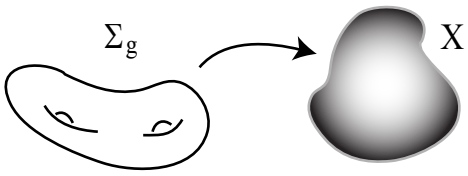
Asymptotic Freedom and the Spectral Index of String Vacua

based on joint work [hep-th/0505204, hep-th/stay_tuned] with Kirill Saraikin and Cumrun Vafa

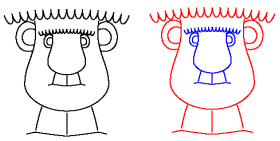
Topological Strings

- determine the dynamics of the 4-dimensional low-energy effective theory
- compute interesting invariants of X

X Calabi-Yau 3-fold



$$Z_{top} = \exp\left(\sum_g \lambda^{2g-2}\mathcal{F}_g\right)$$



A-model

$$\mathcal{F}_g(X,t) = \sum_\beta \mathrm{GW}_g(\beta)\, t^\beta$$

Gromov-Witten invartiants

Kahler moduli

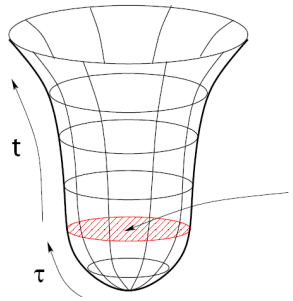
B-model

$$\mathcal{F}_0 : \quad X^I = \int_{A_I}\Omega, \quad F_I = \int_{B^I}\Omega, \quad F_I(X) = \frac{\partial \mathcal{F}_0(X)}{\partial X^I}$$

$\mathcal{F}_1$: holomorphic Ray-Singer torsion

Mirror Symmetry: $Z^A_{top}(X,t) = Z^B_{top}(\widetilde{X},z)$

No-Boundary Proposal [Hartle-Hawking'83]:

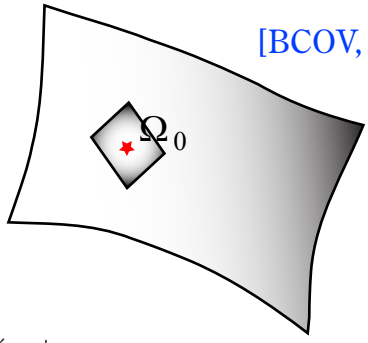


$$\Psi(g_M) = \sum_{W:\ \partial W = M} \int \mathcal{D}g_W \exp[-S_{eucl.}(W)]$$

Z_{top} = Wave Function

depends on Ω_0 (choice of polarization)

[BCOV, Witten]



$\Psi_{top} = Z_{top}$

topological M-theory

Entropic Principle

Z_{top} = Hartle-Hawking wave function for flux compactifications of type II string theory on $M^9 = \mathbf{S}^1 \times \mathbf{S}^2 \times X$

Ooguri-Vafa-Verlinde'05

$$e^{S_*} = |\Psi_{top}|^2$$

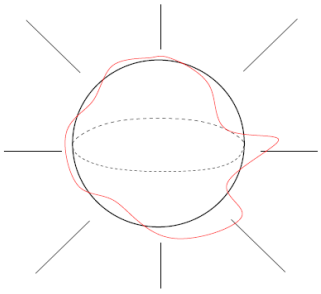
entropy functional

$$S = -i\frac{\pi}{4}\int_{CY}\Omega\wedge\overline{\Omega}.$$

String Cosmology: a Toy Model

$$Z_{\rm top}(t) = Z_{c=1}(t)$$

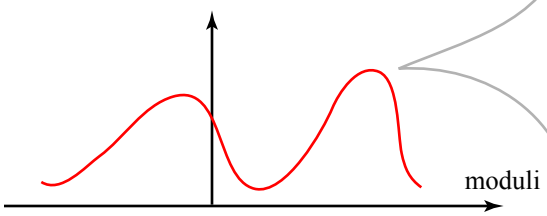
$$\langle\phi_{\vec{k}}\phi_{-\vec{k}}\rangle \sim g_s^2|\vec{k}|$$



Fluctuations of the $\mathbf{S}^3$ inside a Calabi-Yau.

Which String Vacua are Preferred?

$|\Psi_{top}|^2$ = measure (probability density) on the space of flux vacua



"attractor points":

$$\mathrm{Im}a_i = \mathrm{Im}a_i^D = 0,$$

- all but one periods have equal phase
- marginal BPS bound states

$$M_{BPS} = |n_0 + \vec{n}_2\cdot\vec{a} + \vec{n}_4\cdot\vec{a}^D|$$

- asymptotically free low-energy effective theories are preferred!