Stonybrook Talk – August 24, 2006

SPLIT POLAR ATTRACTORS

Work done "with" F. Denef

Outline

1. Introduction
   - A – BPS statecounting
   - B – IIA/CY
   - C – Overview of OSV conj.
2. Describing BPS States
3. BPS Counting Functions
4. Fareytail expansion
5. Polar tems as boundstates
6. Extreme polar states
7. Formal derivation of OSV
8. Counting D6D2D0 States

1. Intro: BPS state counting + Overview

A. Why interesting

- Black hole entropy
- Quantum Corrections
- Nonptve info / duality
- Mathematical structures:

BPS algebras, automorphic forms, enumerative geometry of derived categories

B. Framework of type II / $M_4 \times X_{CY}$

IIA: 4D $\mathcal{N}=2$ sugra

$(h^{1,1}+1)$ $U(1)$'s

Wrapped D-branes have charge $\Gamma \in K^0(X)$

Neglect torsion. Choose E/M splitting

$$D6\,D4\,D2\,D0 \longleftrightarrow \left(p^0, P, q_2, q_0\right) = \Gamma$$

$$\underbrace{H^0 \oplus H^2}_{\text{Mag}} \oplus \underbrace{H^4 \oplus H^6}_{\text{El.}}$$

Note: we will often identify

$H^6(X,\mathbb{Z}) \approx \mathbb{Z}$ by integration.

Goal is to understand:

$$\Omega(\Gamma \quad) := -\frac{1}{2} \mathrm{Tr}_{\mathcal{H}_{BPS}(\Gamma \quad)} F^2 (-1)^F$$

$F = 2J_3$

$$= \mathrm{Tr}\,(-1)^{F'}$$

Key point of today's talk:

$\Omega(\Gamma; t)$ depends on $t \in \mathcal{M}_{VM}$!

This point is often neglected, but is quite crucial.

C. Overview of our attack on the OSV Conjecture

OSV: $$Z_{BH} = \sum_q \Omega(p,q) e^{q\cdot\phi}$$

$$\sim \left| Z_{top}(g_s, t) \right|^2$$

$$g_s = \frac{1}{\phi^0 + i p^0} \qquad t^A = \frac{\phi^A + i p^A}{\phi^0 + i p^0}$$

We only attempt it for: $p^0 = 0$
P = ample

Ingredients:

C1. U-duality makes Z_{BH} modular

C2. $SL(2,\mathbb{Z})$ average of "polar piece" Z_{BH}^-

C3. Interpret polar terms as $D6\overline{D6}$

C4. Identify $Z_{D6} \sim Z_{top}$

In slightly more detail

C1': D4 partition function in backgnd flat RR fields C_1, C_3 on

$$\mathbb{R}^3 \times S^1 \times X$$

$C_0 := \oint C_1 \qquad C := \oint C_3$

$$Z_{D4} = \sum_q \Omega(p,q;t)\, e^{-\beta H_{BPS}(p,q;t) + i C\cdot q}$$

is suitably modular in $\tau = C_0 + i\beta/g_s^{IIA}$

C2: Write it as a Poincaré series

$$Z_{D4} \sim \sum_{A \in SL(2,\mathbb{Z})} Z_{D4}^{-}(A\cdot\tau)$$

Z_{D4}^{-} has negative powers of $q = e^{2\pi i \tau}$, a finite series

C3′: The states enumerated by Z^{-}_{D4} are <u>not</u> single-centered BH's but are boundstates of D6 $\overline{D6}$, at least for $t = B + iJ$, $J \longrightarrow \infty$.

$$Z^{-}_{D4} \sim Z^{-}_{D6}\, Z^{-}_{\overline{D6}} \qquad (\exists \text{ tricky!!})$$

C4: I dentify $Z_{D6} \sim Z_{DT}$ and use

$$Z_{DT} \sim Z_{GW}$$

to get an OSV-like statement.

$\exists$ LOT'S OF SUBTLETIES!!

2. Describing The BPS States

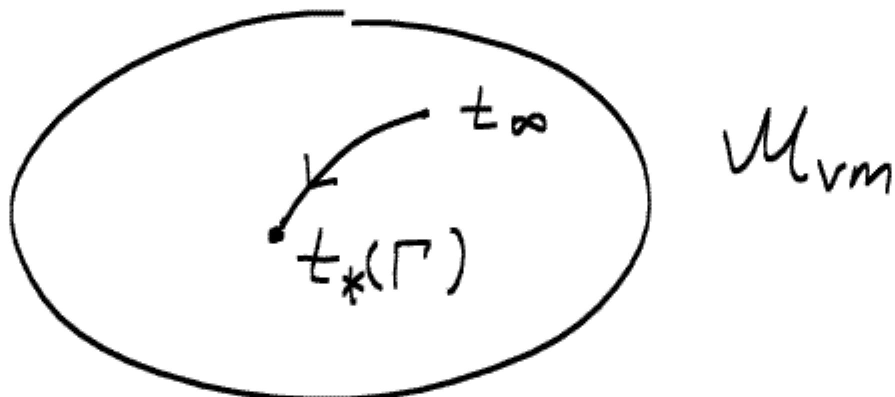
D-branes are always objects in a category

For IIA: Bounded derived category of coherent sheaves.

But we will work at $J \to \infty$, weak coupling where there is a beautiful sugra description.

Γ, t_∞ + spherical symmetry $\Longrightarrow$

Unique BPS solution to sugra



gradient flow of $\log |Z(\Gamma;t)|^2$

Where:

$$Z(\Gamma,t) = \langle \Gamma, \Omega \rangle$$

$\langle\ ,\ \rangle$: Symplectic product on $K^0(X)$

$$\int \mathrm{ch}E\, \overline{\mathrm{ch}E'}\, \hat{A} = \int -p^0 q_0' + p q' - q p' + q_0 p^{0\prime}$$

$\mathrm{IIA}, J \to \infty$: $\quad \Omega \simeq - e^{B+iJ} / \sqrt{J^3}$

$$Z \simeq \left(\tfrac{1}{6} p^0 t^3 - \tfrac{1}{2} p \cdot t^2 + q \cdot t - q_0 \right) / \sqrt{(\mathrm{Im}\, t)^3}$$

Horizon Area $:= 4\pi\, S(\Gamma) = 4\pi \left| Z(\Gamma, t_*(\Gamma)) \right|^2$

$:= \sqrt{\mathcal{D}(\Gamma)}$ ⟷ "Discriminant"

For $p^0 = 0$

$$S(\Gamma) = 2\pi \sqrt{\frac{-\hat{q}_0\, \chi(P)}{6}}$$

$\chi(P) = P^3 + c_2 \cdot P$ = Euler of surface in $|P|$

$$\hat{q}_0 := q_0 - \tfrac{1}{2} \left(D_{ABC} P^C \right)^{-1} q_A q_B$$

<u>But</u>: $\mathcal{D}(\Gamma) < 0$ for some charges

(esp: $\hat{q}_0 > 0$!!)

Find $\mathcal{D}(\Gamma) < 0 \iff Z(\Gamma, t)$ has a zero in $\mathcal{M}_{VM}$

$\Longrightarrow$ Attractor flow is singular

So, No BPS State .... ?

Denef: Not necessarily: Relax spherical Symmetry: BPS eqs then become

(1.) $ds^2 = -e^{2U}(dt+\omega)^2 + e^{-2U} d\vec{x}^2$

(2.) $2e^{U} \operatorname{Im}(e^{-i\alpha}\Omega) = -H$

$= $ Harmonic function $: \mathbb{R}^3 \longrightarrow H^{ev}(X, \mathbb{R})$

$\alpha(x) = \arg(Z(\Gamma, t(x)))$

(3.) $*_3 \, d\omega = \langle dH, H \rangle$

(4.) $\vec{E} + i\vec{B} = \cdots\cdots$

$$(2) \Longrightarrow H(\vec{x}) = \sum_i \frac{\Gamma_i}{|x - x_i|} - 2\operatorname{Im}(e^{-i\alpha}\Omega)_\infty$$

$$(3) \Longrightarrow \sum_{j \neq i} \frac{\langle \Gamma_i, \Gamma_j \rangle}{|x_i - x_j|} = 2\,\mathrm{Im}\left(e^{-i\alpha} Z(\Gamma_i)\right)_\infty$$

Sugra valid $\Longleftrightarrow$ $\mathcal{D}(H(\vec{x})) > 0 \quad \forall\, \vec{x} \in \mathbb{R}^3$

(Note $\Longrightarrow$ $\mathcal{D}(\Gamma_i) \geq 0 \Rightarrow$ Constituents of BPS molecule have regular attractor points)

$\sim$ Need to bend the rules a bit if some Γ_i are purely electric

<u>Example</u>: Lots of intuition from 2-centered case

$$\Gamma_1 \xrightarrow{\vec{R}} \Gamma_2$$

$$R = \frac{1}{2}\langle \Gamma_1, \Gamma_2 \rangle \frac{|Z_1 + Z_2|_\infty}{\mathrm{Im}(Z_1 \bar{Z}_2)_\infty}$$

$$\vec{J} = \frac{1}{2}\langle \Gamma_1, \Gamma_2 \rangle \hat{R}$$

Remarks

1. <u>Halos</u>: Simple generalization
   $\Gamma_2, \cdots, \Gamma_N$ all mutually local

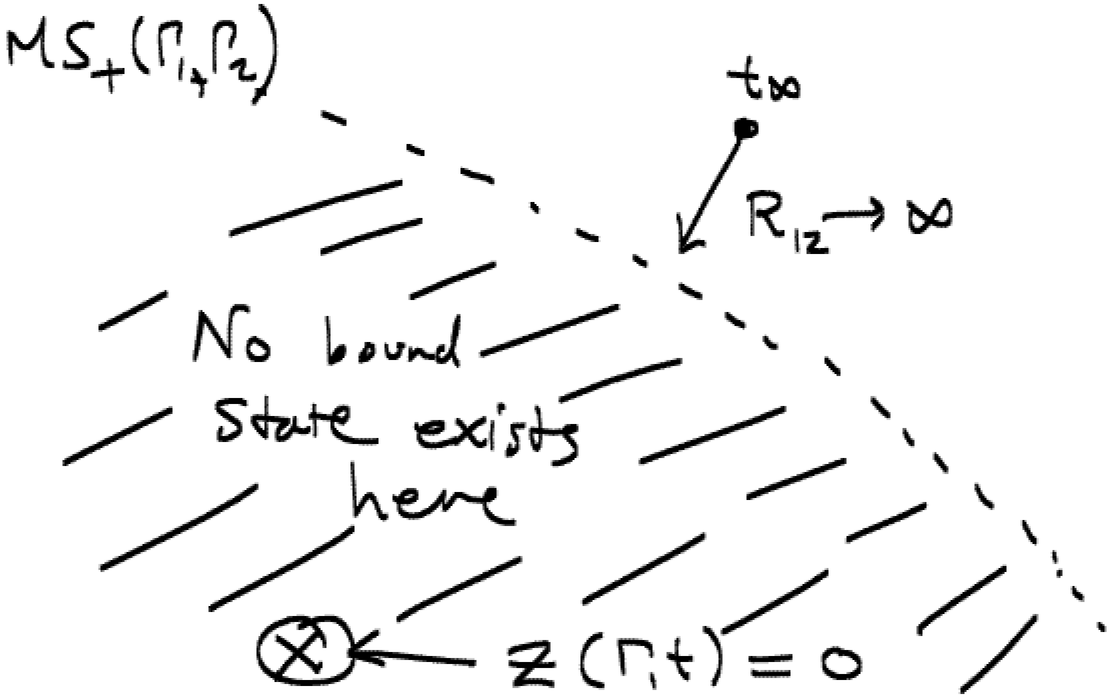
   $$\Gamma_j = \lambda_j \Gamma_2 \qquad \lambda_j > 0 \quad j = 2, \cdots, N$$

Then

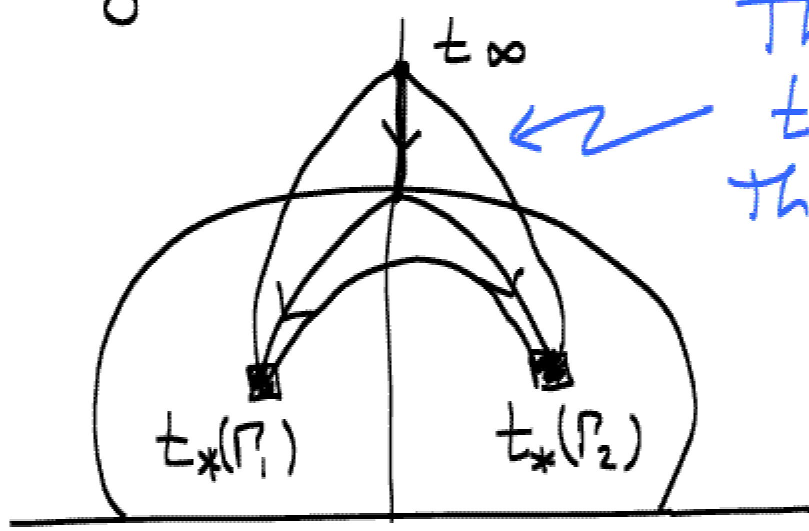
$\leftarrow \Gamma_j$

2. Marginal Stability : $\Gamma = \Gamma_1 + \Gamma_2$

$$MS_+(\Gamma_1, \Gamma_2) = \{ t \mid Z_1 / Z_2 \in \mathbb{R}_+ \}$$



3. Image of $t(\vec{x})$



The image of $t(\vec{x})$ is a thickened "tree"

<u>Def:</u> A "Split Attractor Flow" is a piecewise attractor flow tree connected at walls of marginal stability.

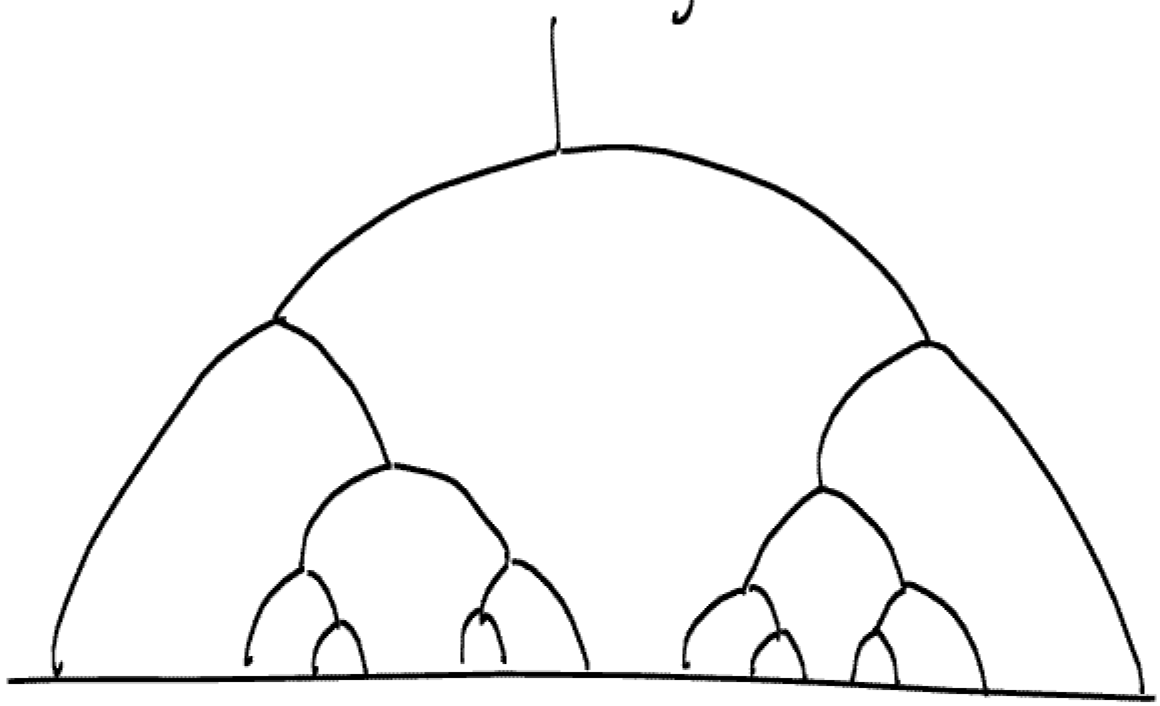
Split Attractor Conjecture (Denef)

(components of)

(a.) Multicentered BPS solutions are in 1-1 correspondence with S.A.F.'s

(b.) For a fixed (t_∞, Γ) there are a finite number of S.A.F.'s

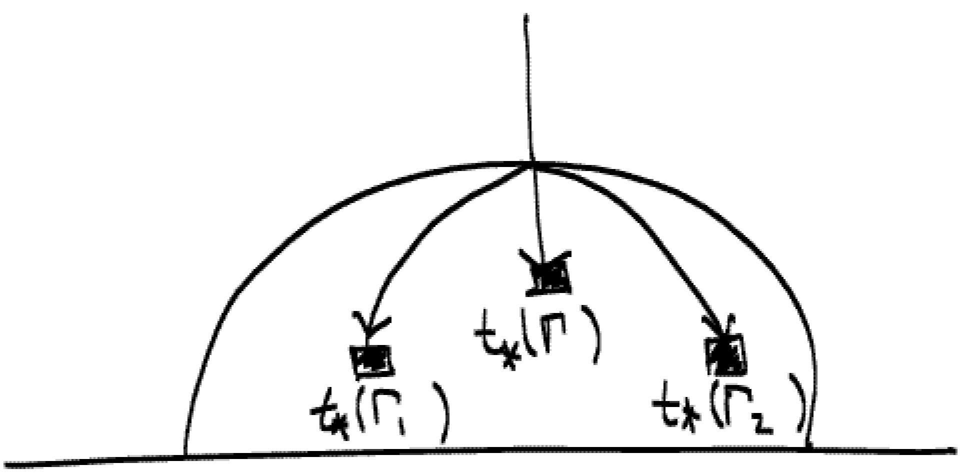
Note: these SAF's can be very elaborate, e.g. we constructed an inft. "fractal family"



Two other significant & surprising examples

1. $(0, P, 0, q_0) = (r, \tfrac{1}{2}P, q, \tfrac{1}{2}q_0)$
$+(-r, \tfrac{1}{2}P, -q, \tfrac{1}{2}q_0)$

For an appropriate range of q, q_0 $\exists$ both single-centered and two-centered soln's



$\Rightarrow$ Compare entropies

$S(\Gamma)$ vs $S(\Gamma_1) + S(\Gamma_2)$

$\exists$ family of such

$$\lambda\Gamma = \lambda(0, P, 0, q_0) = \left(r, \tfrac{\lambda}{2}P, q(\lambda), \tfrac{\lambda}{2}q_0\right)$$

$$+ \left(-r, \tfrac{\lambda}{2}P, -q(\lambda), \tfrac{\lambda}{2}q_0\right)$$

with

$$S(\lambda\Gamma) = \lambda^2 S(\Gamma)$$

but

$$S(\Gamma_1^\lambda) = S(\Gamma_2^\lambda) = \lambda^3 S(\Gamma_1)$$

$\Rightarrow$ 2-centered solution dominates entropy!!

$\Rightarrow$ Too many states for BH entropy!!

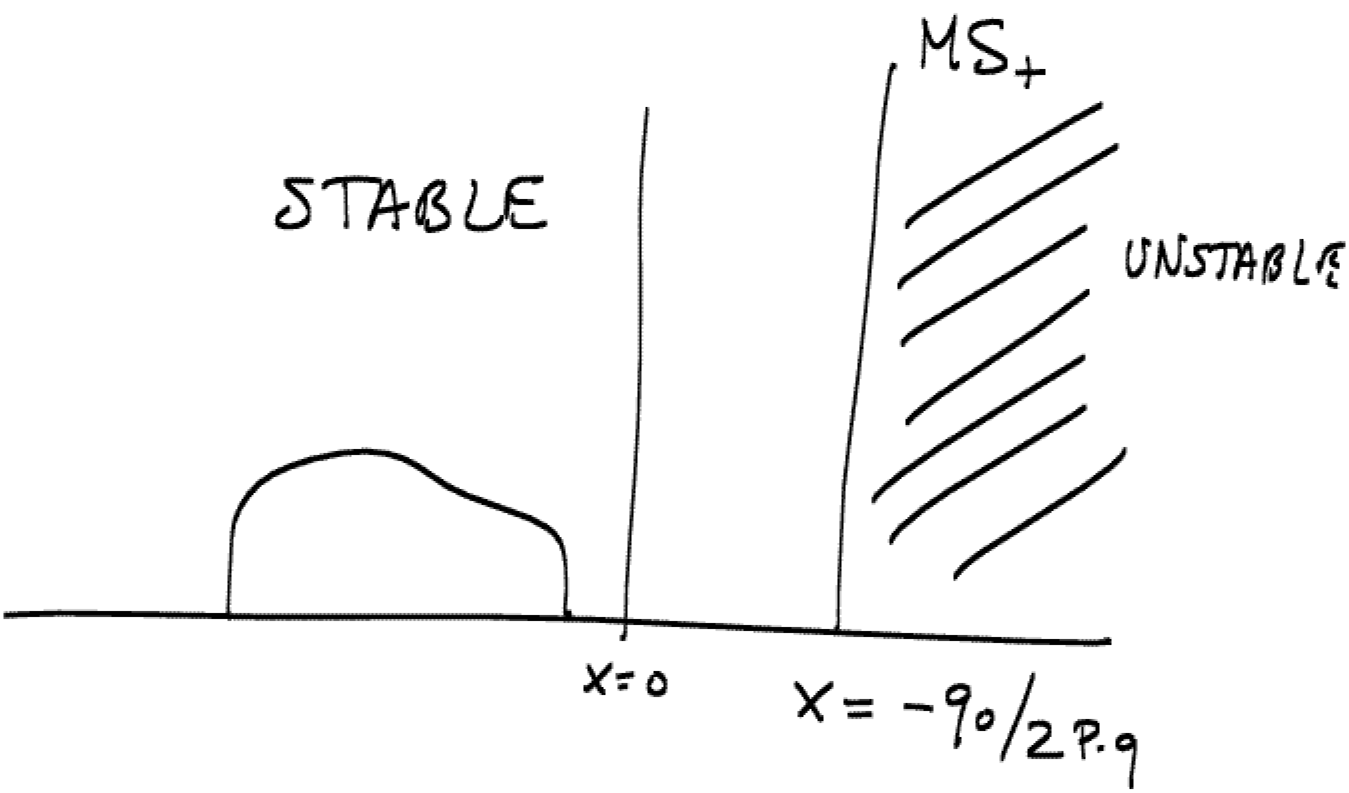
We'll return to this.....

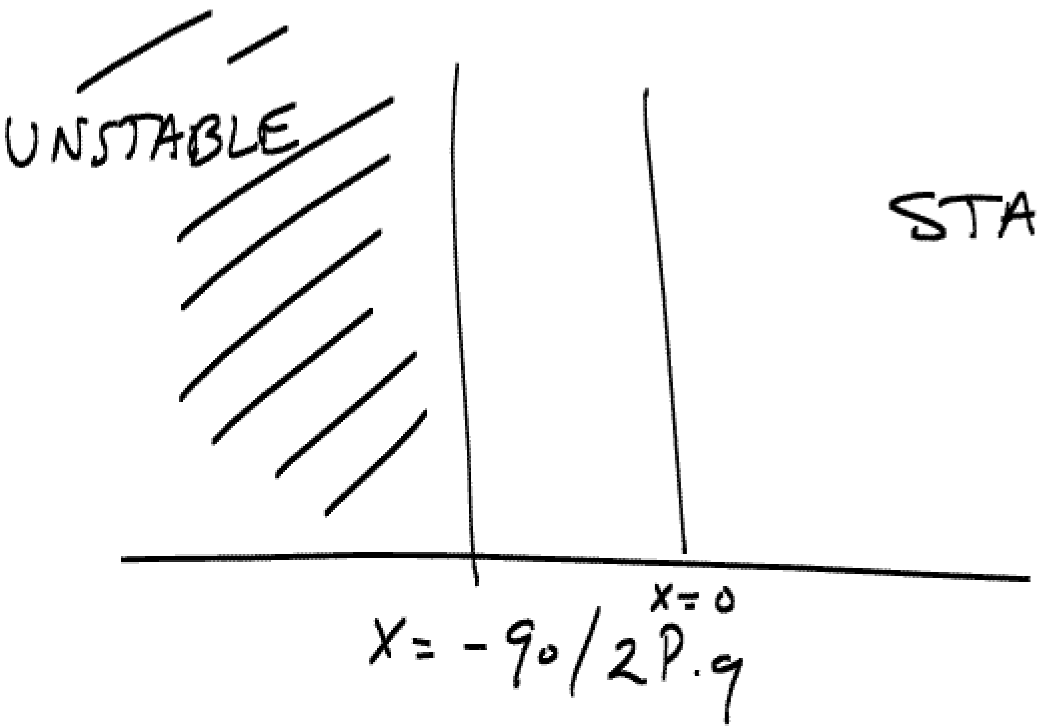
2.. $\Gamma_1 = (1, 0, -\beta, n)$

$\Gamma_2 = (0, 0, q, q_0)$

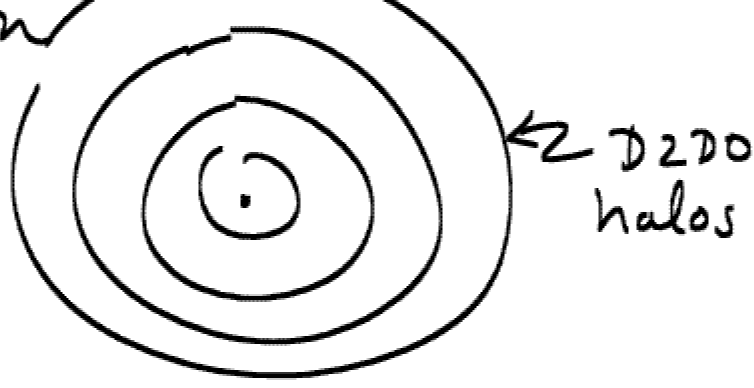
Set $t = (x + iy)P$ Then the

$\mathrm{Im}\, Z_1 Z_2^* = 0$ curve is – for $P \cdot q < 0$ –





$\Rightarrow$ Typical D6 D2D0 boundstate looks like an onion



$\boxed{\text{3. BPS Counting Functions}}$

D4D2D0 boundstates ~ count by a suitable partition function of the D4-brane theory in presence of <u>flat</u> RR fields

$$Z_{D4} = \mathrm{Tr}_{\mathcal{H}_{D4}} (-1)^{F'} e^{-\beta H + i C \cdot q}$$

Description of BPS states:

Choose P dual to ample divisor.

D4 wraps $\Sigma \in |P|$

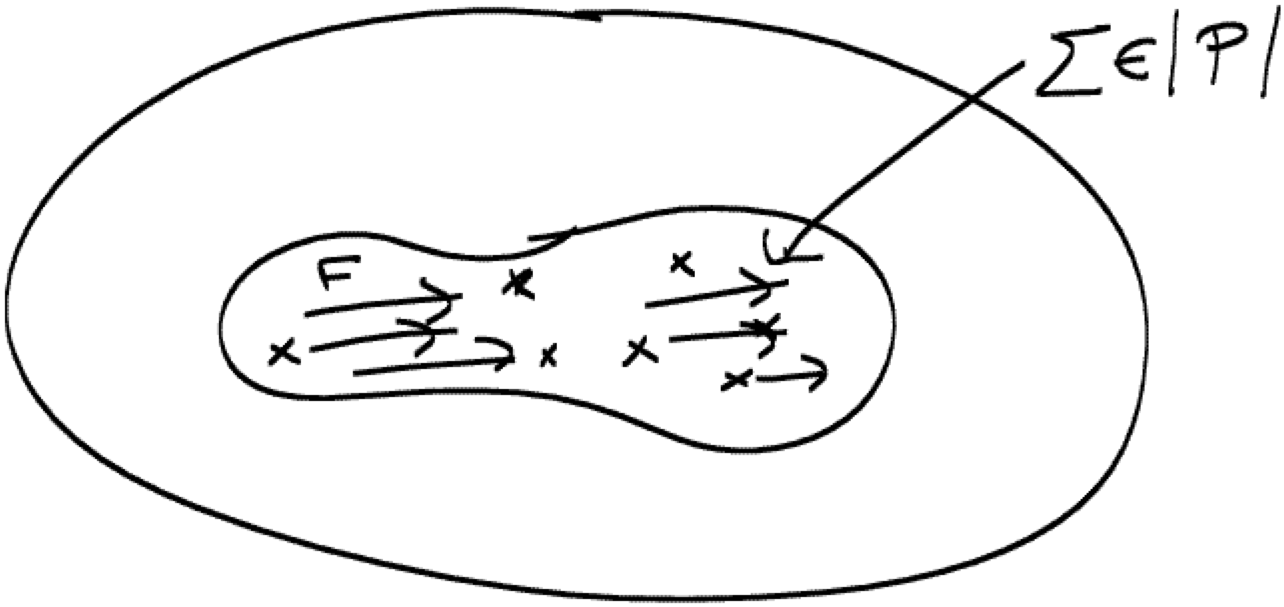
$$b_2(P) = \chi(P) + 2 = P^3 + c_2 \cdot P + 2 \gg b_2(X)$$

Lots of fluxes in $U(1)$ g.t. on Σ.

$$[F] \in H^2(\Sigma, \mathbb{Z})$$

For generic F, $F^{2,0} = 0$ fixes moduli of Σ "open string flux vacua"

In addition we can have pointlike instantons of $ch_2(I) = N$.



Mathematically

$$d(F,N) = \chi\left[\begin{array}{ccc} \mathrm{Sym}^N\Sigma & \to & \mathcal{M}(P,F,N) \\ \downarrow & & \downarrow \\ \text{smooth point} & \hookrightarrow & NL(P,F) \end{array}\right]$$

NL = "Noether-Lefshetz locus" of $\Sigma \in |P|$ where $F^{2,0} = 0$.

$$Z_{D4} = \sum_{F,N} d(F,N)\, e^{-\frac{\beta}{g_s}|Z| - i C_0 q_0 - i C\cdot q}$$

$$q_0 = q_0(F,N) = \frac{\chi(P)}{24} + \frac{1}{2}F^2 - N$$

$$q_A = \int_P J_A \cdot F \qquad A = 1, \ldots, h^{1,1}(X)$$

U-duality $\Longrightarrow$

$$Z(\tau, \bar\tau, C) := \sum d(F,N)\, e^{2\pi i \tau\left(N + \frac{1}{2}F_-^2 - \frac{\chi(P)}{24}\right)}$$

$$e^{-2\pi i \bar\tau \frac{1}{2}F_+^2 - 2\pi i F \cdot C}$$

is a Jacobi form.

Relation to OSV's Z_{BH}:

Put $\tau = \bar\tau = i\phi^0$, ϕ^0 real

$C = i\Phi$

$$Z\Big|_{\substack{OSV \\ sub}} = \sum d(F,N)\, e^{2\pi\phi^0 q_0 + 2\pi \Phi \cdot q}$$

Theta function decomposition:

It's useful to seperate out the $\bar{\tau}, C$ dependence exactly:

$$L_X = \iota^* H^2(X,\mathbb{Z}) \subset H^2(P,\mathbb{Z})$$

$$d(F+\ell_X, N) = d(F,N) \qquad \ell_X \in L_X$$

$$\mathcal{Z}(\tau,\bar{\tau},C) = \sum_{\gamma \in L_X^*/L_X} \Theta_\gamma(\tau,\bar{\tau},C)\, H_\gamma(\tau)$$

N.B. Singleton decomposition in AdS_3 interpretat'n

$$\underbrace{H_\gamma(\tau) = \sum_{\substack{F \in L_X^\perp + \gamma^\perp \\ N}} d(F,N)\, e^{-2\pi i \tau \hat{q}_0}}_{\text{Nontrivial info is here}}$$

$$\hat{q}_0 = \hat{q}_0(F,N) = q_0(F,N) - \frac{1}{2}q^2$$

$$= \frac{\chi(P)}{24} + \frac{1}{2}(F^{\perp})^2 - N \leq \frac{\chi(P)}{24}$$

Finite # of terms with negative power of $q = e^{2\pi i\tau}$

Remark : Immediate consequence : "Strong coupling OSV"

$$\Omega(P, Q_0, Q) = \oint d\tau \, H_{\gamma_Q}(\tau) e^{2\pi i \tau \hat{Q}_0}$$

s.p. $\tau_* = i\sqrt{\dfrac{-\chi(P)}{24\hat{Q}_0}}$

$\hat{Q}_0 \longrightarrow -\infty$, $\tau \longrightarrow 0$ Modularity $\Rightarrow$

$$Z_{OSV}(\phi^0, \Phi) \approx -i I_p \phi^0 \sum_{S \in H^2(X,\mathbb{Z})} e^{\frac{2\pi}{\phi^0}\left[\frac{\chi(P)}{24} - \frac{1}{2}(\Phi + iS)^2\right] + i\pi P S}$$

- Strong g_{top} is the regime of MSW.

- We would like to understand what happens at weak g_{top}.

e.g. we would like to scale the charges (P, Q) uniformly, not asymmetrically as with $\hat{Q}_0 \to -\infty$ at fixed P.

So we need a better description of $H_\gamma(\tau)$

4. Farey Facts

Suppose $\quad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$

$$f(A\cdot\tau) = j(A,\tau)\, f(\tau)$$

$$j(A,\tau) = \omega_A\, (c\tau+d)^{w} \qquad w \leq 0$$

$$f(\tau) = \sum_{n\geq 0} c_n\, e^{2\pi i (n-\Delta)\tau}$$

$$= \underbrace{f^{-}(\tau)}_{n-\Delta<0} + f^{+}(\tau)$$

Then

$$f(\tau) = \sum_{A}{}' \, j(A,\tau)^{-1} f^{-}(A\cdot\tau)$$

When suitably regularized

Apply to $H_\gamma(\tau)$. Upshot:

$$\mathfrak{z}(\tau,\bar{\tau},C) = \sum_{A}{}' \, j(A,\tau)^{-1}\, \mathfrak{z}^{-}(A\cdot\tau, A\cdot\bar{\tau}, A\cdot C)$$

Suitably regulated.....

Here $\mathcal{Z}^-$ only counts polar terms, i.e. terms with $\hat{q}_0 = q_0 - \frac{1}{2}q^2 > 0$.

$$\boxed{\text{Thus we can get all degeneracies } \Omega(P,Q;t) \text{ given only a knowledge of the polar terms } \mathcal{Z}^-}$$

$$\mathcal{Z} = \mathcal{Z}^- + \mathcal{Z}^-\Big|_{\tau \to -1/\tau}$$
$$+ \sum_{d \neq 0} \mathcal{Z}^-\Big|_{\tau \to -\frac{1}{\tau + d}}$$
$$+ \cdots\cdots$$

Plugging into

$$\Omega(P,Q) = \oint d\tau\, H_\gamma(\tau)\, e^{2\pi i \tau \hat{Q}_0}$$

gives an exact expression for the degeneracies.

Later on we'll see that

$\tau \to -\frac{1}{\tau+d}$ are the most important mod. tmns.

After making the OSV substitution we get

$$Z\Big|_{\tau\to -1/\tau} \sim \phi^0 e^{-\frac{\pi}{\phi^0}\Phi^2} \sum_{\hat{q}_0>0} \Omega(P,Q)\, e^{\frac{2\pi}{\phi^0} q_0 + \frac{2\pi i}{\phi^0} q\cdot\Phi}$$

So we're on the right track

So we need to understand $\Omega(P,Q;t)$ better for the polar states.

$\boxed{\text{5. Polar Terms as Bound States}}$

Recall $S = 2\pi \sqrt{\frac{-\hat{q}_0 \chi(P)}{24}}$

$*$ $\boxed{\text{Polar states are always split attractor states !}}$

Claim: For the polar states there is a factorization of degeneracies.

$$\boxed{\Omega(\Gamma) = \sum_{\Gamma=\Gamma_1+\Gamma_2} (-1)^{\langle\Gamma_1,\Gamma_2\rangle - 1} |\langle\Gamma_1,\Gamma_2\rangle| \, \Omega(\Gamma_1)\Omega(\Gamma_2)}$$

Γ_1, Γ_2 might or might not be polar

Reasons for this:

(1.) Macroscopic Approach a wall $MS_+(\Gamma_1, \Gamma_2)$

2-centered boundstates separate

So it is reasonable to expect this part of Hilbert space $\dim \mathcal{H}_\alpha$

$\sim \Omega(\Gamma_1)\Omega(\Gamma_2)$

However: There is a spin degeneracy

$J_{12} \sim \langle \Gamma_1, \Gamma_2 \rangle$

leading to sign and prefactor.

(2.) Microscopic This component$^{(\alpha)}$ of the moduli of susy config's is expected to have the form

$$\begin{array}{ccc} \mathbb{C}P^{k-1} & \longrightarrow & \mathcal{M}_\alpha \\ & & \downarrow \\ & & \mathcal{M}(\Gamma_1)\times\mathcal{M}(\Gamma_2) \end{array} \qquad k = \langle \Gamma_1, \Gamma_2 \rangle$$

$g_s \to 0$ we have a quiver picture

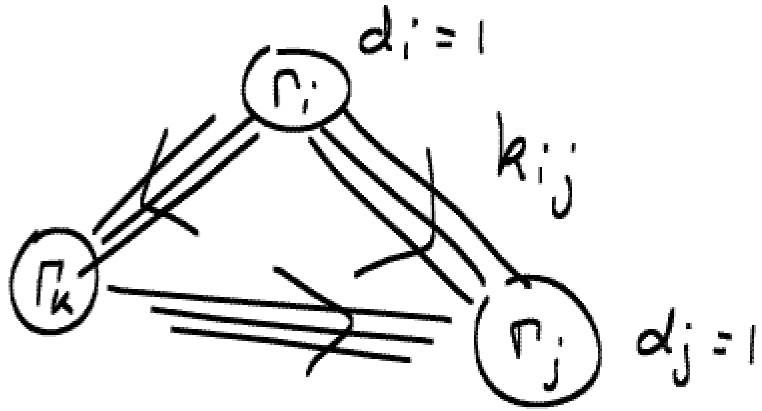
In general

$\Gamma_i \sim$ simple objects

Near locus where $Z(\Gamma_i, t)$ are parallel

$$k_{ij} = \langle \Gamma_i, \Gamma_j \rangle$$

Susy Moduli $\sim$ quiver:



etc.

For a large class of quivers without loops we found the moduli space indeed has the above structure

Even more dramatic: For quivers with loops with generic superpotential we found factorization.

Remark: Similar formulae in Joyce paper

This does not quite give the desired factorization because

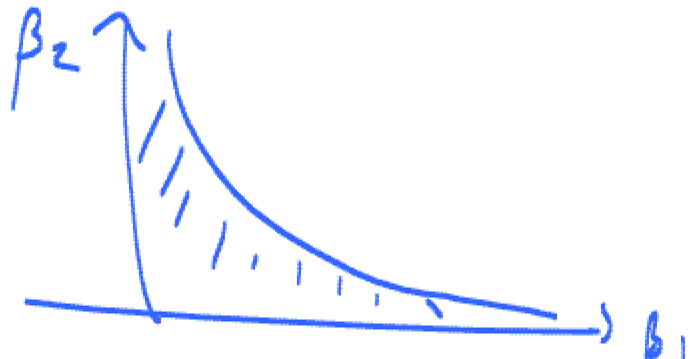
(1.) Extra spin degeneracy

(2.) Γ_1, Γ_2 are correlated

(Revealed by numerical search of Split attractors on the quintic with

$$\Gamma_1 = e^{P/2}\left(1 - \beta_1 + n_1 dV\right)$$

$$\Gamma_2 = -e^{-P/2}\left(1 - \beta_2 + n_2 dV\right)$$



Barring miracles, we do not have factorization. What we can hope for is some kind of effective factorization in a limit of interest.

6. Extreme Polar States

From $\Omega(P,Q) = \oint d\tau\, e^{2\pi i \tau \hat{Q}_0} H_\gamma(\tau) \cdots$

The contribution of

$$e^{-2\pi i (A\tau)\hat{q}_0} \quad \text{to} \quad \Omega(P,Q)$$

$$\sim \exp\left(\frac{4\pi}{c}\sqrt{\hat{q}_0 |\hat{Q}_0|}\right)$$

$\Rightarrow$ Focus on terms in the Fareytail with $c=1$ and terms in $\mathfrak{Z}^-$ with $\hat{q}_0$ maximal.

$$\hat{q}_0 = \frac{\chi(P)}{24} + \frac{1}{2}(F^\perp)^2 - N$$

$\Rightarrow$ $F^\perp, N$ "small"

N.B. Values of $\hat{q}_0$ in a "discretuum" for large P.

We arrived at the following Conjecture for what these "extreme polar states" look like:

<u>Conjecture</u>: For suitably small ε:

All polar states with

$$\frac{\chi(P)}{24} - \hat{q}_0 < \varepsilon P^3$$

are split attractors for $\Gamma = \Gamma_1 + \Gamma_2$

$$\Gamma_1 = e^{S_1}\left(1 - \beta_1 + n_1\, dV\right) = e^{S_1}\,\Gamma(\beta_1, n_1)$$

$$\Gamma_2 = -e^{S_2}\left(1 - \beta_2 + n_2\, dV\right) = -e^{S_2}\,\Gamma(\beta_2, n_2)$$

where

(a.) $S_1 - S_2 = P$

(b.) $\beta_i = [\sigma_i]$ for holomorphic curves $\sigma_i \subset X$

(c.) $\beta_i \cdot P < \varepsilon' P^3$, $n_1 - \chi_h(\sigma_1) < \varepsilon' P^3$
$n_2 + \chi_h(\sigma_2) > \varepsilon' P^3$

Heuristics: we have a D6 brane bound to a gas of D2, D0's:

$$\mathrm{ch}\, I_1 = \Gamma(\beta_1, n_1) \qquad \mathrm{ch}\, I_2^* = -\Gamma(\beta_2, n_2)$$

Evidence for this conjecture:

A. Picture of moduli space $\mathcal{M}(P,F,N)$ for small values of F,N:

7. Formal derivation of OSV

If we identify $\Omega(\Gamma(\beta,n)) = N_{DT}(\beta,n)$

$$\sum_d Z^-\left(\frac{-1}{\tau+d}, \cdots\right)\Big|_{OSV}$$

$$= \sum_{\phi^0\to\phi^0+id} \sum{}' \left(I_p - P\cdot(\beta_1+\beta_2)+n_1-n_2\right) N_{DT}(\beta_1,n_1)\, N_{DT}(\beta_2,n_2)$$

$$\times\, e^{\frac{2\pi}{\phi^0}q_0 + \frac{2\pi i}{\phi^0}q\cdot\Phi}$$

Define

$$Z_{DT}(u,v) = \sum N_{DT}(n,\beta)\, u^n v^\beta$$

Then compute $q_0(\Gamma_1+\Gamma_2)$, $q_2(\Gamma_1+\Gamma_2)$ for above Γ_1, Γ_2

After some algebra we find

$$\simeq I_P \sum_{\phi^0 \to \phi^0 + i\alpha} \sum_{S \in H^2(X,\mathbb{Z})} e^{\frac{2\pi}{\phi^0}\chi(P) - \frac{\pi}{\phi^0}(\Phi - iS)^2 P}$$

$$\times Z_{DT}\left(e^{\frac{2\pi}{\phi^0}}, e^{-\frac{2\pi i}{\phi^0}(\Phi - iS)}\right)$$

$$\times Z_{DT}\left(e^{-\frac{2\pi}{\phi^0}}, e^{-\frac{2\pi i}{\phi^0}(\Phi + iS)}\right)$$

where $S := \frac{1}{2}(S_1 + S_2)$

BUT!!

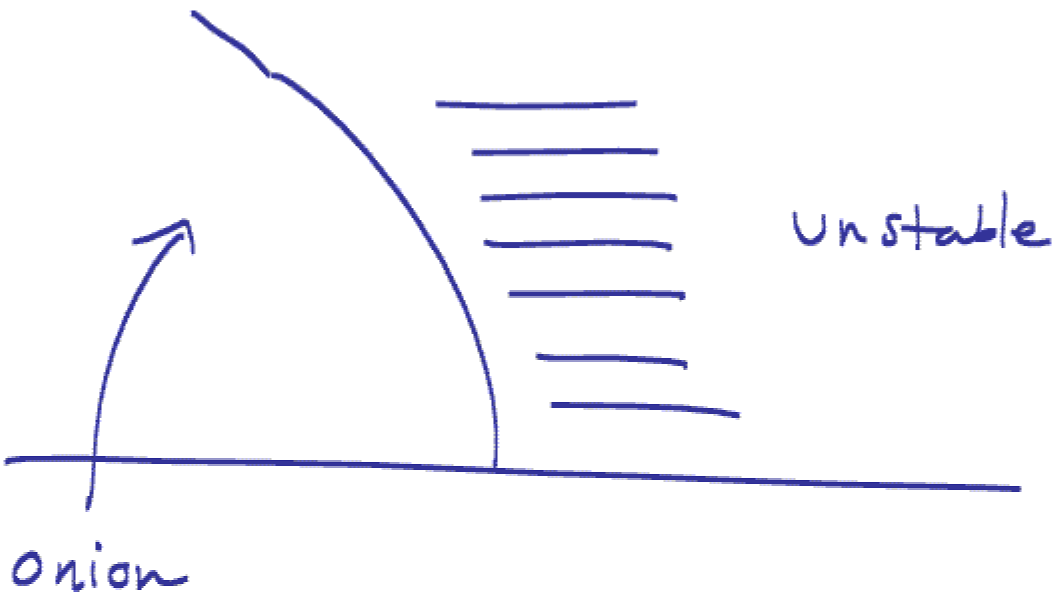
1. The above sum must be truncated, and the domains don't really factorize..
2. We have not been careful about Z_{DT} vs. Z'_{DT}
3. There is an important subtlety in passing to Z_{DT}.

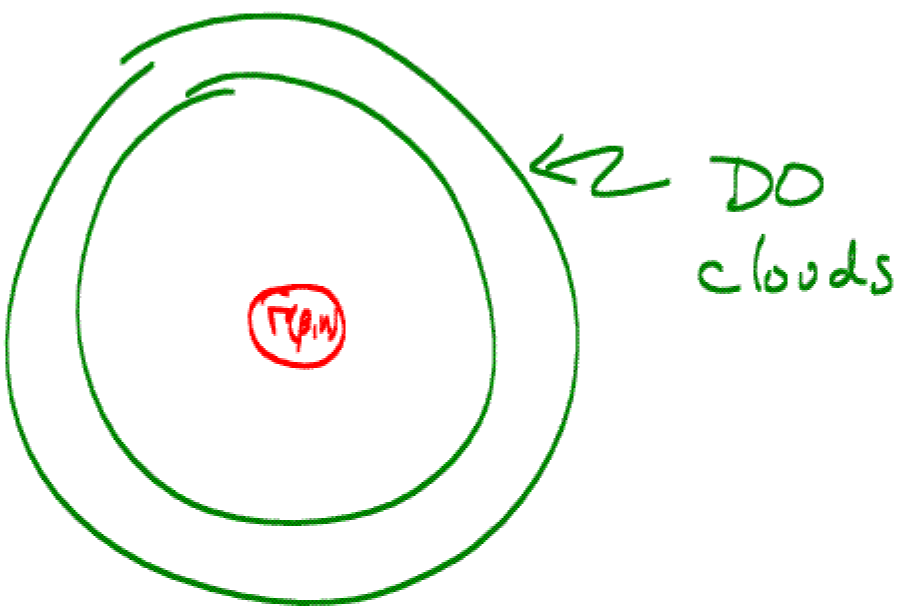
8. Counting D6D2D0 States

A. D6$\overline{\text{D0}}$ and McMahon

B. D6 $\overline{\text{D2D0}}$ and DT

A.





(1.) D0 charge n , $n = 1, \dots\dots$

(2.) LL degeneracy

$$j = \frac{1}{2}\left(\langle \Gamma(\beta, n), n\Gamma_{D0} \rangle - 1 \right) = \frac{n-1}{2}$$

deg. label $= m = 0, \dots\dots, n-1$

(3.) Internal DOF $\omega \in H^*(X)$

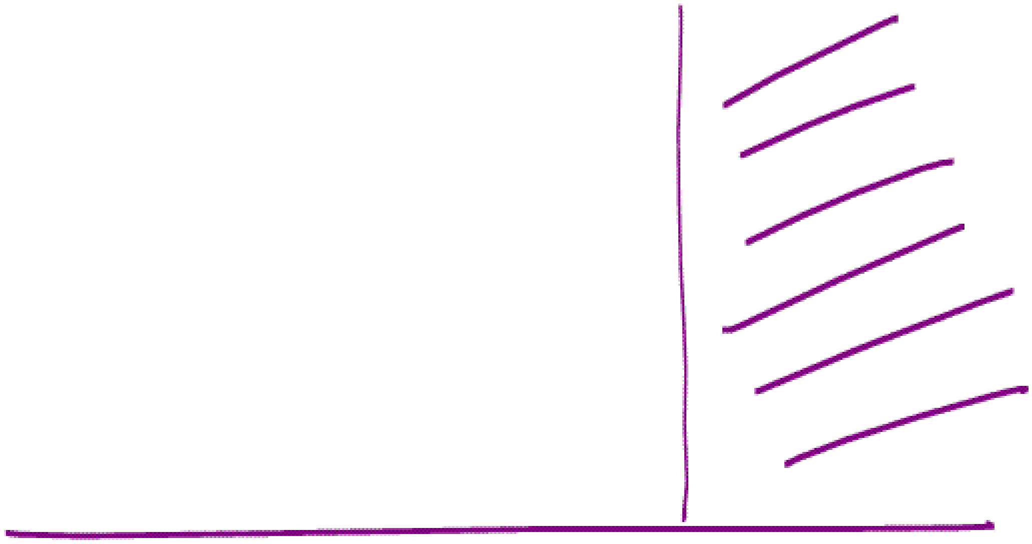
$\Longrightarrow$ Fock space

$\alpha^{+}_{n,m,\omega_e}$: bosonic $\quad \psi_{n,m,\omega_o}$: fermionic

$$Z_{D6\overline{D0}} = \sum d_N u^N = \left(M(-u) \right)^{\chi(X)}$$

D0 cloud binds either to D6 or to $\overline{D6}$ depending on B-field

B. Recall MS line



$x = -q_0/2P\cdot q_2$

$$\Gamma = \Gamma(\beta, n) + (0, 0, q_2, q_0)$$

Define:

$$Z'_{D6\,\overline{D2D0}} = 1 + \sum_{\substack{\beta, n \\ \beta \neq 0}} \lim_{y \to \infty} \Omega\big(\Gamma(\beta,n), (x+iy)P\big) u^n v^\beta$$

Wall crossing

$$Z' \longrightarrow \left(1 - u^{q_0} v^{q}\right)^{-|q_0|\Omega(q)} Z'$$

$$\Omega(q) = \sum (-1)^{2j_L+2j_R}(2j_L+1)(2j_R+1)\, N_{q,j_L,j_R}$$

$\Rightarrow$ Z' is a nontrivial function of x !!

Conjecture:

$$\lim_{x\to\infty} Z' = Z'_{DT}$$

$$\lim_{x\to -\infty} Z' = \overline{Z'_{DT}}$$

Can be justified by M-theory

Conclusion: OSV is not true in the strictest sense, but something like it might well be true — but we have many obscure points still to be resolved.