Entanglement, quantum critical phenomena and efficient simulation of quantum dynamics

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Entanglement
in quantum phase transitions

- Scaling of entanglement in critical and non-critical spin chains.
- Emergence of universality at a quantum critical point.
- Connection to conformal field theory, irreversibility of RG flow.
- Entanglement in spin lattices.
- Failure of the DMRG method.

Efficient classical simulation of quantum dynamics

- Critical and non-critical spin chains.
- Non-critical spin lattices in 2D, 3D.
Entanglement in quantum phase transitions

Latorre, Rico, Vidal, quant-ph/0304098

Measures of entanglement in a quantum spin chain

T=0, ground state

concurrence (entanglement between two spins)

Osterloh, Amico, Falci and Fazio, Nature 2002
Osborne and Nielsen, Phys. Rev. A 2002

Our approach:

entropy $S_L$ of a block of L spins
(Entanglement between block of spins and rest of the chain)
Entanglement in quantum phase transitions

**XY model** with magnetic field

[including **XX model** and **Ising model**]

\[
H_{XY} = \sum_{l=0}^{N-1} \left( \frac{(1+\gamma)}{2} \sigma^x_l \sigma^x_{l+1} + \frac{(1-\gamma)}{2} \sigma^y_l \sigma^y_{l+1} + \lambda \sigma^z_l \right)
\]

Ground state:

- **XY model with magnetic field**: Barouch and McCoy, Phys. Rev. A (1971)

Gaussian in fermionic modes (efficient description)
Entanglement in quantum phase transitions
Scaling of entanglement in critical and non-critical spin chains

Ising model for different values of the magnetic field $\lambda$
Entanglement in quantum phase transitions

Emergence of Universality

\[ S_L \approx \frac{1}{3} \log L \]

\[ S_L \approx \frac{1}{6} \log L \]

Critical XX

Critical XXZ

Critical Ising

Critical XY

ASYMPTOTICS
Jin and Korepin, quant-ph
Entanglement in quantum phase transitions

**Extra bonus!**

- **Connection to conformal field theory**
  
  \[ S_L \approx \frac{c + c}{6} \log L \]
  
  - geometric entropy
  
  - central charge
  
  - Srednicki, PRL 71 (1993)
  
  

- **C-theorem**
  
  - Entanglement decreases under RG flow

- **Spin lattices in D>1 dimensions**
  
  \[ S_L \approx L^{D-1} \]
  
  - "Area" law
  
  
  - Zamolodchikov, JETP Lett (1986)
  
  

- **Failure of White’s DMRG numerical method in 2D,3D**

  - # of eigenvectors of \( \rho_L \)
  
  \[ m \approx 2^{S_L} \]

  - non-critical  |  critical
  
  | 1D  | ✔   | ✔   | ✗   |
  
  | 2D, 3D | ✗   | ✗   | ✗   |
Efficient classical simulation of quantum dynamics

Measure of multipartite entanglement

Schmidt decomposition

\[ |\Psi\rangle = \sum_{\alpha=1}^{\chi_A} \lambda_{\alpha} |\Phi_{\alpha}^{[A]}\rangle |\Phi_{\alpha}^{[B]}\rangle \]

Schmidt rank

\[ \chi = \max_A \chi_A \]

\[ E_{\chi} \equiv \log_2 \chi \]

- Only vanishes for product (i.e. unentangled) states
- Additive under tensor product
- Non-increasing under LOCC (even under SLO)
Efficient classical simulation of quantum dynamics

Decomposition of N-qubit states

Standard decomposition

\[ |\Psi\rangle \]

\[ \Gamma^{[1]} \Lambda \Gamma^{[2]} \Lambda \cdots \Gamma^{[N]} \Lambda \]

\[ C_{i_1 i_2 \Lambda i_l \Lambda i_N} \]

\[ 2^N \text{ coefficients} \]

New decomposition

\[ l = 1, \Lambda, N \]

\[ i = 0, 1 \]

\[ \alpha = 1, \Lambda, \chi \]

\[ \beta = 1, \Lambda, \chi \]

\[ N \exp E_{\chi} \text{ coefficients} \]
Efficient classical simulation of quantum dynamics

Non-critical spin chain

\[ \chi \equiv \max_L \chi_L \]

saturation of \( \chi_L \) \( \rightarrow \) \( O(N) \) parameters to describe \( N \) spins
### Efficient classical simulation of quantum dynamics

<table>
<thead>
<tr>
<th>N spins</th>
<th>cost of simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-critical 1D system</td>
<td>$O(N)$</td>
</tr>
<tr>
<td>Critical 1D system</td>
<td>$O(N^q)$, $q &gt; 1$</td>
</tr>
<tr>
<td>Non-critical 2D system</td>
<td>$O(N \exp \sqrt{N})$</td>
</tr>
<tr>
<td>Critical 2D system</td>
<td>$O(N \exp N^{2/3})$</td>
</tr>
<tr>
<td>Non-critical 3D system</td>
<td></td>
</tr>
<tr>
<td>Critical 3D system</td>
<td></td>
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</tbody>
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Alternative method for non-critical systems: $O(N)$
Classical simulation of quantum dynamics
- Critical and non-critical spin chains.
- Non-critical spin lattices in 2D, 3D.

Entanglement in Quantum Many-Body Physics

Entanglement in quantum phase transitions
- Scaling of entanglement in critical and non-critical spin chains.
- Emergence of universality at a quantum critical point.
  - Conformal field theory
  - Monotonicity under RG flow
  - 2D, 3D systems
  - Failure DMRG method