Holographic Action for Self-dual Fields

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The problem of formulating a quantum self-dual field is an important part of the formulation of string theory and supergravity. It is very subtle. It was pointed out some time ago by Marcus and Schwarz [2] that there is no simple Poincaré invariant action principle for the self-dual gauge field. Since then, much has been written about the action and the quantization of the self-dual field. Nevertheless, we believe the last word has not yet been said on this problem. The main point of my talk is to describe a new approach to the formulation of an action principle for self-dual fields.

What is ... a self-dual field? Consider a $(4\ell+2)$-dimensional space-time manifold M equipped with a Lorentzian metric of signature $-+\cdots+$. The Hodge $*$ squares to $+1$ on the middle dimensional forms $\Omega^{2\ell+1}(M)$, making it possible to impose a self-duality constraint on a field strength $\mathcal{F}\in\Omega^{2\ell+1}(M)$:

$$*_g\mathcal{F}^+ = \mathcal{F}^+. \tag{1}$$

When we impose (1) the Bianchi identity and equation of motion coincide

$$d\mathcal{F}^+ = 0. \tag{2}$$

A classical field theory describing the self-dual particle is completely specified by these two equations. The quantum theory, however, is problematic:

- As we have noted, folklore states there is no straightforward Lorentz covariant action.
- An important aspect of the quantum theory is Dirac quantization. In the string theory literature many authors attempt to impose a Dirac quantization condition of the form

$$\mathcal{F}^+ \in \Omega^{2\ell+1}_{\mathbb{Z}}(M), \tag{3}$$

i.e. $\mathcal{F}^+$ is a closed form with integral periods. However this quantization condition is *incompatible* with the self-duality constraint (1) since the self-duality condition (1) varies continuously with the metric g.

As we will see, both of these difficulties are nicely overcome by the holographic approach.

Holographic approach. To formulate the holographic approach one has to work in Eucledean field theory. Let (X, g_E) be a compact Riemannian $(4\ell+2)$-dimensional manifold. Now the Hodge star operator $*_E$ squares to -1 on the space of $(2\ell+1)$-forms. So the self-dual form becomes *imaginary anti-self-dual* (IASD):

$$*_E \mathcal{F}^+ = -i\mathcal{F}^+. \tag{4}$$

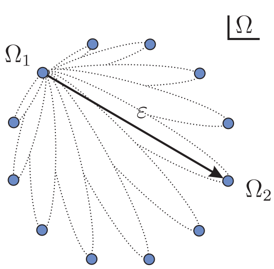
The holographic approach is based on the work by Edward Witten [3, 4], who stressed that the best way to formulate a self-dual theory is to rely on a Chern-Simons theory in one higher dimension. The partition function of IASD field on $(4\ell+2)$-manifold X as a function of an external current is a wave function of abelian spin Chern-Simons theory on $(4\ell+3)$-manifold Y where X is a boundary component of Y.

One advantage of the holographic approach is that it takes proper account of topological aspects ignored in other discussions. These are not — as is often stated — minor topological subtleties, but can lead to qualitative physical effects. Even in the simplest example of a chiral scalar in $1+1$ dimensions, that chiral scalar is equivalent to a chiral Weyl fermion. Accordingly, one cannot formulate the theory without making a choice of spin structure. A higher dimensional analog of the spin structure we called QRIF (Quadratic Refinement of Intersection Form). QRIF is map Ω from the integral cohomology $H^{2\ell+1}(X;\mathbb{Z})$ to $U(1)$ satisfying the equation

$$\Omega(a_1 + a_2) = \Omega(a_1)\Omega(a_2)(-1)^{(a_1 \cup a_2)[X]} \tag{5}$$

for all a_1, $a_2 \in H^{2\ell+1}(X;\mathbb{Z})$. A choice of solution of this equation is the choice of the theory of self-dual field.

What are the possible choices of Ω? Two solutions of (5) differ by a homomorphism from $H^{2\ell+1}(X;\mathbb{Z})$ to $\mathbb{R}/\mathbb{Z}$. By Poincaré duality it follows that any two solutions Ω_1 and Ω_2 are related by $\Omega_2(a) = \Omega_1(a)\, e^{i\pi(a\cup\varepsilon)[X]}$ where $\varepsilon \in H^{2\ell+1}(X, \mathbb{R}/\mathbb{Z})$. If we want Ω to take values ± 1 then ε is 2-torsion, i.e. $2\varepsilon = 0$.



Now, associated to Ω is an important invariant. Note that since the bilinear form $(a \cup b)[X]$ vanishes on torsion classes, Ω is a homomorphism from $\mathrm{Tor}\, H^{2\ell+1}(X;\mathbb{Z})$ to $\mathbb{R}/\mathbb{Z}$. Since there is a perfect pairing on torsion classes it follows that there is a $\mu \in \mathrm{Tor}\, H^{2\ell+2}(X;\mathbb{Z})$ such that

$$\Omega(a_T) = e^{2\pi i T(a_T,\mu)} = e^{2\pi i(\alpha\cup\mu)[X]} \tag{6}$$

for all torsion classes a_T. In the second equality we have written out the definition of the torsion pairing $T(a,\mu)$, namely, if $a_T = \beta(\alpha)$ where β is the Bockstein map then we can express it as a cup product. If we choose Ω to be $\mathbb{Z}_2$-valued then μ is 2-torsion. Note that if $\Omega_2(a) = \Omega_1(a)e^{i\pi(a\cup\varepsilon)[X]}$ then $\mu_2 = \mu_1 + \beta(\varepsilon)$.

Thus, the set of $\mathbb{Z}_2$-valued solutions Ω is a torsor for the group of 2-torsion points $G = (H^{2\ell+1}(X,\mathbb{R}/\mathbb{Z}))_2$. The set of solutions with a fixed value of μ is a torsor for the 2-torsion points in the identify component $G_0 = \mathcal{W}_2^{2\ell+1}(X)$. The group G_0 is isomorphic to $\bar{H}^{2\ell+1}(X;\mathbb{Z})/2\bar{H}^{2\ell+1}(X;\mathbb{Z})$ where $\bar{H}^{2\ell+1}(X;\mathbb{Z})$ denotes the reduction of the cohomology group modulo torsion.

Partition function. The main result of the paper [1] is that the partition function for IASD field can be written in the form of functional integral:

$$\mathscr{Z}^{\Omega}(a^+,a^-;\Sigma) = e^{i\pi\omega(\varepsilon_1,\varepsilon_2)}e^{i\pi\omega(a^-,a^+)}\vartheta^{\varepsilon+\sigma(\check{A}_\bullet,\Sigma)}(a^+) \tag{7}$$

where $\varepsilon \in \Omega_d^{2\ell+1}(X)$ is defined by Ω and a choice Lagrangian decomposition of the field space (will be explained below) and

$$\vartheta^{\eta}(a^+) = \exp\Big[i\pi(\eta,\mathcal{F}^-(\eta)) + \frac{\pi}{2}B(a^+,a^+) + 2\pi i\omega((a,\mathcal{F}^-(\eta))\Big] \times \int_{\bar{V}_1/V_{12}} \mathscr{D}R\, \exp\Big[i\pi\omega(R,\mathcal{F}^-(R)) - 2\pi i\omega(a+\eta,\mathcal{F}^-(R))\Big]. \tag{8}$$

All notations will be explained in details in the next section. Let's briefly go through the equation (8): R is a closed $(2\ell+1)$-form, $\mathcal{F}^-$ is a linear operator, ω is a symplectic form. So $\omega(R,\mathcal{F}^-(R))$ is the quadratic action — a Eucledean action for the self-dual field. The functional integral is Gaussian and thus can be calculated exactly:

$$\mathscr{Z}^{\Omega}(a^+,a^-)e^{i\pi\omega(\varepsilon_1,\varepsilon_2)}e^{i\pi\omega(a^-,a^+)}\mathcal{N}_g\,\vartheta^{\varepsilon}_{\Gamma_1\oplus\Gamma_2}(a^+) \tag{9}$$

where $\vartheta^{\varepsilon}_{\Gamma_1\oplus\Gamma_2}(a^+)$ is a classical theta function and $\mathcal{N}_g$ is a metric dependent factor coming from integration over topologically trivial fields. For example, for $X = T^2$ with the metric $ds^2 = \frac{1}{\tau_2}|d\sigma_1 - \tau d\sigma_2|^2$ $\mathcal{N}_g = 1/\eta(\tau)$ where $\eta(\tau)$ is Dedekind eta function. Not much is known about this metric dependent factor $\mathcal{N}_g$ on general manifold X. Equation (8) is the first equation appeared in the literature which allows to calculate $\mathcal{N}_g$.

The partition function (7) satisfies quantum Gauss law. In the infinitesimal form it yields *quantum equation of motion* (compare with (2)):

$$d\langle\mathcal{F}^-(R-\varepsilon-\sigma)\rangle_{\check{A},\Sigma} = \delta(\Sigma) - F(\check{A}). \tag{10}$$

Phase space. The phase space of CS theory can be identified with $V_{\mathbb{R}} = \Omega^{2\ell+1}(X)$. The symplectic form is defined by

$$\omega(v,w) = \int_X v\wedge w. \tag{11}$$

Note that ω is integral valued on $\Omega^{2\ell+1}_{\mathbb{Z}}(X)$. To quantize CS theory we need to choose a polarization of $V_{\mathbb{R}}$. A choice of Riemannian metric g_E on X defines a compatible complex structure on $V_{\mathbb{R}}$, $J = -*_E$.

Using J we decompose the complexification of $V_{\mathbb{R}}$ as

$$V_{\mathbb{R}} \otimes \mathbb{C} \cong V^+ \oplus V^-.$$

imaginary anti self-dual   imaginary self-dual

Any vector $R^{\pm}$ of the complex vector space $V^{\pm}$ can be *uniquely* written as

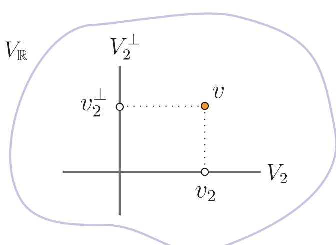
$$R^{\pm} = \frac{1}{2}(R \pm i *_E R) \tag{12}$$

for some real vector $R \in V_{\mathbb{R}}$. A symplectic form ω defines a Hermitian form H on $V^+ \times V^+$:

$$H(v^+, w^+) := 2i\,\omega(v^+, \overline{w^+}) = g(v,w) + i\omega(v,w) \tag{13}$$

where $g(v,w) = \int_X v \wedge *w$. In our notation H is $\mathbb{C}$-linear in the first argument and $\mathbb{C}$-antilinear in the second: $H(u,v) = \overline{H(v,u)}$.

To solve the quantum Gauss law we need in addition a $\mathbb{C}$-bilinear form B on $V^+ \times V^+$. However, *there is no natural* $\mathbb{C}$*-bilinear form on* $V^+ \times V^+$. Thus we have to make a choice — this is the source of *all* difficulties we had with the SD form.



$\mathbb{C}$-bilinear forms on $V^+ \times V^+$ are in one-to-one correspondence with Lagrangian subspaces of $V_{\mathbb{R}}$. A choice of Lagrangian subspace $V_2 \subset V_{\mathbb{R}}$ defines an orthogonal coordinate system on $V_{\mathbb{R}}$: $\mathbb{R} = V_2 \oplus V_2^{\perp}$ where $V_2^{\perp} = *_E V_2$. Any vector $v \in V_{\mathbb{R}}$ can be uniquely written as $v = v_2 + v_2^{\perp}$.

Actually in the solution of the Gauss law appears only the combination $(H - B)$ which is defined by

$$(H - B)(v^+, w^+) = -2i\omega(v, \mathcal{F}^-(w)) \tag{14}$$

where $\mathcal{F}^-(w) = w_2^{\perp} + i *_E w_2^{\perp}$. $(H - B)$ restricted to the diagonal is almost the classical action. The problem is that $(H - B)$ is *not* a symmetric form. Note however that if v, w are elements of a Lagrangian subspace V_1 then $(H - B)|_{V_1^+ \times V_1^+}$ defines a symmetric form. So the Eucledean action is

$$S_E(R) = -i\pi\omega(R, \mathcal{F}^-(R)) = \pi \int_X (R_2^{\perp} \wedge *R_2^{\perp} - iR_2 \wedge R_2^{\perp}) \quad \text{for} \quad R \in V_1. \tag{15}$$

What is V_1? A Lagrangian subspace V_1 is defined by the following procedure:

1. A choice of V_2 defines a Lagrangian subspace $\Gamma_2 \subset \Gamma = H^{2\ell+1}_{dR}(X)$. We assume that V_2 is such that Γ_2 contains a lattice $\bar{\Gamma}_2 \subset \bar{\Gamma}$ where $\bar{\Gamma}$ is the image of integral cohomology under the natural projection onto de Rham cohomology.
2. Choose a complementary Lagrangian subspace $\Gamma_1 \subset \Gamma$ such that $\Gamma_1 \oplus \Gamma_2 = \Gamma$ and $\bar{\Gamma}_1 \oplus \bar{\Gamma}_2 = \bar{\Gamma}$.
3. Define $\bar{V}_1 = \{R \in \Omega^{2\ell+1}_{d,\mathbb{Z}}(X) \,|\, [R]_{dR} \in \bar{\Gamma}_1\}$. V_1 is a Lagrangian subspace, $\bar{V}_1$ is an isotropic subspace.

Note that by construction $S_E(R)$ vanishes on $R \in V_{12}$ where

$$V_{12} = V_1 \cap V_2 = \{ \text{ exact forms in } V_2 \}. \tag{16}$$

The transformation $R \mapsto R + v_{12}$ should be considered as a gauge transformation.

Lorentzian action and its properties. Let (M, g) be a Lorentzian manifold. The action in the Lorentzian signature can obtained from (15) by Wick rotation:

$$S_L(R) := \pi \int_X R \wedge \mathcal{F}^+(R) = \pi \int_M \left(R_2^\perp \wedge *R_2^\perp + R_2 \wedge R_2^\perp\right) \tag{17}$$

where $\mathcal{F}^+(R) = R_2^\perp + *R_2^\perp$. This action depends on a choice of Lagrangian subspace V_2. For a Riemannian manifold a choice of V_2 automatically defines the Lagrangian decomposition $V_\mathbb{R} = V_2 \oplus *_E V_2$. For a Lorentzian manifold this is not true, and we need *to constrain* the choice of V_2 by the requirement[1]

$$V_2 \cap *V_2 = \{0\}. \tag{18}$$

The action (17) has the following properties

1. Variation of the action with respect to $R \mapsto R + d\delta c$ for $\delta c \in \Omega^{2\ell}_{cpt}(M)$ is

$$\delta S_L(R) = -2\pi \int_M \delta c \wedge \mathcal{F}^+(R) \tag{19}$$

   where $\mathcal{F}^+(R) = R_2^\perp + *R_2^\perp$. This yields equations of motion: $R \in V_1$ and $d\mathcal{F}^+(R) = 0$.
2. Metric dependence

$$\delta_g S_L(R) = \pi \int_M (\delta g^{-1} g)^\mu{}_\nu \, \mathcal{F}^+ \wedge (dx^\nu \wedge i(\tfrac{\partial}{\partial x^\mu})\mathcal{F}^+) = \int_M \delta g^{\mu\nu} T_{\mu\nu}(\mathcal{F}^+) \tag{20}$$

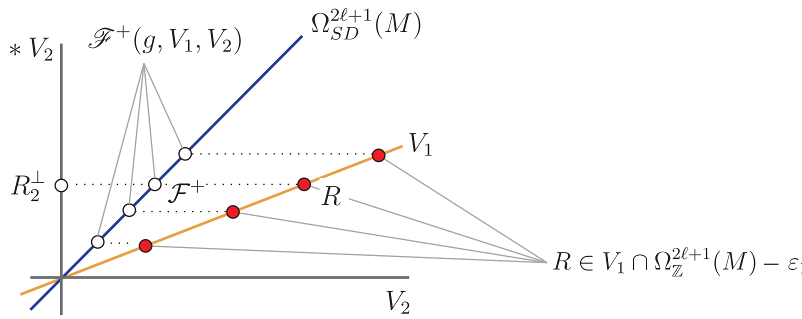
   where $T_{\mu\nu}(\mathcal{F}^+)$ is the standard stress-energy tensor for self-dual field.

[1] In principle, V_2 can be an arbitrary Lagrangian subspace satisfying the constraint (18). Recall that on any Lorentzian manifold M there exists a nowhere vanishing timelike vector field ξ. It can be used to define a Lagrangian subspace $V_2 \subset \Omega^{2\ell+1}(M)$:

$$V_2(\xi) := \{\omega \in \Omega^{2\ell+1}(M) \,|\, i_\xi \omega = 0\}.$$

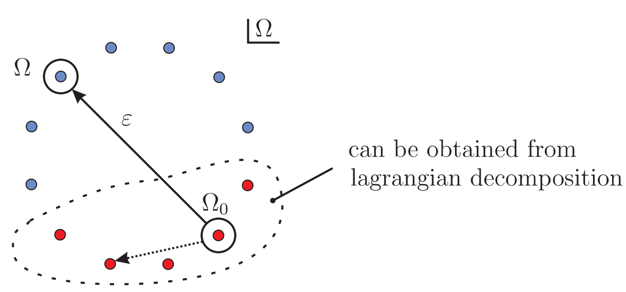
3. (Classical) diffeomorphism invariance. Note that a diffeomorphism from a connected component of the identity $f \in \mathrm{Diff}_0^+(M)$ preserves the Lagrnagian subspace V_1: $f^*V_1 = V_1$. For vectorfield ξ with a compact support one finds
$$\delta_\xi S_L = -2\pi \int_M i_\xi R \wedge d\mathcal{F}^+(R) + \int_M \nabla^\mu \xi^\nu T_{\mu\nu}(\mathcal{F}^+). \quad (21)$$
Thus on equations of motion we have (classical) conservation of the stress-energy tensor.

Dirac quantization. Let us now return to the second conundrum surrounding (3). We see from (17) that we should distinguish between R and the self-dual flux $\mathcal{F}^+(R)$. Thus the way out is to understand the quantization condition in a broader sense: there is an abelian group $\mathscr{F}^+(g, V_1, V_2)$ with nontrivial connected components inside the space of *closed* self-dual forms $\Omega^{2\ell+1}_{SD}(M)$, and



the classical self-dual field $\mathcal{F}^+(R)$ takes values only in this group: $\mathcal{F}^+(R) := R_2^\perp + *R_2^\perp$ where $R = R_2^\perp + R_2$ and $[R]_{dR} \in \Gamma_1 - [\varepsilon_1]$. Here $\varepsilon_1 \in V_1$ and $\varepsilon_2 \in V_2$ are characteristics. They are determined by Ω and Lagrangian subspaces V_1 and V_2 in the following way.

A choice of Ω_0 with $\mu = 0$ is naturally determined by a Lagrangian decomposition of $\bar{H}^{2\ell+1}(X;\mathbb{Z}) = \bar{\Gamma}_1 \oplus \bar{\Gamma}_2$. Any $R \in \Omega^{2\ell+1}_{\mathbb{Z}}(X)$ can be written as $R = R_1 + R_2$ where $R_1 \in \bar{V}_1$ and $R_2 \in V_2 \cap \Omega^{2\ell+1}_{\mathbb{Z}}(X)$. Since $V_1 \cap V_2 \neq \{0\}$ this decomposition is not unique and two different decompositions are related by adding exact forms in V_2. Now define
$$\Omega_{\Gamma_1 \oplus \Gamma_2}(R) := e^{i\pi\omega(R_1, R_2)}. \quad (22)$$
Since R_1 and R_2 are closed it follows that $\Omega_{\Gamma_1 \oplus \Gamma_2}(R)$ does not depend on a particular choice of decomposition $R = R_1 + R_2$. Moreover $\Omega_{\Gamma_1 \oplus \Gamma_2}$ takes values in $\{\pm 1\}$.



Given $\Omega_{\Gamma_1 \oplus \Gamma_2}$ we can parameterize all solutions with $\mu = 0$ by $[\varepsilon] \in \Omega_d^{2\ell+1}(X)/\Omega_{\mathbb{Z}}^{2\ell+1}(X)$. So Ω_0 can be written as

$$\Omega_0(R) = e^{i\pi\omega(R_1,R_2)+2\pi i\omega(\varepsilon,R)}. \tag{23}$$

If we want $\Omega_0(R)$ to take values in $\{\pm 1\}$ then ε is quantized $[\varepsilon] \in \Omega_{\frac{1}{2}\mathbb{Z}}^{2\ell+1}(X)/\Omega_{\mathbb{Z}}^{2\ell+1}(X)$. In this case a simple calculation shows that only *even* solutions ($\mathrm{arf}(\Omega) = +1$) can be obtained by a choice of Lagrangian decomposition.

Example: Chiral scalar on $\mathbb{R}^2$. Consider a Lorentzian manifold $M = \mathbb{R}^2$ equipped with the metric $ds^2 = e^{2\varphi(x,t)}(-dt^2 + dx^2)$. Choose the Lagrangian subspace V_2 as

$$V_2 = \{dt\,\omega_t(x,t)\} \quad \Rightarrow \quad V_2^\perp = \{dx\,\omega_x(x,t)\}. \tag{24}$$

The Lagrangian subspace V_1 is just the space of all exact 1-forms $\Omega^1_{\mathrm{exact}}(M)$. So $R \in V_1$ decomposes as

$$R = \underbrace{dx\,R_x}_{R_2^\perp} + \underbrace{dt\,R_t}_{R_2}. \tag{25}$$

The action (17) takes the form

$$S_L(R) = \pi \int_{\mathbb{R}^2} dtdx\,R_x(R_x + R_t).$$

Now for $R = d\phi$ the action becomes

$$S_L(\phi) = \pi \int_{\mathbb{R}^2} dtdx\,\Big[(\partial_x\phi)^2 + (\partial_x\phi)(\partial_t\phi)\Big]. \tag{26}$$

This action for the chiral scalar has been proposed before [6]. The equation of motion is

$$(\partial_x + \partial_t)\partial_x\phi = 0. \tag{27}$$

Thus the general solution is $\phi(x,t) = f(t) + \phi_L(x-t)$. It seems that we get an extra degree of freedom represented by an arbitrary function of time $f(t)$. However the self-dual field $\mathcal{F}^+$ depends only on $\phi_L(x-t)$. Indeed, substituting the solution to the definition of $\mathcal{F}^+$ one finds

$$\mathcal{F}^+(\phi) = (dx - dt)\phi_L'(x-t) \tag{28}$$

where ϕ_L' denotes the derivative of ϕ_L with respect to the argument.

The extra degree of freedom $f(t)$ is the gauge degree of freedom (16), and it can be removed by the gauge transformation $R \mapsto R - df(t)$ where $-df(t) \in V_1 \cap V_2$.

RR fields of type IIA/IIB supergravity. In paper [8] we show that action for Ramond-Ramond fields of type IIA/IIB supergravity can be obtained from Chern-Simons theory in 11 dimensions. There are two Chern-Simons theories in 11-dimensions: Chern-Simons-A and Chern-Simons-B whose quantization on 11-manifold with boundary yields partition function of RR fields of type IIA and type IIB supergravity respectively.

For RR fields of type IIA the phase space is $V_{\mathbb{R}} = \Omega^{ev}(M)$. So any element $v \in \Omega^{ev}(M)$ can be written as a sum of forms of even degree: $v = v_0 + v_2 + v_4 + \cdots + v_{10}$. The symplectic form is

$$\omega_0(v,w) = \int_X (-v_0 w_{10} + v_2 w_8 - v_4 w_6 + v_6 w_4 - v_8 w_2 + v_{10} w_0); \tag{29}$$

In the large volume limit there is a natural choice of lagrangian subspace V_2 consisting of forms of higher degree:

$$V_2 = (\Omega^6 \oplus \Omega^8 \oplus \Omega^{10})(M). \tag{30}$$

It defines a lagrangian subspace $\Gamma_2 \subset H^{ev}_{d_H}(M)$ inside the twisted cohomology. Let Γ_1 be an arbitrary complementary subspace. V_1 is defined by

$$V_1 = \{R \in \Omega(M)^{ev}_{d_H} \,|\, [R]_{d_H} \in \Gamma_1\}. \tag{31}$$

$R \in V_1$ decomposes as

$$R = \underbrace{R_0 + R_2 + R_4}_{R_2^{\perp}} + \underbrace{R_6 + R_8 + R_{10}}_{R_2}. \tag{32}$$

Thus the self-dual flux $\mathcal{F}^+(R)$ is

$$\mathcal{F}^+(R) = R_0 + R_2 + R_4 + \ell_s^{-2} * R_4 - \ell_s^{-6} * R_2 + \ell_s^{-10} * R_0 \tag{33}$$

The equation of motion $d_H \mathcal{F}^+(R) = 0$ yields the following system of equations:

$$dR_0 = 0, \quad dR_2 - H \wedge R_0 = 0, \quad dR_4 - H \wedge R_2 = 0; \tag{34a}$$

$$d(\ell_s^{-2} * R_4) - H \wedge R_4 = 0, \quad d(\ell_s^{-4} * R_2) + H \wedge *R_4 = 0 \tag{34b}$$

These are the equations of motion for RR fields of type IIA supergravity in the string frame.

The action (17) takes the following form

$$S_{\mathtt{IIA}}(R) = -\frac{\pi}{\ell_s^{10}} \int_M R_0 \wedge *R_0 - \frac{\pi}{\ell_s^{6}} \int_M R_2 \wedge *R_2 - \frac{\pi}{\ell_s^{2}} \int_M R_4 \wedge *R_4 - \pi \int_M (R_{10} \wedge R_0 - R_8 \wedge R_2 + R_6 \wedge R_4). \tag{35}$$

The kinetic terms coincide with the kinetic terms of the RR fields in the string frame. The topological term looks differently as compared to the usual one. Suppose that M is a boundary of 11 manifold Y and K differential character $\check{C}$ extends to a character $\check{\tilde{C}}$ over Y, then

$$e^{i\pi \int_{\partial Y} (R_{10} \wedge R_0 - R_8 \wedge R_2 + R_6 \wedge R_4)} = e^{i\pi \int_Y d(\tilde{R}_{10} \wedge \tilde{R}_0 - \tilde{R}_8 \wedge \tilde{R}_2 + \tilde{R}_6 \wedge \tilde{R}_4)} = e^{i\pi \int_Y \tilde{H} \wedge \tilde{R}_4 \wedge \tilde{R}_4}. \tag{36}$$

Notice that in the presence of sources the right hand side is *not* well defined while the left hand side is.

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