Entanglement, quantum critical phenomena and efficient simulation of quantum dynamics

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outline

Quantum Information
ENTANGLEMENT

↓ ↓ ↓

Quantum
Many-Body Physics

Efficient classical simulation of quantum dynamics

- Critical and non-critical spin chains.
- Non-critical spin lattices in 2D, 3D.

Entanglement in quantum phase transitions

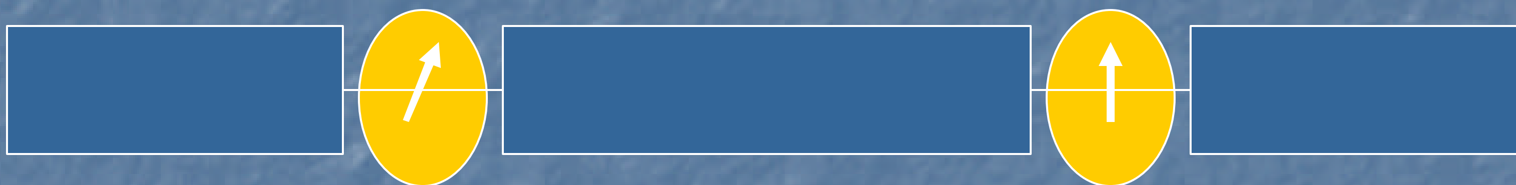
- Scaling of entanglement in critical and non-critical spin chains.
- Emergence of universality at a quantum critical point.
- Connection to conformal field theory, irreversibility of RG flow.
- Entanglement in spin lattices.
- Failure of the DMRG method.

Entanglement in quantum phase transitions

Vidal, Latorre, Rico, Kitaev, quant-ph/0211074 (to appear in Phys. Rev. Lett.)
Latorre, Rico, Vidal, quant-ph/0304098

Measures of entanglement in a quantum spin chain

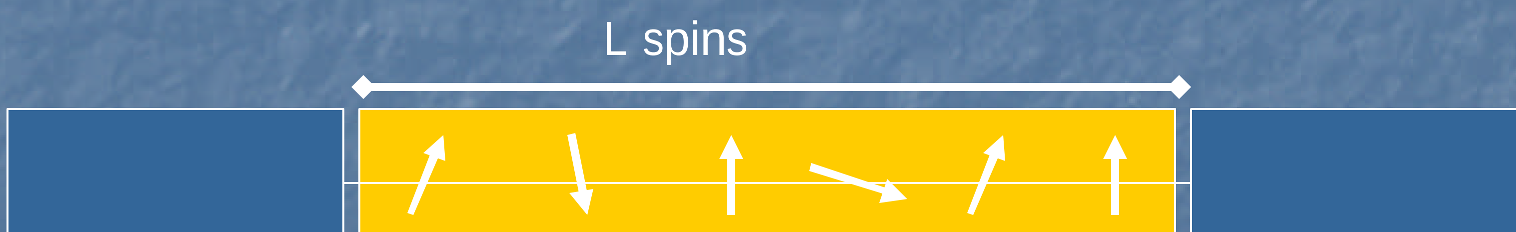
T=0, ground state



concurrence *(entanglement between two spins)*

Osterloh, Amico, Falci and Fazio, Nature 2002
Osborne and Nielsen, Phys. Rev. A 2002

Our approach:



entropy S_L of a block of L spins

(Entanglement between block of spins and rest of the chain)

Entanglement in quantum phase transitions

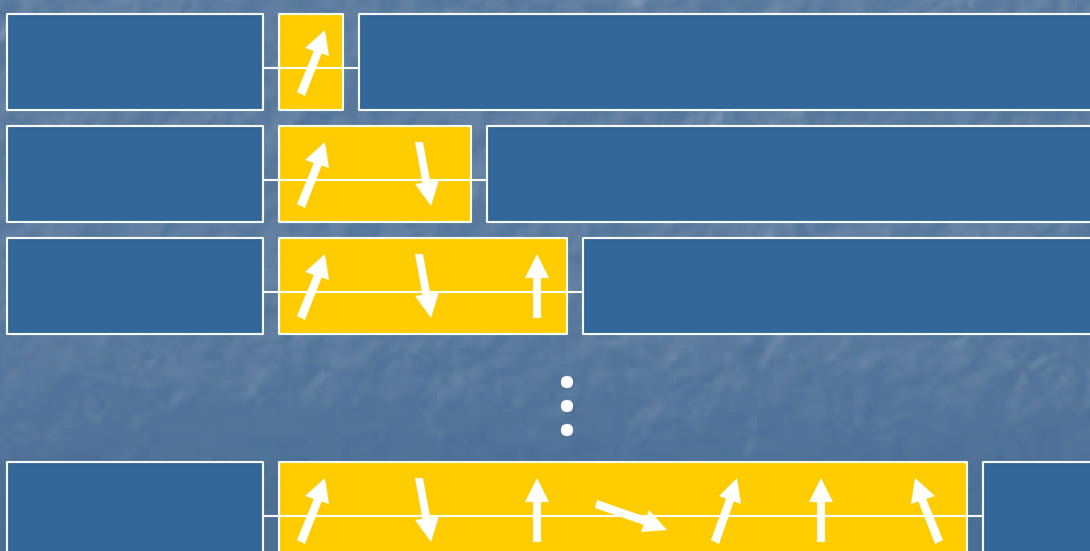
XY model with magnetic field
[including ***XX model*** and ***Ising model***]

$$H_{XY} = \sum_{l=0}^{N-1} \left(\frac{(1+g)}{2} s_l^x s_{l+1}^x + \frac{(1-g)}{2} s_l^y s_{l+1}^y + l\, s_l^z \right)$$

Ground state: $\left| \Psi_g \right\rangle$

XY model: Lieb, Schultz and Mattis, Ann. Phys. (1961)
XY model with magnetic field: Barouch and McCoy, Phys. Rev. A (1971)

gaussian in fermionic modes (efficient description)



r_1 — $S_1 = -tr\left(r_1 \log r_1 \right)$

r_2 — S_2

r_3 — S_3

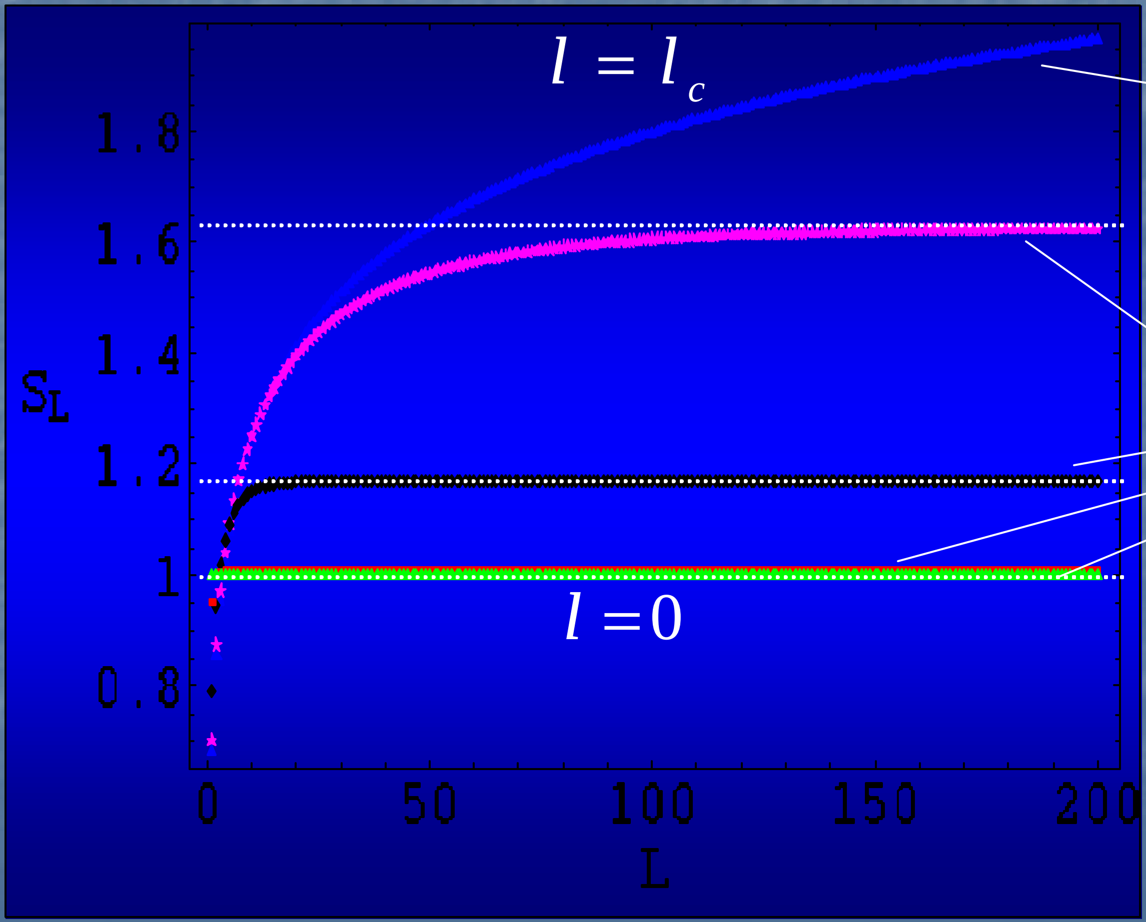
⋮

r_L — S_L

Entanglement in quantum phase transitions

Scaling of entanglement in critical and non-critical spin chains

Ising model for different values of the magnetic field l

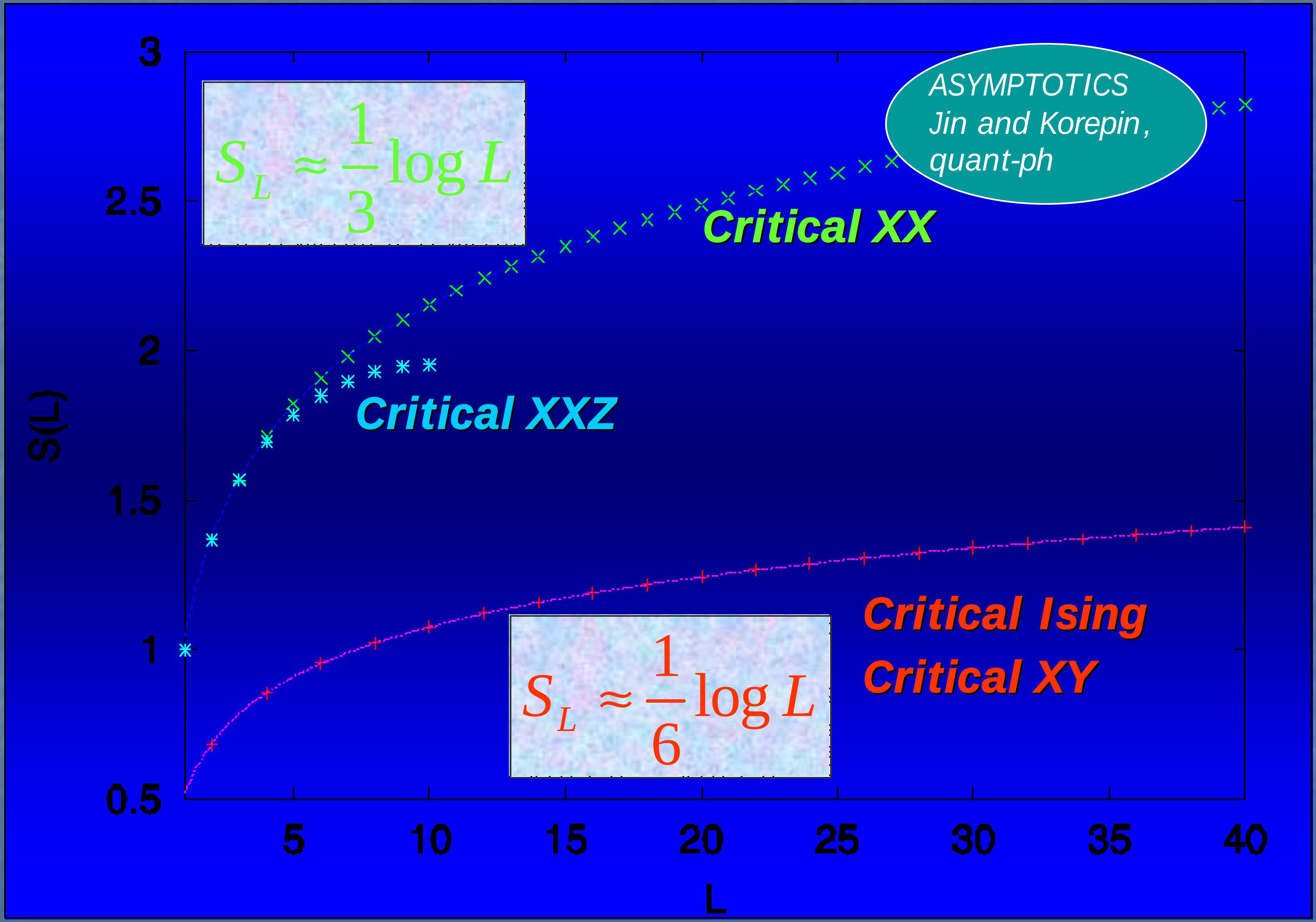


critical chain $S_L \approx \frac{1}{6}\log L$

non-critical chains $S_L \leq S^*(l)$

Entanglement in quantum phase transitions

Emergence of Universality



Entanglement in quantum phase transitions

Extra bonus!

- Connection to conformal field theory

geometric entropy

$$S_L \approx \frac{c+\bar{c}}{6}\log L$$

central charge

Srednicki, PRL 71 (1993)

Holzhey, Larsen, Wilczek, Nucl. Phys. B (1994)

Fiola, Preskill, Strominger, Trivedi, Phys Rev D (1994)

- C-theorem

Entanglement decreases under RG flow

Zamolodchikov, JETP Lett (1986)
Capelli, Friedan, Latorre, Nucl. Phys. B (1991)
Forte, Latorre, Nucl. Phys. B (1998)

- Spin lattices in D>1 dimensions

"Area" law

$$S_L \approx L^{D-1}$$

Srednicki, Phys. Rev. Lett. (1993)

- Failure of White's DMRG numerical method in 2D,3D

# of eigenvectors of ρ_L

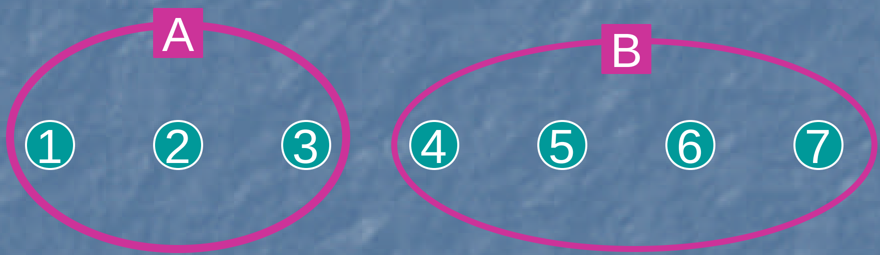
$$m \approx 2^{S_L}$$

	non-critical	critical
1D	✓	✓ ✗
2D, 3D	✗	✗

Efficient classical simulation of quantum dynamics

Vidal, quant-ph/0211074
Vidal, in preparation

Measure of multipartite entanglement



Schmidt decomposition

$$|\Psi\rangle = \sum_{a=1}^{c_A} l_a |\Phi_a^{[A]}\rangle |\Phi_a^{[B]}\rangle$$

Schmidt rank c_A

$$c \equiv \max_A c_A$$

$$E_c \equiv \log_2 c$$

- Only vanishes for product (*i.e.* unentangled) states
- Additive under tensor product
- Non-increasing under LOCC (even under SLO)

Decomposition of N-qubit states

standard decomposition $c_{i_1 i_2 \Lambda\ i_l \Lambda\ i_N}$ — 2^N coefficients

$|\Psi\rangle$

① ② ③ … Ⓛ … Ⓝ N qubits

new decomposition $\Gamma^{[1]}\Gamma^{[2]}\Lambda\ \Gamma^{[l]}\Lambda\ \Gamma^{[N]}$

$\Gamma^{[l]i}_{ab}$

$l = 1, \Lambda\ , N$

$i = 0,1$

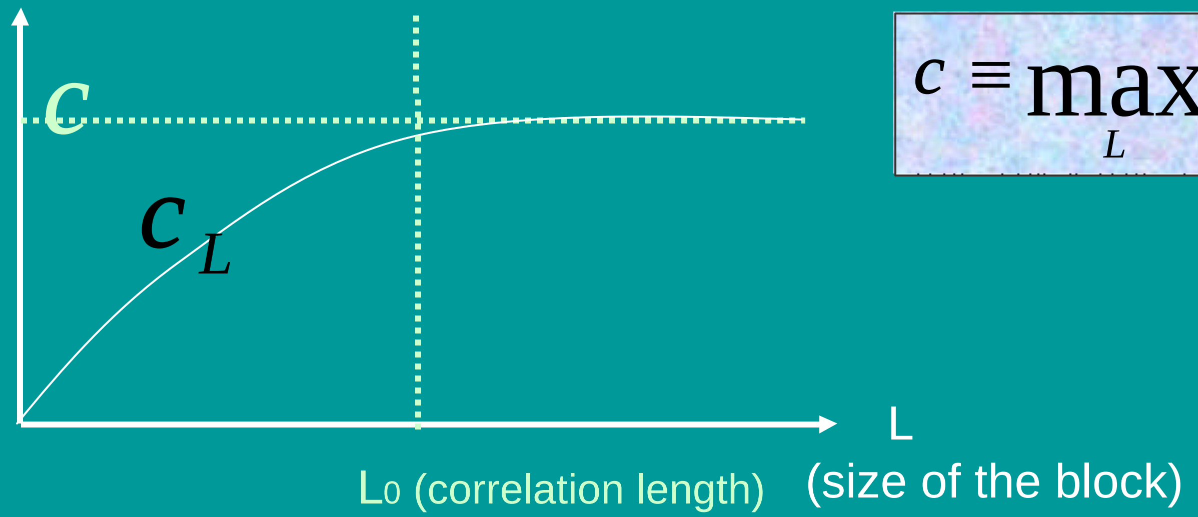
$a = 1, \Lambda\ , c$

$b = 1, \Lambda\ , c$

$N \exp E_c$ coefficients

Efficient classical simulation of quantum dynamics

Non-critical spin chain



$$c \equiv \max_L c_L$$

saturation of c_L ➡ O(N) parameters to describe N spins

Efficient classical simulation of quantum dynamics

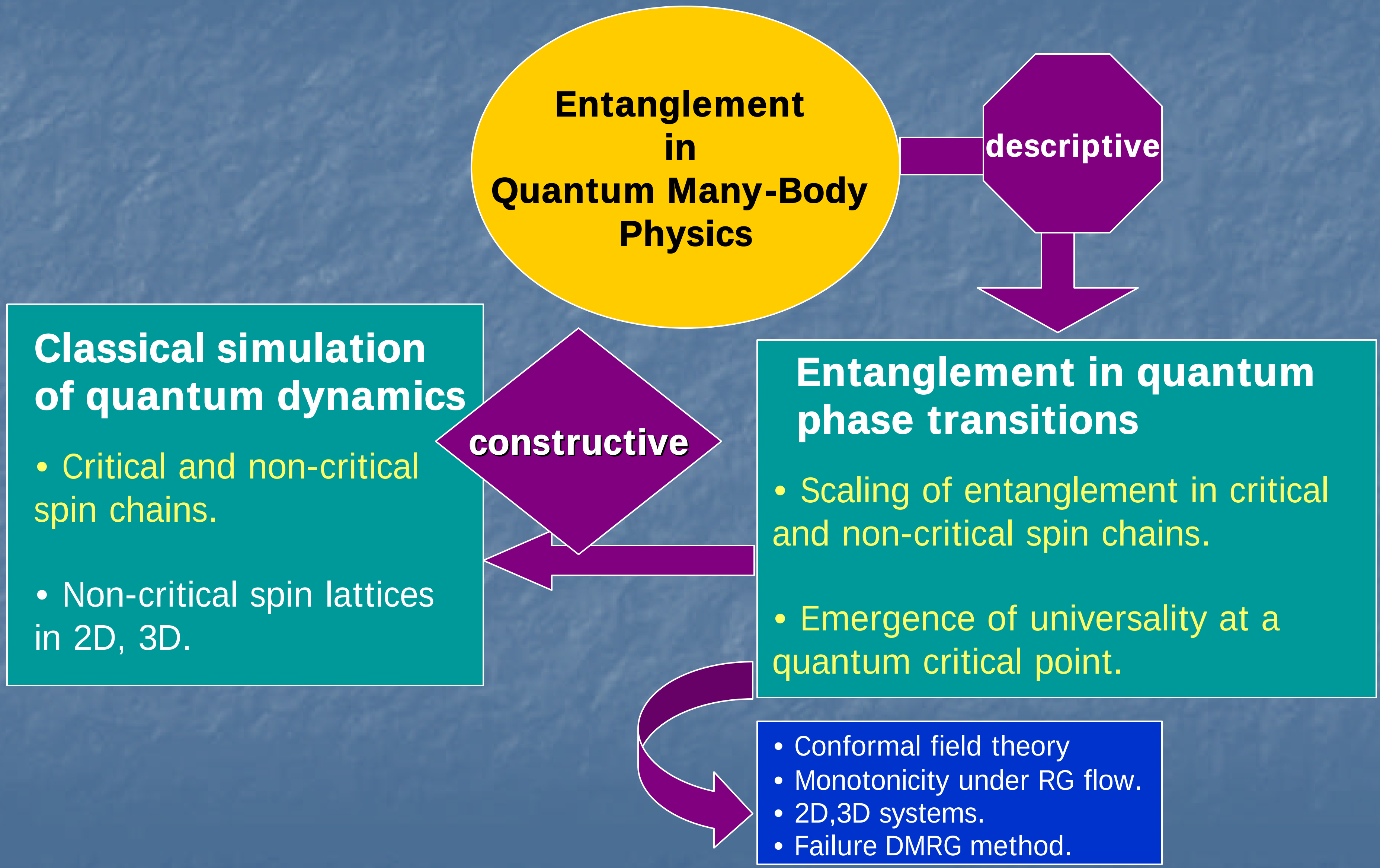
N spins	cost of simulation
Non-critical 1D system	$O(N)$
Critical 1D system	$O(N^q)$ q>1
Non-critical 2D system Critical 2D system	$O(N \exp \sqrt{N})$
Non-critical 3D system Critical 3D system	$O(N \exp N^{2/3})$

alternative method

$O(N)$

for non-critical systems

summary


Entanglement
in
Quantum Many-Body
Physics
descriptive
Entanglement in quantum phase transitions
• Scaling of entanglement in critical and non-critical spin chains.
• Emergence of universality at a quantum critical point.
constructive
Classical simulation of quantum dynamics
• Critical and non-critical spin chains.
• Non-critical spin lattices in 2D, 3D.
• Conformal field theory
• Monotonicity under RG flow.
• 2D,3D systems.
• Failure DMRG method.