Inflation and Cosmic Strings in Heterotic M-theory

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Introduction

For many years, string theory was considered to be inaccessible to experimental observations. The energy scale at which stringy effects may become observable is beyond of what we expect to measure in current experiments. It is hoped that in the years to come new experimental data coming from LHC and various new astronomical experiments might change this. In particular, we believe that the observations of cosmic superstrings might provide the long sought experimental window into string theory. If such strings exist they would have implications for the cosmic microwave background anisotropies, which could be measured in the near future. Cosmic string are generically produced at the end of inflation.

Over the past years, cosmology has had some great successes. The inflationary paradigm and the Standard Big-Bang cosmology (SSB) have been precisely verified by data coming from different sources including WMAP. These experiments have led to a coherent but still puzzling picture of our universe and we are left with the goal of going from answering how to answering why. In particular we want to embed the inflationary paradigm into a fundamental theory, such as string theory.

A model of inflation is based on one or more scalar fields called *inflatons* and different models for inflation differ in the form of the inflaton potential. String theory provides a multitude of scalar fields that can serve as potential inflatons, including the moduli fields that describe the deformations of the size and shape of the internal manifold. In conventional Calabi–Yau compactifications these scalars are massless, so that the moduli fields cannot serve as inflatons. However, the situation is different for flux compactifications of string theory, where generically a potential for (at least some) of these fields is generated in terms of fluxes. This potential stabilizes the moduli and a large number of consistent string theory vacua, called the *landscape* has emerged. One of the goals of string cosmology is to find among this multitude of vacua one which gives rise to inflation and which describes our four-dimensional universe. The broader perspective adopted in the *swampland program* [10] (which analysis the generic features of the moduli space of the string theory landscape) puts constraints on the type of inflationary models that can be derived from string theory. We might hope that among the hundreds of different models of inflation invented to date by cosmologists, only a limited number can be consistently implemented into string theory.

Several different models for inflation have been devised in the context of string theory. The most popular and most studied model is based on an embedding of hybrid inflation into type IIB string theory and it is known as D3-$\bar{\text{D}}$3 brane inflation [1, 2]. In this lecture we will study a different model [3] that describes the embedding of a multiple scalar field model known as assisted inflation [9] into heterotic M-theory. In this model heterotic cosmic strings are produced at the end of the inflationary era [8]. As we shall see, *heterotic* cosmic strings have some rather attractive features, that are not shared by their type II counterparts.

Heterotic cosmic strings: the early days

In 1985, Witten [4] investigated whether the heterotic string at weak coupling could exist as a cosmic string. Besides the fact, that the heterotic string is unstable at weak coupling due to the formation of an axion domain wall, Witten found that the tension is too high and was thus ruled out by experiments. In 1985 the strongest bounds on the cosmic string tension came from the galaxy surveys and COBE but in this lecture we will mostly quote the bounds from the first year of data from WMAP.

Strings of cosmic size crossing our observable universe can, quite predictably, have important observational consequences. Among them, the conical geometry associated with a cosmic string can lead to gravitational lensing, a double image in the sky where the deflection angle of the light ray is given by $\Delta \sim 2\frac{G_N}{\alpha'}$, where G_N is the four-dimensional Newton's constant and α' is related to the string tension as usual $T = \frac{1}{2\pi\alpha'}$. Therefore, measuring the deflection angle would allow for a direct measurement of the string tension.

A cosmic string would also create anisotropies in the CMB with a very specific power spectrum. The recent data from WMAP strongly favors inflation as the main source of density perturbations in the universe but cosmic strings could still contribute to about 10% of the total power in the CMB. The most recent bound from WMAP is $G_N\mu \leq 2 \times 10^{-7}$ where μ is the cosmic string tension and G_N is Newton's constant. This leads to

$$\sqrt{\mu} \leq 5.5 \times 10^{15}, \tag{1}$$

GeV which is roughly the GUT scale.

In the weakly coupled heterotic string, the string tension is set by the string scale $\sqrt{\mu} \sim M_s$, which is fixed to be at 10^{18} GeV. This is ruled out by experimental data by roughly 3 orders of magnitude and so the idea of cosmic superstrings was abandoned early on. The whole subject was revived [5,6] in the context of flux compactifications of the type IIB theory [11], where a hierarchy of scales is generated by the warp factor. In the type IIB context, the string scale can then be lowered as low as the TeV scale, so that the cosmic string tension is within the regime allowed by cosmological data. The cosmic string was fully embedded into flux compactifications of the type IIB theory in [7], where it was shown that the cosmic string tension can range between the TeV scale (10^{12} eV) and the GUT scale (10^{25} eV). Even though this is good news, as the string tension is in the range allowed by cosmic data, it has the drawback of not being very predictive, due to the large range of allowed string tensions. On the other hand, cosmic strings in heterotic M-theory are both below the current experimental bound and yet have a tension that is precisely fixed to be at the GUT scale, leading to a sharp prediction for their experimental observation [8]. Before describing the heterotic M-theory inflationary scenario and the heterotic cosmic string, let me review some basics on cosmology, which will be useful later on.

Basics on Cosmology

At the very early stage of our universe, space-time was singular and the energy density was infinite. This era is known as the *Big Bang* era which is described within the *Standard Big Bang model* of cosmology. It is expected that string theory smoothes out the singularity so that there is a cosmology before the Big Bang.

The most basic assumption used in cosmology is that our universe is isotropic and homogeneous on large scales, so that it can be described as a *perfect fluid*. A perfect fluid has an energy-momentum tensor that is a smoothly varying function of the position and is isotropic in the local rest frame

$$T_{00} = \rho, \qquad T_{ij} = p g_{ij}. \tag{2}$$

This tensor is characterized by three quantities: the mass-energy density ρ, the pressure p and the spatial components of the metric g_{ij}. The mass-energy density ρ and pressure p are related by the *equation of state*

$$p = w\rho \, , \tag{3}$$

where w is a constant that depends on whether the Universe is dominated by relativistic particles (termed radiation) $w = 1/3$, non-relativistic particles (collectively called matter) with $w = 0$ or vacuum energy $w = -1$.

The homogeneity and isotropy of the Universe, together with the observed flatness, fixes the metric to be of the *Friedman-Robertson-Walker* (FRW) form on large scales

$$ds^2 = -dt^2 + a^2(t)[dx^2 + dy^2 + dz^2], \tag{4}$$

where $a(t)$ is the scale factor that depends only on time. Further k is a constant which takes values $k = -1$ for a closed (finite) universe, $k = 0$ (flat universe) and $k = 1$ for an open (infinite) universe.

The evolution of the scale factor is dictated by Einstein's equation

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}, \tag{5}$$

and thus by the dynamics of the theory. Here a cosmological constant Λ has been added to the Einstein equation, as astronomical data indicate that our universe has a small and positive cosmological constant $\Lambda = 10^{-120} \mathrm{M_p}^4 = (10^{-3}\mathrm{eV})^4$.

Inserting the FRW metric into this equation and using the perfect fluid description leads to the Friedmann and acceleration equations respectively

$$H^2 = \frac{1}{3M_p^2}\rho - \frac{k}{a^2} + \frac{\Lambda}{3} \ , \tag{6}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_p^2}(\rho + 3p) + \frac{\Lambda}{3} \ , \tag{7}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter and $M_p = \frac{1}{8\pi G_N}$ is the reduced Planck mass. The cosmological constant appearing above can be interpreted as the density and pressure of the vacuum $\rho = -p = M_p^2 \Lambda$, so that it sometimes does not show up explicitly. The evolution of the scale factor is determined by these equations. If the universe is dominated by radiation, it will expand and the density of radiative matter is diluted by a^{-4} with the scale factor growing in time like $t^{\frac{1}{2}}$. If the universe is dominated by non-relativistic matter then the universe is also expanding but the density is diluted by a^{-3} with the scale factor growing in time like $t^{2/3}$. For both of these cases the universe is expanding but decelerating $\ddot{a} < 0$.

Inflation is defined as an accelerated expansion $\ddot{a} > 0$. From Eq. (7), we see that this is only possible for $p < -\frac{1}{3}\rho$. Since the density is assumed to be positive, it follows that inflation is driven by a material with negative pressure. Depending on the actual equation of state, the accelerated expansion can be exponential $a \sim e^t$ or a power law $a \sim t^n$ with $n > 1$, for example.

The simplest model of inflation is formulated in terms of one dynamical field and is called slow-roll inflation. In this model, we imagine that we have a homogeneous real scalar field with the following action

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \partial_\nu \phi \partial_\mu \phi + V(\phi). \tag{8}$$

Such a scalar field behaves like a perfect fluid with density and pressure given by

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \ , \tag{9}$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{10}$$

where we have suppressed the space derivatives since they are suppressed by factors of a^{-2} as compared to time derivatives. We see here that if the kinetic energy of the inflaton is negligible compared to the potential energy, we can neglect the $\dot{\phi}^2$ term and we obtain a universe dominated by a perfect fluid with $\rho = -p = V(\phi)$. Such a universe would inflate until the *slow-roll condition* ($\dot{\phi}^2 \ll V(\phi)$) is violated and at that point it should reheat. The Friedmann equation together with equation of motion for ϕ are the following

$$H^2 = \frac{1}{3M_p^2}(V(\phi) + \frac{1}{2}\dot{\phi}^2) , \qquad H^2 \approx \frac{1}{3M_p^2}V(\phi) , \tag{11}$$

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi) , \qquad 3H\dot{\phi} \approx -V'(\phi) . \tag{12}$$

The second equation looks like the equation of motion for a harmonic oscillator with the friction given by the Hubble parameter. Different models for inflation differ in the form of the potential $V(\phi)$. The slow-roll approximation is justified as long as the *slow-roll parameters* are small

$$\epsilon(\phi) = \frac{1}{2}M_p^2\left(\frac{V'}{V}\right)^2 \ll 1 , \qquad \eta(\phi) = M_p^2\left|\frac{V''}{V}\right| \ll 1 , \tag{13}$$

where the prime indicates derivative wrt the inflaton field. These are sufficient but not necessary conditions for inflation, that can be easily checked for particular models. The equations (11) and (12) can be solved exactly in the case of an exponential potential $V(\phi) = V_0 e^{-\sqrt{\frac{2}{p}}\frac{\phi}{M_p}}$ where p is a constant not to be confused with the pressure used before. This type of potential describes *power law inflation*. The exact solution is

$$a(t) = a_0 t^p , \qquad \phi(t) = \sqrt{2p}M_p \log\left(\sqrt{\frac{V_0}{3(3p-1)}}\frac{t}{M_p}\right) , \tag{14}$$

and for $p > 1$ it corresponds to an inflating universe. Of course it turns out that unlike the period of inflation we are going through today (which is a very slow exponential inflation), the inflationary period in the early universe was very fast and therefore we expect p to be significantly larger than 1. This might be a fine tuning in some models or one could alternatively achieve a large 'p' by having many scalar fields all with the same potential. An example of such a multi scalar model is called *assisted inflation* [9] in the cosmology literature

$$U = U_0 e^{-\sqrt{\frac{2}{p}}\frac{\varphi_i}{M_p}} , \qquad i = 1, \cdots , N . \tag{15}$$

The equations of motion have a late time attractor where all the fields are the same $\varphi_1 = \varphi_2 = \cdots \varphi_N$. This multiple field model can be mapped to a single field model that looks just like power law inflation but with a modified p

$$a(t) = a_0 t^{p(N)} , \qquad p(N) = Np. \tag{16}$$

With the extra new parameter N, it is possible to achieve inflation, even though the single exponentials were too steep to generate slow roll inflation by themselves. This solution turns out to be important in the heterotic models, as we will see next.

Heterotic M-theory and assisted inflation

Consider heterotic M-theory (also known as strongly coupled $E_8 \times E_8$ heterotic), which can be obtained by compactifying 11d SUGRA on an interval $S^1/\mathbb{Z}_2$. To cancel the anomaly one must introduce on each boundary a *twisted sector* corresponding to each E_8. The heterotic theory at weak coupling constant previously discussed is obtained by taking the length of the interval to be very small.

Scalar fields with an exponential potential (of the form appearing in assisted inflation) naturally arise from the dynamics of N M5-branes in heterotic M-theory (see Fig. (1)). In this set-up super-

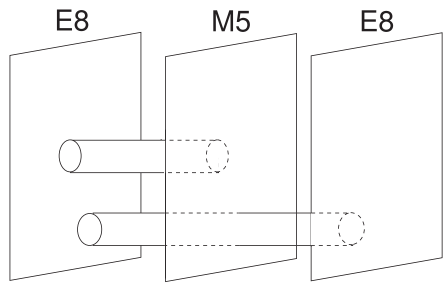

Figure 1: Different M5 branes interact with each other as well as with the boundary through open membrane instantons.

symmetry is spontaneously broken by a superpotential that arises from open membrane instantons that stretch between different M5 branes (called W_{55} below), an M5 brane and the boundary (W_{59} and W_{95}) or between both boundaries, which one can think of as M9-branes (W_{99}) [12]. There are four contributions to the superpotential coming from these interactions

$$W = W_{99} + W_{95} + W_{59} + W_{55}. \tag{17}$$

Suppose that at the beginning of inflation the M5-branes are grouped on a common site, while the boundary M9-branes are far apart. We can then neglect all the interactions with the boundaries and only consider the interaction W_{55}. If one only considers the nearest neighbor (repulsive) interaction (which is justified based on the attractor solution mentioned previously), we get

$$W = W_{55} = h \sum_{i<j} e^{-y_{ij}} \ , \qquad\qquad y_{ij} = y_i - y_j \ , \tag{18}$$

where the y_i is the position on the interval of the i^{th} M5-brane. The scalar potential resulting from this superpotential can be mapped to power law inflation [3] with an exponent

$$p = \frac{4N(N^2-1)}{3st}, \tag{19}$$

where s and t describe the size of the Calabi–Yau and the open membrane instanton respectively, which are assumed to be frozen during the inflationary period. We observe that by choosing N large enough, it is possible to obtain slow-roll inflation. One can get enough e-folds and the correct running of the spectral index for $19 \leq N \leq 195$.

Inflation ends when neighboring M5-branes are separated by a distance of about the orbifold length. At that point the boundaries start moving until the interval reaches the critical length, where the gauge theory on the hidden boundary becomes strongly coupled. Gaugino condensation on the hidden boundary then stabilizes the size of the interval. During inflation, the M5-branes dissolve into the boundaries through a *small instanton transition*, which will lead to reheating and the production of cosmic strings.

Heterotic cosmic strings revisited

Heterotic M-theory contains M2- and M5-branes. Strings of cosmic size are stable if they are either closed or infinitely extended. Consider heterotic M-theory compactified on a Calabi–Yau 3-fold. Due to the Bianchi identify

$$(dG)_{11IJKL} = -\delta(x^{11})(\kappa)^{2/3}(\mathrm{tr}F \wedge F - \frac{1}{2}\mathrm{tr}R \wedge R) \neq 0, \tag{20}$$

the flux component $G_{2,2,0}$ (which is a $(2,2)$ form on the 3-fold) needs to be taken into account. The 11D space-time geometry is then a warped geometry of exactly the same form as for type IIB theories

$$ds_{11}^2 = e^{-\Delta(x_{11})} g_{\mu\nu} dx^\mu dx^\nu + e^{\Delta(x_{11})}(g_{mn} dy^m dy^n + (dx^{11})^2). \tag{21}$$

The warp factor is expressed in terms of the charge of the visible M9-brane

$$e^{\Delta(x_{11})} = (1 - x_{11} Q_v)^{2/3}, \tag{22}$$

with

$$Q_v = \frac{\kappa_{11}^{2/3}}{V_v} \int_{X_v} J \wedge (\mathrm{tr}F \wedge F - \frac{1}{2}\mathrm{tr}R \wedge R). \tag{23}$$

Here J is the Kähler form , F and R are the Yang-Mills and curvature 2-forms respectively and X_v, and V_v denote the Calabi–Yau and it's volume at the location of the visible M9-brane. Similarly as in the type IIB theory, the presence of the warp factor in Eq. (21) warranties that the tension of the cosmic string can be lowered to the observable regime. It turns out, however, that in the strongly coupled heterotic theory the fundamental scale is the GUT scale $M_{GUT} = 3 \times 10^{16}$ GeV (precisely at the upper bound on the constraint imposed by WMAP data). This strongly constrains the possible cosmic string candidates emerging from heterotic M-theory.

There are different configurations of M2- and M5-branes leading to strings in the four-dimensional external space-time: the M5-brane can either wrap a Calabi–Yau 4-cycle or a Calabi–Yau 3-cycle times the interval, while the membrane can wrap the interval. Each of these configurations can be checked for 3 criteria which, if satisfied, could lead to the observation of cosmic strings: the tension of the string should be in agreement with WMAP data, the string should be stable and a production mechanism for the string after inflation needs to be available. It is possible to check (though we will not present the details of the calculation here) that the first candidate, namely the M5-brane wrapping a Calabi–Yau 4-cycle has a tension of the order M_{GUT}^2. Further, this string is stable. The axion, which potentially can create an axion domain wall that makes the string collapse, is removed via the Higgs effect. Finally, this cosmic string is produced via a small instanton transition taking place when the M5-brane dissolves on the M9-boundary. The details of this production mechanism are work in progress.

To conclude, cosmology provides a fascinating arena to search for experimental evidence of string theory. It is fair to say that this area of modern string theory research still needs to be developed in much more detail than what is known at this point. Many unexpected surprises may appear on our way.

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