Refining the Topological Vertex

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Introduction

This talk is about the continuing pursuit[1] for a *more refined* topological vertex. I will be more concrete with what I mean by "more refined" in the sequel. In this short summary I will go over the basic construction rather than talking about the technical details for which I will give references.

The topological vertex formalism turns out to be a very powerful way to compute topological string partition functions. If we consider the type IIA string theory on $\mathbf{R}^4 \times X$, where X is a Calabi-Yau 3-fold (CY3-fold), the effective theory on the transverse 4 dimensional subspace has $\mathcal{N} = 2$ supersymmetry with exactly calculable F-terms of the following type

$$\int d^4x F_g(\omega)\, R_+^2\, F_+^{2g-2}\,, \quad g \geq 1 \tag{1}$$

where R_+^2 is the contraction of the self-dual part of the Riemann tensor, and F_+ is the self-dual part of the graviphoton field strength. The function $F_g(\omega)$ which we want to compute is the A-model topological string amplitude of M. ω is the Kähler class of M and can written as a linear combination in the basis α_i of $H^{1,1}(X, \mathbf{Z})$. If we denote the coefficients in this expansion by t_i, then $F_g(t_i)$'s are holomorphic functions of t_i. The prepotential of the theory is given by the genus zero amplitude for which the F-term looks like

$$\int d^4x F_+^i \wedge F_+^j \partial_{t_i} \partial_{t_j} F_0 \tag{2}$$

[1] by Amer Iqbal, CK, Cumrun Vafa, to appear

$F_g(\omega)$'s have an important meaning from the geometric point of view, they are the generating functions of the "number of genus g maps into M". They can be written as

$$F_g(\omega) = P_g(\omega) + \sum_{\beta \in H_2(M,\mathbf{Z})} N_{g,\beta} e^{-\int_\beta \omega} \tag{3}$$

where P_0 and P_1 are cubic and linear polynomials in t_i, respectively, and they vanish identically for higher genus g. $N_{g,\beta}$ are the so-called Gromov-Witten invariants and they are rational numbers. The topological string amplitudes can be compactly organized into the generating function

$$F(t_i, \lambda_s) = \sum_{g=0}^{\infty} \lambda_s^{2g-2} F_g(t_i)\,, \tag{4}$$

where λ_s is the constant self-dual graviphoton field strength.

From the worldsheet point of view, the genus g amplitude, F_g, is the generating function of the "number" of maps from a genus g Riemann surface to CY3-fold X. However, the target space viewpoint provides a more physical interpretation of the generating function $F(t_i, \lambda_s)$, which we now review [1]. Recall that in M-theory compactification on CY3-fold X we get a 5-dimensional field theory with eight supercharges. The particles in this theory come from quantization of the wrapped M2-branes on various 2-cycles of X. If we consider compactifying one direction, then we can interpret the particles as wrapped D2-branes and the KK modes as bound D0-branes. In this case, integrating out these various charged particles gives rise to the F-terms in the effective action. The contribution of a particle of mass m and in representation $\mathcal{R}$ of the $SU(2)_L \times SU(2)_R$ (the little group of massive particles in 5D) to F is given by

$$S = \mathrm{logdet}(\Delta + m^2 + 2e\,\sigma_L \mathcal{F}) \;=\; \int_\epsilon^\infty \frac{ds}{s} \frac{\mathrm{Tr}_{\mathcal{R}}(-1)^{\sigma_L+\sigma_R} e^{-sm^2} e^{-2se\sigma_L\mathcal{F}}}{(2\sinh(se\mathcal{F}/2))^2}\,, \tag{5}$$

where σ^L is the Cartan of $SU(2)_L$ and arises because the graviphoton field strength is self-dual. e is the charge of the particle, and is equal to its mass and we identify the graviphoton field strength $\mathcal{F} = \lambda_s$. The mass of the particle is given by the area of the curve on which the D2-brane is wrapped. An extra subtlety arises due to D0-branes. In the lift to M-theory, we see that a wrapped M2-brane comes with momentum in the circle direction, and therefore, if we denote the mass of the M2-brane wrapping a curve class $\Sigma \in H_2(X,\mathbf{Z})$ by T_Σ then the mass of the M2-brane with momentum n is given by taking T_Σ to $T_\Sigma + 2\pi i n/\lambda$. Let us denote by $N_\Sigma^{(j_L,j_R)}$ the number of BPS states coming from an M2-brane wrapped on the holomorphic curve Σ, and the left-right spin content under $SU(2)_L \times SU(2)_R$ given by (j_L, j_R). Then the total contribution coming from all particles is obtained by summing over the momentum, the holomorphic curves and the left-right spin content,

$$\begin{aligned} F &= \sum_{\Sigma\in H_2(X,\mathbf{Z})}\sum_{n\in\mathbf{Z}}\sum_{j_L,j_R} N_\Sigma^{(j_L,j_R)}\int_\epsilon^\infty \frac{ds}{s}\frac{Tr_{(j_L,j_R)}(-1)^{\sigma_L+\sigma_R}e^{-sT_\Sigma-2\pi i n}e^{-2s\sigma_L\lambda_s}}{(2\sinh(s\lambda_s/2))^2}\,, \\ &= \sum_{\Sigma\in H_2(X,\mathbf{Z})}\sum_{k=1}^\infty\sum_{j_L,j_R} N_\Sigma^{(j_L,j_R)}e^{-kT_\Sigma}\frac{\mathrm{Tr}_{(j_L,j_R)}(-1)^{\sigma_L+\sigma_R}e^{-2k\lambda_s\sigma_L}}{k(2\sinh(k\lambda_s/2))^2}\,, \\ &= \sum_{\Sigma\in H_2(X,\mathbf{Z})}\sum_{k=1}^\infty\sum_{j_L} N_\Sigma^{j_L}e^{-kT_\Sigma}\frac{\mathrm{Tr}_{j_L}(-1)^{\sigma_L}e^{-2k\lambda_s\sigma_L}}{k(2\sinh(k\lambda_s/2))^2}\,, \end{aligned} \tag{6}$$

where

$$N_\Sigma^{j_L} = \sum_{j_R} N_\Sigma^{(j_L,j_R)}(-1)^{2j_R}(2j_R+1)\,. \tag{7}$$

It is useful to define a different basis of $SU(2)_L$ representations given by $I_g = (2(0)+(\frac{1}{2}))^g$ such that in terms of this basis

$$\sum_{j_L} N_\Sigma^{j_L}[j_L] = \sum_{g=0}^\infty n_\Sigma^g I_g\,. \tag{8}$$

The coefficients n_Σ^g are integers given by

$$\sum_{g=0}^\infty n_\Sigma^g(-1)^g(q^{1/2}-q^{-1/2})^{2g} = \sum_{j_L} N_\Sigma^{j_L}(q^{-j_L}+\cdots+q^{+j_L})\,. \tag{9}$$

In terms of these integers one can write F as

$$F = \sum_{\Sigma\in H_2(X,\mathbf{Z})}\sum_{k=1}^\infty\sum_{g=0}^\infty n_\Sigma^g e^{-kT_\Sigma}\frac{\mathrm{Tr}_{I_g}(-1)^{\sigma_L}e^{-2k\lambda_s\sigma_L}}{k(2\sinh(k\lambda_s/2))^2}\,, \tag{10}$$

It is easy to show that

$$\mathrm{Tr}_{I_g}(-1)^{\sigma_L}e^{-2k\lambda_s\sigma_L} = \left(\mathrm{Tr}_{I_1}(-1)^{\sigma_L}e^{-2k\lambda_s\sigma_L}\right)^g = (2\sinh(k\lambda_s/2))^{2g}\ . \tag{11}$$

Thus we get

$$F = \sum_{\Sigma\in H_2(X,\mathbf{Z})}\sum_{k=1}^\infty\sum_{g=0}^\infty \frac{n_\Sigma^g}{k}(2\sinh(k\lambda_s/2))^{2g-2}e^{-kT_\Sigma}\,. \tag{12}$$

The target space point of view allows the topological string amplitudes to be written in terms of integers n_Σ^g which give the BPS degeneracies of the states coming from wrapped D2-branes. The fact that F has this particular form with integer n_Σ^g has been confirmed for many non-compact toric threefolds.

Non-compact toric threefolds and generalized partition functions

In this section we consider the case of non-compact toric CY3-folds. These CY3-folds are extremely interesting not only because they are "simple" enough so that exact calculation of the A-model partition function can be done, but also because they give rise to gauge theories via geometric engineering [2].

From the discussion of the last section, we see that if the graviphoton field strength is not self-dual, $F := F_+ + F_-$, then we can write the contribution that comes from integrating out the particle in representation $\mathcal{R}$ of $SU(2)_L \times SU(2)_R$ as

$$S := \int_\epsilon^\infty \frac{ds}{s} \frac{\mathrm{Tr}_{\mathcal{R}}(-1)^{\sigma_L+\sigma_R} e^{-sm^2} e^{-2se(\sigma_L F_+ + \sigma_R F_-)}}{(2\sinh(seF_+/2))(2\sinh(seF_-/2))}\,. \tag{13}$$

Summing over the contributions from all particles as before we get

$$F(\omega, t, q) = \tag{14}$$

$$\sum_{\Sigma\in H_2(X,\mathbf{Z})} \sum_{n=1}^{\infty} \sum_{j_L, j_R} \frac{(-1)^{2j_L+2j_R} N_\Sigma^{(j_L,j_R)} \left((t\,q)^{-nj_L} + \cdots + (t\,q)^{nj_L}\right)\left((\frac{t}{q})^{-nj_R} + \cdots + (\frac{t}{q})^{nj_R}\right)}{n(t^{n/2} - t^{-n/2})(q^{n/2} - q^{-n/2})}\, e^{-nT_\Sigma}, \tag{15}$$

where we have defined $t = e^{F_+}, q = e^{F_-}$. The integers $N_\Sigma^{(j_L,j_R)}$ give the degeneracy of particles with spin content (j_L, j_R) and charge Σ and are the number of cohomology classes with spin (j_L, j_R) of the moduli space of a D-brane wrapped on a holomorphic curve in the class Σ [1]. Because the D-brane has a $U(1)$ gauge field living on its worldvolume, the moduli space of supersymmetric configurations includes not only the curve moduli but also the moduli of the flat connections on the curve coming from the gauge field. Since the moduli space of flat connections on a smooth curve of genus g is T^{2g}, the moduli space of the D-brane is a T^{2g} fibration over the moduli space of the curve. The total space is a Kähler manifold and the Lefshetz action by the Kähler class is the diagonal $SU(2)_D \subset SU(2)_L \times SU(2)_R$ action on the moduli space. The $SU(2)_L \times SU(2)_R$ action on the moduli space is such that $SU(2)_L$ acts on the fiber direction and the $SU(2)_R$ acts in the base direction.

In the previous section, when discussing generic CY3-folds, we summed over the $SU(2)_R$ action by taking the graviphoton field strength to be self-dual. This was essentially due to the fact that $N_\Sigma^{(j_L,j_R)}$ can change as we change the complex structure; the supersymmetry algebra allows such pairings between neighboring j_R's to give a non-reduced multiplet. But $N_\Sigma^{j_L}$, which sums over all j_R's with alternating signs, remains invariant. For the case of non-compact toric CY3-folds, there are no complex structure deformations. Therefore, one would expect no jumps in the $N_\Sigma^{(j_L,j_R)}$ degeneracies, and so one would hope to be able to compute these as well. We will come back to this after our discussion of the topological vertex.

Topological Vertex

In this section, I want to briefly review the basics of the topological vertex[3, 4] and its connection to $3D$ partitions. The amplitude in the topological A-model is the sum over the holomorphic maps from the genus g Riemann surface Σ_g to a target CY (weighted by the area of the surface). If the target space is a non-compact toric CY, the geometric information can be encoded in so-called toric diagrams. These are tri-valent graphs on the plane showing the degeneration loci of the toric fibration. The topological vertex is based on dividing the geometry into basic building blocks, computing the amplitude for these blocks seperately and gluing them with an appropriate algorithm to get the amplitude for the desired toric CY. Each trivalent vertex corresponds to a $\mathbf{C}^3$ patch. The division will be realized by making use of a stack of brane/anti-brane pairs along the compact degeneration loci. The maps from the Riemann surface Σ_g are cut as well, so they have boundaries on the branes and are wrapping around the $\mathbb{P}^1$'s along each edge of the vertex. The windings on the boundary can be translated to representation theory language using the Frobenius formula, and we end up with vertices depending on three representations of $U(\infty)$, which we denote by $C_{\lambda\mu\nu}$. This is the crucial ingredient to relate the topological vertex to the $3D$ partitions.

After "chopping" a general toric diagram, obviously, we do not get a number of identical vertices. However, all possible vertices are related to each other by $SL(2,\mathbf{Z})$ transformation. Once we know how to compute the amplitude for a given one, we know all others. Another interesting property of the topological vertex is its cyclic symmetry with respect to the representations along each edge.

$3D$ **Partitions**

A fascinating duality has been proposed between a classical model of crystal melting and the topological vertex[5]. This duality allowed a combinatorical interpretation of the topological vertex. The model for the melting crystal is actually quiete trivial. Imagine that we are stacking small boxes in the postive octant O^+ of $\mathbf{R}^3$ (with coordinates (x_1, x_2, x_3)) and label the boxes by the lattice points. Then melting basically corresponds to removing these small boxes. The only rule we impose on removing boxes is that a box is allowed to be removed only if it does not have any pair of boxes on opposite sides. Removal of each box contributes the factor $q = e^{-\mu/T}$ to the Boltzman weight of the configuration, μ being the chemical potential. If we start with a configuration of $N \times N \times N$ boxes then there is only one box we are allowed to move, the one labeled by (N, N, N). For the second box we have three candidates, and so on. The partition function for this model in $3D$ is defined as

$$Z = \sum_{3D\,\text{partitions}} q^{\#\,\text{of boxes}} = \sum_{n=0}^{\infty} (\#\,\text{of partitions of}\, n\, \text{boxes})\, q^n \qquad (16)$$

The above partition function can actually be computed and is equal to McMahon function:

$$Z_{3D} = \prod_{n=1}^{\infty} \frac{1}{(1-q^n)^n} \qquad (17)$$

The partition functions for the lower dimensional crystals are also known, Z_{1D} is particularly easy to understand since there is only one partition for any given number of boxes:

$$Z_{1D} = \frac{1}{1-q} \tag{18}$$

$$Z_{2D} = \prod_{n=1}^{\infty} \frac{1}{1-q^n} \tag{19}$$

Let me explain what the relationship is between our classical model of crystal melting and the topological vertex $C_{\lambda\mu\nu}$. I will ignore details about framing factors. Consider first a finite dimensional box of size $N_1 \times N_2 \times N_3$[2] where we place our $3D$ partition. As I pointed out earlier the topological vertex can be labeled with the three representations along each degeneration locus. We focus on the generating function for $3D$ partitions which asymptotically approaches the $2D$ partitions λ, μ and ν when we slice along each direction with the planes $x_i = N_i, i = 1, 2, 3$. Following the notation of [5] $P_{N_1,N_2,N_3}(\lambda, \mu, \nu)$ denotes the generating function of the $3D$ partition in the finite sized box with the given asymptotes and we are interested in the infinite limit

$$P(\lambda, \mu, \nu) = \lim_{N_1,N_2,N_3 \to \infty} q^{-N_1|\lambda| - N_2|\mu| - N_3|\nu|} P_{N_1,N_2,N_3}(\lambda, \mu, \nu) \tag{20}$$

where $|\mu|$ is the number of boxes in μ. The duality tells us, up to framing factors, this formal expansion in q is the topological vertex itself. The expansion in q is very easy to understand: the generating function is weighted by $q^{\#\,\text{of boxes}}$. Assume all the boxes behind the asymptotic $2D$ partition in all three directions are removed, and we completely ignore them, that is why we have the factor in the limit. Start the expansion with 1, then the coefficient of q is equal to the number of places we can put a box, then the coefficient in front of q^2 is the number of distinct ways we can put two boxes, and so on, subject to our rule of melting. I will give an equivalent condition which would be easier to check. As an example, imagine all representations being fundamental, then we have 3 diffrent ways for the first box, 9 different ways to put two boxes etc.

A nice representation of $3D$ partitions is in terms of an array of non-negative integers $\pi_{i,j}$ where $\pi_{i,j}$'s are the number of boxes at the (i, j) position and they are subject to the following condition to ensure our rule of removing boxes:

$$\pi_{i,j} \geq \pi_{i+r,j+s} \quad r, s \geq 0 \tag{21}$$

Our model for crystal melting assumes the same contribution for all boxes in the crystal. The generalization of this model has been worked out by mathematicians [6]. We need to go over diagonal slicing to understand the generalization better. One way related to this slicing is the transfer matrix approach. $3D$ partitions can be sliced diagonally by planes which are perpendicular

[2]All N's are assumed to be large, and we should keep in mind that we will be looking the limit when all of them go to infinity

to the base and whose projection onto the base are given by a set of equations parametrized by $a \in \mathbf{Z} : x_2 - x_1 = a$, Fig.1. We end up with a number of $2D$ partitions or Young diagrams $\eta(a)$ which satisfy the *interlacing condition*: in general, let μ and ν be two distinct $2D$ patitions. We say μ and ν interlace, denoted by $\mu \succ \nu$, if

$$\mu_1 \geq \nu_1 \geq \mu_2 \geq \nu_2 \geq \ldots \tag{22}$$

where μ_i stands for the number of boxes on the ith column of the $2D$ partition. Our model requires the following interlacing conditions for the diagonal slices

$$\eta(a+1) \succ \eta(a), \quad a < 0, \tag{23}$$
$$\eta(a) \succ \eta(a+1), \quad a \geq 0. \tag{24}$$

Since I do not want to talk about the technical details I will only introduce creation and annihilation operators via their action on a $2D$ partition and refer the reader for their more complete introduction and treatment to [5, 6]:

$$\prod_i \Gamma_-(x_i)|\lambda\rangle = \sum_\mu s_{\lambda/\mu}(x_1, x_2, \ldots)|\mu\rangle, \tag{25}$$
$$\prod_i \Gamma_+(x_i)|\lambda\rangle = \sum_\mu s_{\mu/\lambda}(x_1, x_2, \ldots)|\mu\rangle. \tag{26}$$

where $s_{\lambda/\mu}(x_1, x_2, \ldots)$ are the skew Schur functions [7]. These operators satisfy the commutation relation

$$\Gamma_+(x)\Gamma_-(y) = (1 - xy)\Gamma_-(y)\Gamma_+(x). \tag{27}$$

The relevance to creation and annihilation operators becomes more transparent once we realize that

$$s_{\lambda/\mu}(1) = \begin{cases} 1, & \text{if}\, \lambda \succ \mu \\ 0, & \text{otherwise.} \end{cases} \tag{28}$$

Then their more general action reduces to

$$\Gamma_-(1)|\lambda\rangle = \sum_{\lambda \succ \mu} |\mu\rangle, \tag{29}$$
$$\Gamma_+(1)|\lambda\rangle = \sum_{\mu \succ \lambda} |\mu\rangle. \tag{30}$$

The $3D$ generating function can be expressed now in terms of these operators as

$$Z_{3D}(q) = \left\langle \left(\prod_{t=0}^{\infty} q^{L_0} \Gamma_+(1) \right) q^{L_0} \left(\prod_{t=-\infty}^{-1} \Gamma_-(1) q^{L_0} \right) \right\rangle \tag{31}$$

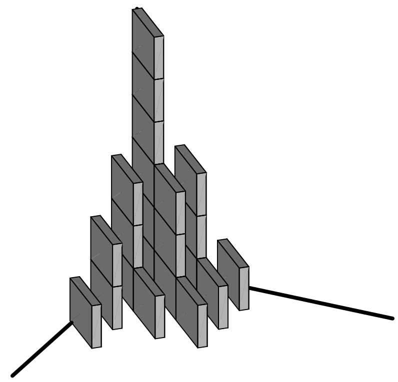

Figure 1: Diagonal slicing

with $q^{L_0}|\mu\rangle = q^{|\mu|}|\mu\rangle$.

The intuitive way of understanding the form of the partition function in terms of the creation and annihilation operators is straightforward: we start with the slice at $a = -\infty$ with the empty set and act on this slice with $\Gamma_-(1)$ to create all possible partitions as a sum. On the next slice (we go from $a = -\infty$ to 0), we apply the creation operator on this sum, we again create all possible partitions such that they interlace the partitions in the previous slice. We keep acting with $\Gamma_-(1)$, until we hit the main slice $a = 0$. The main slice is where we start applying the annihilation operator $\Gamma_+(1)$ which destroys the previously created partitions, essentially by "creating" $2D$ partitions on the slice a that are interlaced by the partitions on the previous slice $a-1$, for positive a's. This procedure, with the operators q^{L_0}'s, gives the sum of $q^{|\pi|}$ over all possible $3D$ partitions satisfying the interlacing condition that we stated before. Note that Γ_+ acting on the vacuum gives zero, so we can move the Γ_+'s to the right to act on the vacuum, and use the commutation relations between $\Gamma_\pm$'s each time we pass them through each other.

The generalization of the generating function in [6] allows us to assign a different weight to each diagonal slice $\eta(a)$. The partition function is defined as

$$Z_{3D}(\mathbf{q}) = \sum_\pi \prod_a q_a^{|\eta(a)|}. \tag{32}$$

Note this definition reduces to our previous definition in case all weights are set to be equal to each other. This partition function has a very nice representation in terms of the creation and annihilation operators we have introduced[3]

$$Z_{3D}(\mathbf{q}) \quad = \quad \left\langle \prod_{k=-\infty}^{0} \Gamma_-(x^+_{-k+\frac{1}{2}}) \prod_{k=1}^{\infty} \Gamma_+(x^-_{-k+\frac{1}{2}}) \right\rangle \tag{33}$$

[3]Becuase of limited space, I again have to refer the reader for the details in the actual computation as well as for the notation to the original work, also to appear in our paper.

$$= \prod_{k_1=1}^{\infty}\prod_{k_2=1}^{\infty}(1 - x^+_{k_1-\frac{1}{2}} x^-_{-k_2+\frac{1}{2}}). \tag{34}$$

with $x^+_{k-\frac{1}{2}} = \prod_{i=0}^{k-1} q_i$ and $x^-_{-k+\frac{1}{2}} = \prod_{i=1}^{k-1} q_{-i}$ for $k \geq 1$. Here $x^{\pm}$ is related to the different weights we assign to the slices.

The natural question is whether the generalization of $3D$ partition allows us to contruct a new topological vertex which can be used to compute the partition function when the graviphoton field is not self-dual and reduces to the usual topological vertex when we impose the self-duality. The answer turns out to be 'yes'. Note that the generalized $3D$ partitions have infinitely many parameters associated with each diagonal slice, but the non-self-duality of graviphoton field introduces only two distinct parameters. Hence, we had to find the correct way to assign one of these two parameters to each slice. It became clear that the assignment can be motivated physically and it depends non-trivially on the representation along the preferred direction[4]. So the cyclic symmetry of the vertex is not preserved[5]. Once the vertex is constructed, the gluing algorithm for this new vertex should be found as well, especially for a vertex with a preferred direction. For our algorithm, we were able to show that for different geometries, the partition functions obtained by gluing along the preferred direction or along one of the non-preferred ones are only different representations of the same function. In some cases, known identities can be used, whereas in more complicated cases, for which no such identities were present as far as we know, expanding formally both expressions up to a certain order supplied strong evidence. A very non-trivial check is performed for many different geometries with success: we computed the left-right spin content of the BPS states for certain curves. In many cases, there are more than one state for a fixed left-spin with different right-spins. The spin content can be written in an appropriate basis to read off the Gopakumar-Vafa invariants, and we were able to cover the invariants in the literature.

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References

[1] R. Gopakumar and C. Vafa, "M-theory and topological strings. I," arXiv:hep-th/9809187. R. Gopakumar and C. Vafa, "M-theory and topological strings. II," arXiv:hep-th/9812127.

[2] S. Katz, A. Klemm and C. Vafa, "M-theory, topological strings and spinning black holes," Adv. Theor. Math. Phys. **3**, 1445 (1999) arXiv:hep-th/9910181.

[4] The way we slice the $3D$ partition basically selects a preferred direction

[5] In our paper, this point will be investigated in more detail

[3] A. Iqbal, “All genus topological string amplitudes and 5-brane webs as Feynman diagrams,” arXiv:hep-th/0207114.

[4] M. Aganagic, A. Klemm, M. Marino and C. Vafa, “The topological vertex,” Commun. Math. Phys. **254**, 425 (2005) arXiv:hep-th/0305132.

[5] A. Okounkov, N. Reshetikhin and C. Vafa, “Quantum Calabi-Yau and classical crystals,” arXiv:hep-th/0309208.

[6] A. Okounkov and N. Reshetikhin, “Random skew plane partitions and the Pearcey process,” arXiv:math.CO/0503508.

[7] I. G. Macdonald, “Symmetric functions and Hall polynomials,” (second edition, 1995), Oxford Mathematical Monographs, Oxford Science Publications.