Non-supersymmetric Meta-stable Vacua from Brane Configurations

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Meta-stable vacua in SQCD with massive flavors

Recently, Intriligator, Seiberg and Shih [1] showed the existence of meta-stable, SUSY breaking vacua in SQCD with massive flavors. Consider $SU(N_c)$ SYM with N_f flavors Q and $\tilde{Q}$ of mass m. When the theory is in the free-magnetic range $N_c + 1 \leq N_f < \frac{3}{2}N_c$, we can use the IR free magnetic description. The magnetic theory has an $SU(N = N_f - N_c)$ gauge group, N_f flavors q and $\tilde{q}$, a singlet $N_f \times N_f$ meson Φ and superpotential

$$W = h \operatorname{Tr} (q \, \Phi \, \tilde{q}) - h\mu^2 \operatorname{Tr} \Phi \tag{1}$$

(where the traces run over flavor indices), $h = \Lambda/\hat{\Lambda}$ and $\mu^2 = -m\hat{\Lambda}$. The scale $\hat{\Lambda}$ is defined in terms of the dynamical scales of the electric and magnetic theories. The expression above corresponds to equal masses for all flavors. Generalizing it to the generic case is straightforward.

This theory breaks supersymmetry at tree level due to the F-term of Φ (the so-called rank condition). There is classical moduli space of minima, parametrized by the vevs

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \end{pmatrix} \qquad q = \begin{pmatrix} \varphi_0 \\ 0 \end{pmatrix}, \qquad \tilde{q}^T = \begin{pmatrix} \tilde{\varphi}_0 \\ 0 \end{pmatrix}, \qquad \text{with } \tilde{\varphi}_0 \, \varphi_0 = \mu^2 \mathbf{1_N}. \tag{2}$$

All pseudo-moduli (classical flat directions not corresponding to Goldstone directions) are lifted by the one-loop Coleman-Weinberg effective potential. There non SUSY minimum occurs at

$$\Phi_0 = 0 \qquad \varphi_0 = \tilde{\varphi}_0 = \mu^2 \mathbf{1_N} \tag{3}$$

For generic flavor masses m_i $(i = 1, \ldots, N_f)$, the expectation values of φ_0 and $\tilde{\varphi}_0$ are determined by the N larger masses. Field configurations in which some of their entries correspond to some of the N_c smaller masses are classically unstable. When the masses are much smaller than the dynamical scale of the gauge theory, the lifetime of the meta-stable vacuum is parametrically large. The vacuum energy is $V_{meta} = \sum_{i=1}^{N_c} \hat{\Lambda}^2 |h^2 m_i^2|$, where the sum is over the N_c smallest masses.

Type IIA configuration

An efficient way to realize supersymmetric gauge field theories in string theory is to embed them as the effective gauge theory on the world-volume of configurations of D- and NS-branes. A comprehensive review of these constructions can be found in [5]. Many field theory features are easily visualized in terms of the geometry of the brane configuration. In this talk, we review our work in [2]. Related ideas can be found in [3, 4].

Fig.1 shows the Type IIA brane configuration describing the meta-stable vacuum in $SU(N_c)$ SQCD with N_f massive flavors. All the branes share the 0123 directions, in which the 4d gauge theory lives. The positions of D6-branes in the 45 plane are proportional to the masses. It is straightforward to see many of the field theory properties from this setup. Among them, we have:

- **Pseudo-moduli:** they correspond to the motion of the D4'-branes in the 89 directions.

- **Vacuum energy:** it is accounted for by the increased length of the D4'-branes with respect to the SUSY massless configuration.

- **Classical instabilities:** consider the configuration where a dual quark vev is given by one of the N_c smallest masses. There is one D6-brane on which two D4-brane pieces (one connecting to the NS and other to the NS'-branes) coincide at a non-supersymmetric angle. The open string tachyon at their intersection is precisely the unstable mode that appears in the field theory analysis.
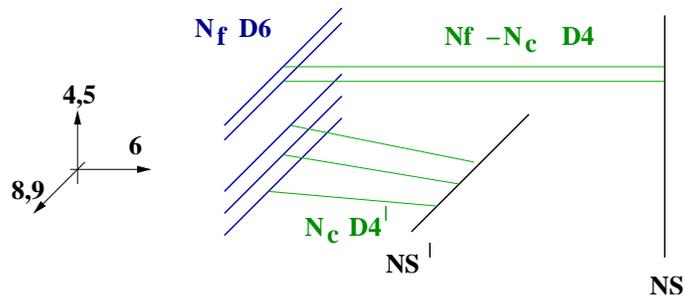

Figure 1: The non-supersymmetric type IIA configuration reproducing the non-supersymmetric ISS field theory minimum for arbitrary flavor masses.

Pseudo-moduli stabilization

The mechanism that stabilizes the pseudo-moduli is clear in the type IIA perspective. The fields that contribute to the field theory 1-loop Coleman-Weinberg potential are classically massive fields whose mass depends on the pseudo-moduli. In the type IIA brane configuration, the pseudo-moduli are the geometric positions of D4'-branes in 89. Clearly, the classically massive fields whose mass depends on these positions are D4-D4' and D4'-D4' open strings. In the case in which the smaller N_c masses are equal, the D4'-D4' open strings are not sensitive to the breaking of supersymmetry and do not contribute, hence only the D4-D4' states contribute. The Coleman-Weinberg potential then corresponds to the annulus diagram with boundaries on the D4 and D4'-branes. Hence the lifting of pseudo-moduli is an effect that cannot be detected from the study of the D4/NS or D4'/NS' systems in isolation, but which arises from their interaction.

Due to the complicated geometry (and the presence of the NS and NS'-branes) this diagram cannot be computed for arbitrary locations of the D4'-branes. In the small distance regime, its result should reproduce the field theory result. On the other hand, in the large distance regime, the annulus diagram corresponds to the exchange of supergravity modes (graviton, dilaton and 5-form exchange) between the D4 and D4'-branes. Being non-supersymmetric, it is expected that the gravitational exchange overcomes the RR-form repulsion (which is smaller due to the misalignment of the D-branes) and lead to a net attraction, which pushes the D4'-branes towards the origin in 89. This is the string theory view of the lifting of the pseudo-moduli, in the large field region of pseudo-moduli space.

M-theory lift

The type IIA configuration that we have described can be lifted to M-theory. In the lift, D4-branes and the NS or NS'-branes on which they end become different parts of a single smooth M5-brane, wrapped on a complex curve in the ambient space (which is given by an N_f-centered Taub-NUT geometry, corresponding to the M-theory lift of the N_f D6-branes). The Taub-NUT geometry is hyper-Kähler and therefore admits a $\mathbb{P}^1$ of complex structures. We focus on the case in which the N_c lowest masses are equal. The M5-brane factorizes into two components, corresponding to the lifts of the D4/NS and the D4'/NS' systems. The two components are holomorphic in two different complex structures in this geometry. Being holomorphic in some complex structure, each component is volume minimizing by itself. However, the complete system can be regarded as an M5-brane on a singular (i.e. reducible) non-holomorphic curve, which is therefore not volume-minimizing as a whole.

In order to consider the lift of the D4/NS system, let us introduce an adapted complex structure, in which the corresponding M5-brane curve is holomorphic. In fact the system is locally $\mathcal{N} = 2$ supersymmetric, hence we may stick to the usual conventions for lifts of configurations of 4d $\mathcal{N} = 2$ theories. Let us introduce $v = x_4 + ix_5$, $w = x_8 + ix_9$, and describe the ambient M-theory Taub-NUT

geometry as the complex manifold

$$yz = \left[\prod_{i=N_c+1}^{N} (v - \mu_i) \right] (v - \mu)^{N_c} \tag{4}$$

where $(\mu_{N_c+1}, \ldots, \mu_{N_f})$ correspond to the N largest mass parameters and $\mu < \mu_i$ is the common mass parameter for the N_c lightest flavors.

In these complex coordinates, the holomorphic curve corresponding to the D4/NS system has the structure

$$z - \prod_{i=N_c+1}^{N} (v - \mu_i) = 0$$
$$w = 0 \tag{5}$$

The lift of the D4'/NS' component is also easily described in the case of a common mass for the lightest flavors. The system is locally $\mathcal{N} = 1$ supersymmetric, so it is described by a holomorphic curve in adapted complex coordinates. Intuitively we introduce $v' = x'_4 + ix'_5$, where x'_4 and x'_5 parametrize the 2-plane orthogonal to the x'_6 direction along which the D4'-branes stretch. Similarly we need to introduce new complex parameters μ'_i, μ' which encode the positions of the D6-branes in the v' direction. In the complex structure adapted to the D4'/NS' system, the Taub-NUT geometry is described by

$$y'z' = \left[\prod_{i=N_c+1}^{N} (v' - \mu'_i) \right] (v' - \mu')^{N_c} \tag{6}$$

In these complex coordinates, the holomorphic curve describing the lift of the D4'/NS' system is

$$z' - \prod_{i=1}^{N_c} (w' - w'_i) = 0$$
$$v' = 0 \tag{7}$$

The M-theory state we have constructed reduces at vanishing string coupling to the type IIA configuration we have studied. Interestingly, it captures various features of the IIA configuration such as its pseudo-moduli. Nevertheless, it is important to notice that this M-theory lift possesses non-holomorphic boundary conditions. The asymptotic behavior of our M-theory curve differs from the one of the M-theory lift of the supersymmetric configuration (in particular it does not preserve supersymmetry even asymptotically). As a result, it cannot be interpreted as a state with spontaneously broken supersymmetry in a supersymmetric 4d theory. Instead, it corresponds to a state in a theory with a Lagrangian that breaks supersymmetry explicitly. This issue was investigated in detail in [4].

Symplectic and orthogonal gauge groups

It is straightforward to carry out a similar discussion for the non-supersymmetric minima in the $SO(N_c)$ and $USp(N_c)$ theories with N_f massive flavors. The construction of the magnetic theories combines the same ingredients as for the $SU(N_c)$ theory, plus an additional O4-plane, stretching along the directions 01236. The O4-plane flips its charge as it crosses the NS and NS'-branes.
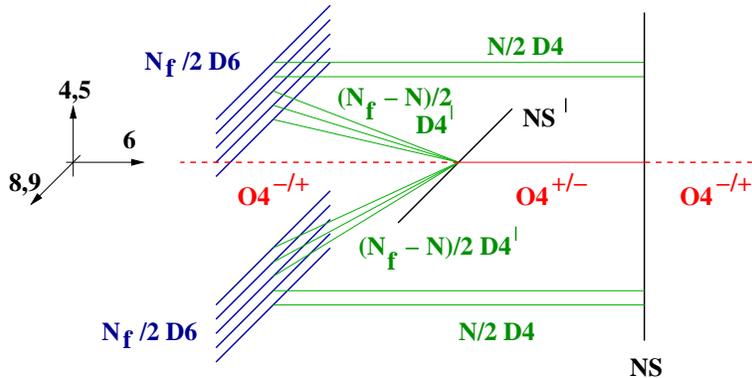

Figure 2: The type IIA brane configuration for the non-supersymmetric minimum of the $SO(N_c)$ and $USp(N_c)$ gauge theories with N_f massive flavors. For clarity we have shown the situation for different flavor masses. The red line corresponds to the O4-plane, and its change of charge as it crosses the NS- and NS'-branes is shown as a change between dashed and solid. For the $SO(N_c)$ theory we have $N = N_f - N_c + 4$, and it corresponds to choosing the O4-plane to have negative charge in the middle interval, and positive on the semi-infinite pieces. For the $USp(N_c)$ theory we have $N = N_f - N_c - 4$ and the O4-plane charge is positive in the middle interval.

References

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