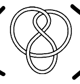
Simons Workshop in Mathematics and Physics 2006

Surface Operators in Gauge Theory and Categorification

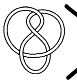
Sergei Gukov based on [hep-th/0412243, math.GT/0505662, hep-th/0512298] & joint work with E.Witten

- Line operators:

- in Chern-Simons theory

$\langle$  $\rangle = q^5 + q^{-5}$

- Surface operators in 4D gauge theory

$\langle$  $\rangle$ = vector space

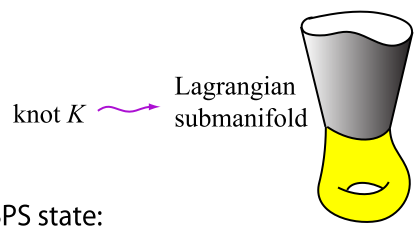
Physical Interpretation

$\mathcal{H} = \mathcal{H}_{BPS}$ space of BPS states

M-theory on $\mathbb{R}^5 \times$ (conifold)

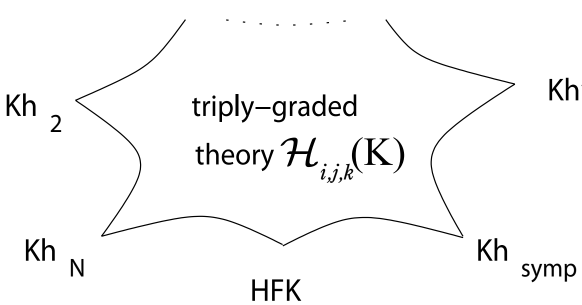
$\cup \qquad \cup$

M5-brane on $\mathbb{R}^3 \times$ Lagrangian



BPS state:
membrane ending on the Lagrangian five-brane

Unification of Knot Homologies



Open Questions and Further Directions

- generalization to other groups and representations
- the role of matrix factorizations
- finite N (stringy exclusion principle)
- realization in topological gauge theory

→ *- boundaries, corners, ...*
- surface operators
- braid group actions on D-branes

Categorification

Number $\underset{\text{dimension}}{\overset{\text{Categorification}}{\rightleftarrows}}$ Vector Space $\underset{\text{Grothendieck Group}}{\overset{\text{Categorification}}{\rightleftarrows}}$ Category

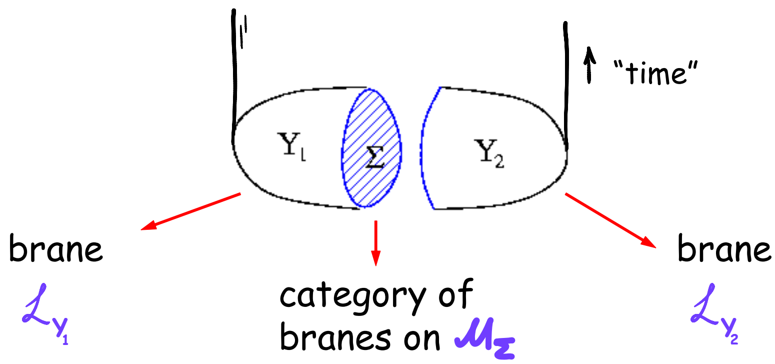
I.Frenkel

doubly graded homology theory, such that graded Euler characteristic = polynomial knot invariant

General Picture of Knot Polynomials and Knot Homologies

G	Knot Polynomial	Knot Homology
U(1|1)	Alexander	knot Floer homology **HFK(K)**
"SU(1)"	——	Lee's deformed theory **H′(K)**
SU(2)	Jones	Khovanov homology $H^{Kh}(K)$
SU(N)	$P_N(q)$	sl(N) homology $HKR^N(K)$

Gauge Theory and Categorification



⇨ A-model: $\mathcal{H}_Y = HF_*^{symp}(\mathcal{M}_\Sigma; \mathcal{L}_{Y_1}, \mathcal{L}_{Y_2})$

Topological Twists of SUSY Gauge Theory

- N=2 twisted gauge theory (Abelian monopoles):

  $\chi(\mathcal{H}) = \Delta(q)$ Alexander polynomial

- N=4 twisted super-Yang-Mills (adjoint non-Abelian monopoles):

  $\mathcal{H}^{i,j}$ doubly-graded knot homology

- Partial twist of 5D super-Yang-Mills:

  $\chi(\mathcal{H}) = Z_{\text{Vafa-Witten}}$ $\quad \mathcal{H} = H^*(\mathcal{M}_{\text{instanton}})$