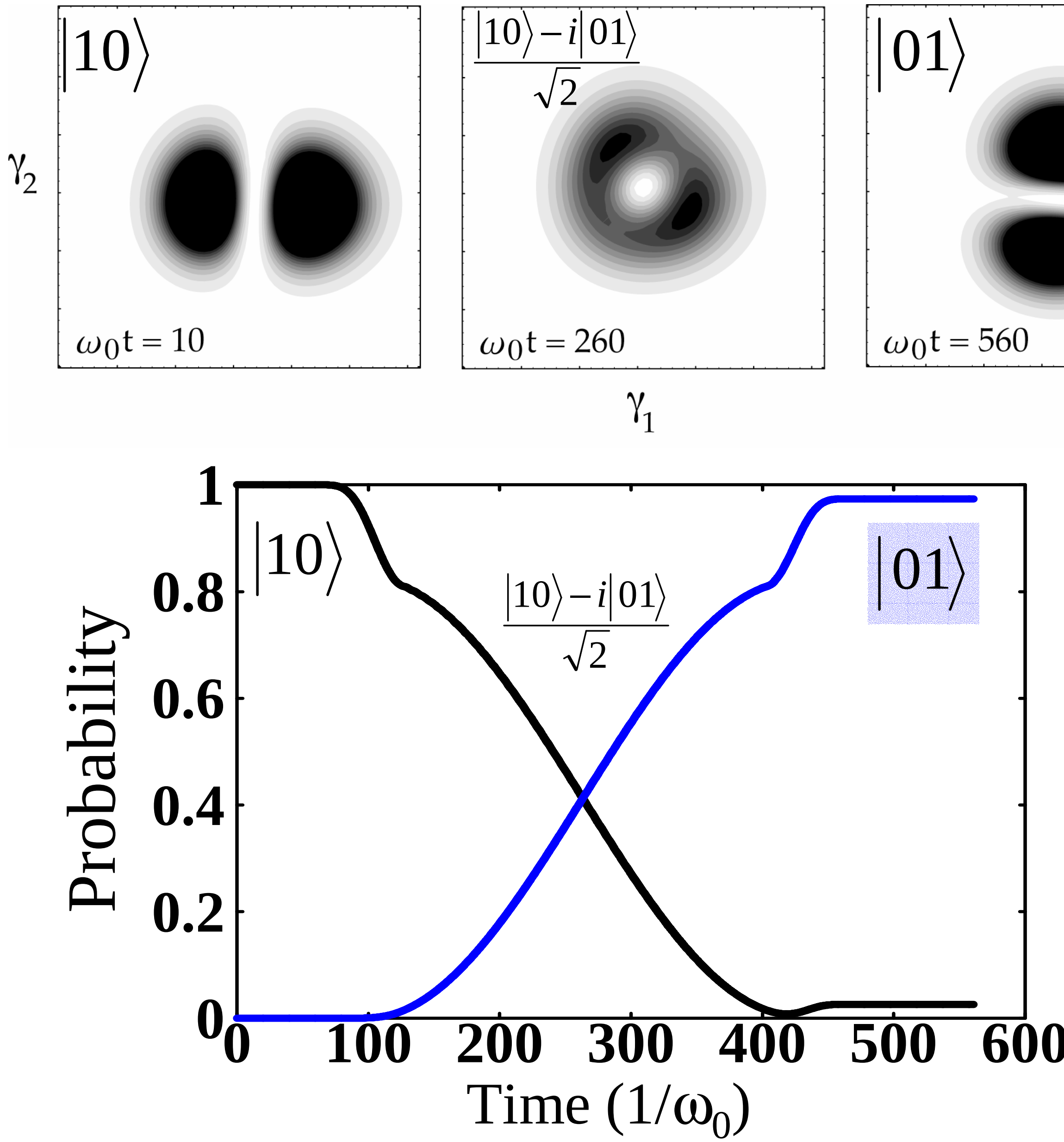
$$U_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_1 & -i\sin\theta_1 & 0 \\ 0 & -i\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 0 & e^{-i\theta_2} \end{pmatrix}$$

Energy Spectrum

$N_s = 4,\ \zeta = 0.01$



Swap Gate Simulation



Fidelity and Leakage

Fidelity is the probability that the gate operation is successful.

Leakage is the probability that the gate drives transitions out of the two-qubit basis.

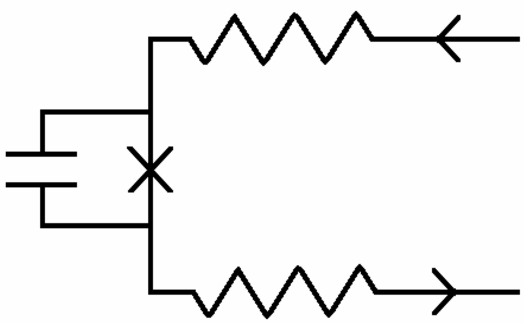
	Fidelity	Leakage	Time(ns)
Phase	0.996	0.003	14.85
Swap	0.972	0.006	10.7

$\omega_0/2\pi = 6$ GHz

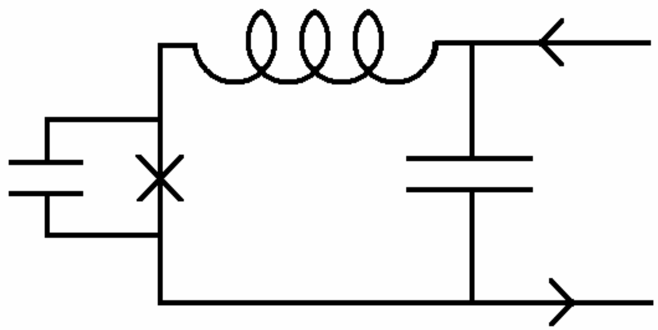
Experimental Demonstration

- The nanosecond pulse times can be generated with conventional electronics.
- Energy levels can be determined spectroscopically.
- Coherence requires characteristic (“RC”) dissipation times $\geq 1\ \mu s$:
    - $C=6pF \Rightarrow R \geq 160\ k\Omega$ at GHz frequencies.
    - BUT, bias lines typically have $R \approx 50\ \Omega$!

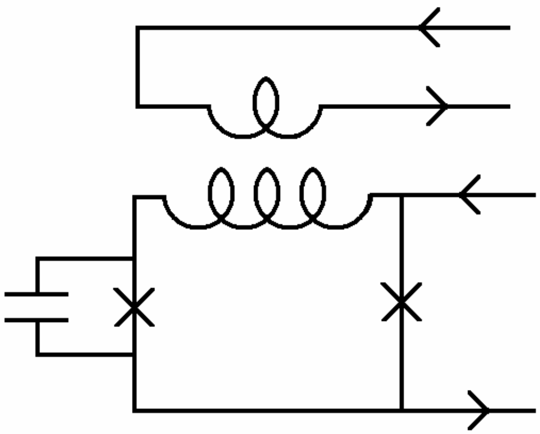
Achieving Coherence:
Impedance Transformers



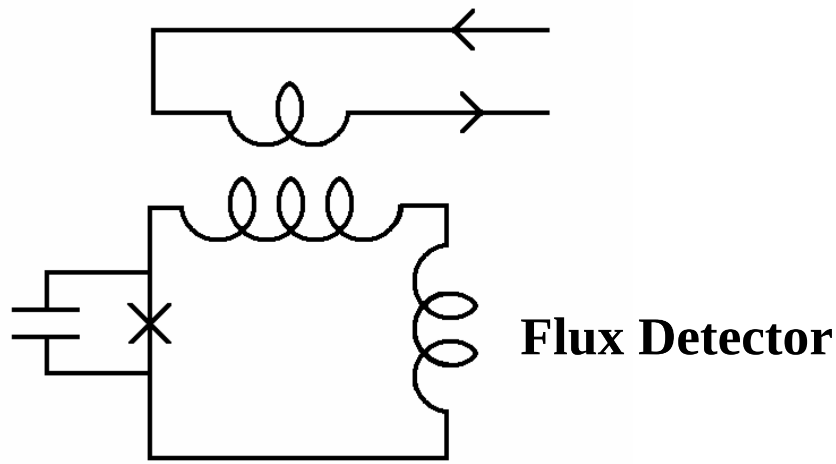
Resistor (Gubrud 2001)



LC isolator (Berkley 2002)



Junction (inductor) (Martinis 2002)

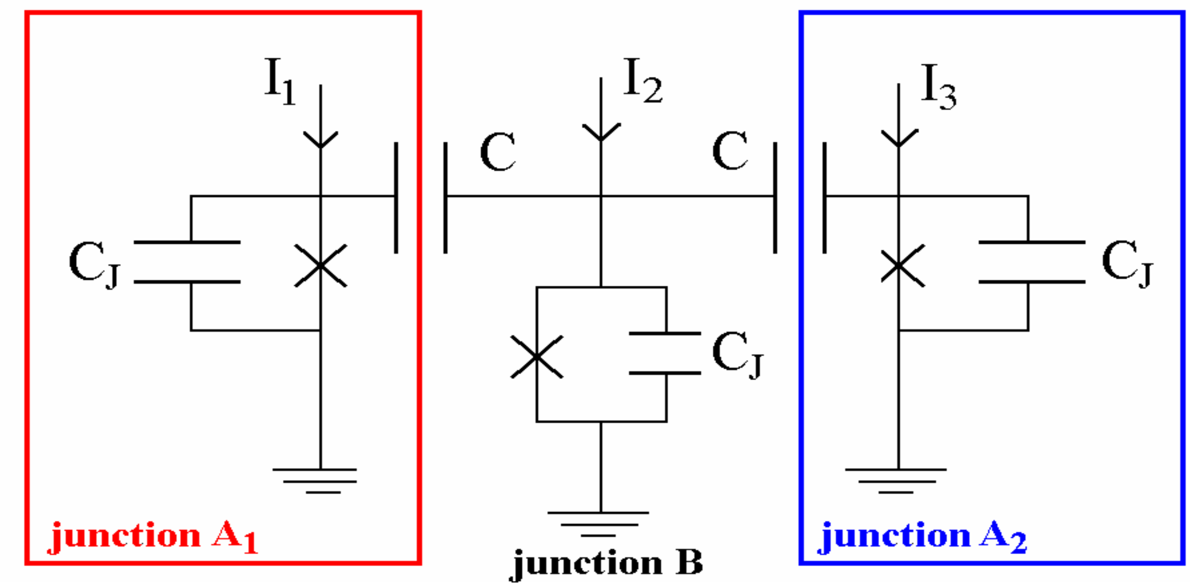


Inductor (tunable T_1) (Martinis 2003)

Properties and Optimizations

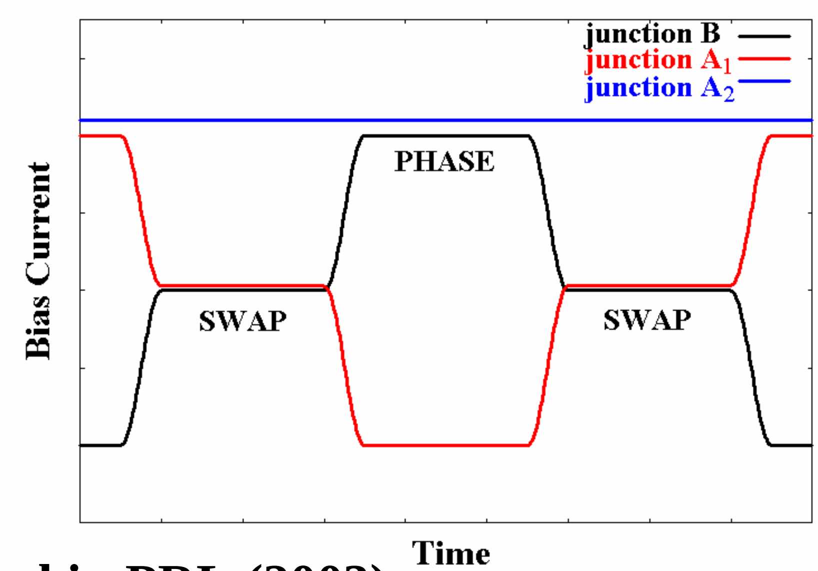
- Fixed coupling requires undesirably large detuning of junctions.
    - Non-identical qubit energy levels.
    - Possibly large tunneling rates.
- Swap gate uses delicate energy level structure.
    - Sensitive to bias current noise.
    - Difficult with non-identical junctions.
- Gate times are close to time scales for high-fidelity single-qubit operations.

Three-Junction Scheme



•Junction B is used to entangle junctions A_1 and A_2.

•Before and after the operation, junction B is in its ground state.



Cf. Blais, Maassen van den Brink, Zagoskin PRL (2003)

Conclusion

- Designed and numerically simulated two fundamental quantum logic operations, each with fidelity F>0.97.
- Explored multiple junction schemes for controlled coupling.
- Experimental demonstration of these logic gates is possible.

Strauch *et al.*, quant-ph/0303002