A New Infinite Class of Supersymmetric AdS_3 Solutions

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Explict examples of the AdS/CFT correspondence, where detailed calculations can be performed, are very useful in developing our understanding of the correspondence. A good example is the discovery of an infinite class of Sasaki-Einstein manifolds in [1, 2], each of which gives rise to a supersymmetric solution of type IIB string theory with an AdS_5 factor. This discovery combined with the subsequent discovery of the corresponding N=1 super conformal field theories [3, 4] has led to much follow up work. This work is also of interest to geometers, since the $Y^{p,q}$ metrics provided the first explicit examples of non-homogeneous Sasaki-Einstein metrics and the very first in the so-called irregular class.

On the other hand, we expect that the vast majority of solutions of string/M-theory with AdS factors cannot be written down in explicit form and more sophisticated tools will be required to study them. It is therefore important, as a first step, to have a precise understanding of the underlying geometry behind the correspondence. This then provides the setting to address general questions such as topological criteria for existence of solutions, the geometric structure of moduli spaces of solutions, the geometrical manifestation of a-maximisation (see the talk of James Sparks) and so on.

Therefore, I have been pursuing a two-fold programme whose aims are:

1. to systematically characeterise or "classify" the geometries giving rise to supersymmetric solutions of D=10 and D=11 supergravity with AdS factors and hence are dual to supersymmetric conformal field theories.

2. to use this insight to construct additional explicit examples.

G-structures have proved to be very powerful tools for for classifying supersymmetric solutions (see [5] for a review), and in particular those with an AdS factor. This approach has also been very helpful in finding explicit supersymmetric solutions and in particular AdS solutions. The $Y^{p,q}$ Sasaki-Einstein manifolds were first discovered by systematically classifying the most general class of solutions of D=11 supergravity with an AdS_5 factor [1]. More precisely the metric is taken to be a warped product of AdS_5 with a compact six-manifold M_6. Amongst other things, the result includes the fact that M_6 has a canonical $SU(2)$ structure.

By simply assuming that M_6 is a complex manifold a rich set of explicit solutions were also found in [1]. Topologically, M_6 are all S^2 bundles over a four-dimensional base space B_4. Furthermore, B_4 could be (i) Kahler-Einstein with positive curvature and hence CP^2, $S^2 \times S^2$ or a del-Pezzo surface dP_k, $k = 3, \ldots 8$. Alternatively it could be a product of two two-dimensional Kahler-Einstein manifolds (with, in general, different curvatures): (ii) $H^2 \times S^2$ (we can also replace H^2 with a Riemann surface of genus greater than two: H^2/Γ) (iii) $S^2 \times S^2$ or (iv) $S^2 \times T^2$. After dimensional reduction and T-duality, the solutions with $B_4 = S^2 \times T^2$ give rise to the $AdS_5 \times Y^{p,q}$ solutions of type IIB supergravity [1, 2].

It is worth noting that the CFTs dual to the solutions in categories (i)-(iii) are still unknown. It is also interesting to note that the solutions in category (ii) with $B_4 = (H^2/\Gamma) \times S^2$ are a two parameter family of supergravity solutions, which includes a special solution, first found in [6], that describes M fivebranes wrapping a holomorphic Riemann surface inside of a Calabi-Yau three-fold.

Following these results, there is strong motivation to classify other supersymmetric AdS_n geometries of D=11 supergravity and of type IIB supergravity and then to attempt to find new explicit solutions. The classification of AdS_5 geometries of type IIB supergravity was carried out in [7]. As yet no new explicit solutions have been found. The classification of AdS_5 geometries with $N = 2$ supersymmetry, AdS_4 geometries with $N = 1$ and $N = 2$ supersymmetry, and AdS_3 geometries with $N = (0, 2)$ supersymmetry was carried out in [8]. A new method, still using G-structures, was developed to carry out these classifications and this is discussed in detail in [8]. Known solutions corresponding to fivebranes wrapped on supersymmetric cycles were recovered, but it is only in the latter case, AdS_3 geometries with $N = (0, 2)$ supersymmetry, where additional explicit solutions were found.

In fact the new infinite class of solutions in this latter class [9, 10] are very analogous to the general $N = 1$ AdS_5 solutions of D=11 supergravity of [1, 2] discussed above. The metric is a warped product of AdS_3 with an eight-manifold M_8. Topologically M_8 is an S^2 bundle over a six-dimensional base manifold B_6. It was found that B_6 can either be (i) $KE_4^+ \times H^2$ or $S^2 \times S^2 \times H^2$, (ii) $KE_4^- \times S^2$ or $H^2 \times H^2 \times S^2$ (iii) $KE_4^+ \times T^2$ or $S^2 \times S^2 \times T^2$. Here $KE_4^\pm$ refers to a four-dimensional Kahler-Einstein metric with positive or negative curvature, respectively. As above, we can also replace H^2 with a Riemann surface of genus greater than two: H^2/Γ. The previously known wrapped brane solution, corresponding to a fivebrane wrapping a Kahler four-cycle in a Calabi-Yau four-fold [11] lies in class (ii).

The solutions in class (iii) containing a T^2 factor can be dimensionally reduced and T-dualised to obtain an infinite new class of solutions of type IIB supergravity that are warped products of AdS_3 with a seven manifold M_7, and only non-vanishing five-form flux. Note that M_7 contains either a KE_4^+ or a $S^2 \times S^2$ factor. Demanding that the solutions are globally defined and that the five-form flux is properly quantised leads to the conclusion that the solutions are specified by three integeres p, q and N.

It would be very interesting to identify the conformal field theories dual to these new solutions. Since the solutions have only non-vanishing five-form flux, we expect that these conformal field theories arise from some configuration of wrapped or intersecting D3-branes. For example D3-branes wrapping two-cycles inside of Calabi-Yau four-folds preserve the same amount of supersymmetry.

We know that the two-dimensional CFTs have $(0, 2)$ supersymmetry and that the isometries of the KE_4^+ or $S^2 \times S^2$ factor give rise to global symmetries in the CFT. There will be additional global symmetries arising from the non-trivial cohomology of M_7. The central charge can also be calculated and we find

$$c = \frac{9pq^2(p + mq)}{3p^2 + 3mpq + m^2q^2} \frac{Mq}{h^2} N^2 \tag{1}$$

where m, M are determined by the choice of the KE_4^+ or $S^2 \times S^2$ factor. In particular for CP^2 we have $(m, M) = (3, 1)$, for $S^2 \times S^2$ we have $(m, M) = (2, 2)$ and for dP_k we have $(m, M) = (1, 9 - k)$. In addition h is the highest common factor of M and q. Finally, we note that since dP_k for $k = 5, 6, 7, 8$ have continuous moduli, so will the CFTs. The solutions with $S^2 \times S^2$ and CP^2 will admit exactly marginal β deformations and the corrresponding supergravity solutions can be constructed using the technique of [15].

The focus of [9, 10] was on solutions with compact M_8. We also briefly discussed some non-compact solutions that appear to correspond to certain defect CFTs. Recall that such defect CFTS were first noticed by considering probe branes with world-volumes containing AdS factors that are embedded in the background of some AdS solutions of string/M-theory [12, 13] (see also [14]). The non-compact solutions we have found can be interpreted as the back-reacted geometry of such probe branes.

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References

[1] J. P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, "Supersymmetric AdS(5) solutions of M-theory," Class. Quant. Grav. **21**, 4335 (2004) [arXiv:hep-th/0402153].

[2] J. P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, "Sasaki-Einstein metrics on S(2) x S(3)," Adv. Theor. Math. Phys. **8**, 711 (2004) [arXiv:hep-th/0403002].

[3] M. Bertolini, F. Bigazzi and A. L. Cotrone, "New checks and subtleties for AdS/CFT and a-maximization," JHEP **0412** (2004) 024 [arXiv:hep-th/0411249].

[4] S. Benvenuti, S. Franco, A. Hanany, D. Martelli and J. Sparks, "An infinite family of superconformal quiver gauge theories with Sasaki-Einstein duals," JHEP **0506** (2005) 064 [arXiv:hep-th/0411264].

[5] J. P. Gauntlett, "Classifying supergravity solutions," Fortsch. Phys. **53** (2005) 468 [arXiv:hep-th/0501229].

[6] J. M. Maldacena and C. Nunez, "Supergravity description of field theories on curved manifolds and a no go theorem," Int. J. Mod. Phys. A **16** (2001) 822 [arXiv:hep-th/0007018].

[7] J. P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, "Supersymmetric AdS(5) solutions of type IIB supergravity," Class. Quant. Grav. **23**, 4693 (2006) [arXiv:hep-th/0510125].

[8] J. P. Gauntlett, O. A. P. Mac Conamhna, T. Mateos and D. Waldram, "AdS spacetimes from wrapped M5 branes," arXiv:hep-th/0605146.

[9] J. P. Gauntlett, O. A. P. Mac Conamhna, T. Mateos and D. Waldram, "Supersymmetric AdS(3) solutions of type IIB supergravity," arXiv:hep-th/0606221.

[10] J. P. Gauntlett, O. A. P. Conamhna, T. Mateos and D. Waldram, "New supersymmetric AdS(3) solutions," arXiv:hep-th/0608055.

[11] J. P. Gauntlett, N. Kim and D. Waldram, "M-fivebranes wrapped on supersymmetric cycles," Phys. Rev. D **63** (2001) 126001 [arXiv:hep-th/0012195].

[12] C. Bachas and M. Petropoulos, "Anti-de-Sitter D-branes," JHEP **0102** (2001) 025 [arXiv:hep-th/0012234].

[13] A. Karch and L. Randall, "Open and closed string interpretation of SUSY CFT's on branes with boundaries," JHEP **0106** (2001) 063 [arXiv:hep-th/0105132].

[14] K. Skenderis and M. Taylor, "Branes in AdS and pp-wave spacetimes," JHEP **0206** (2002) 025 [arXiv:hep-th/0204054].

[15] O. Lunin and J. M. Maldacena, "Deforming field theories with U(1) x U(1) global symmetry and their gravity duals," JHEP **0505** (2005) 033 [arXiv:hep-th/0502086].