Chiral Symmetry Breaking From String Theory

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This talk is based on work done with A. Parnachev [1].

Over the past few years there has been considerable progress in studying QCD-like theories using holography. The gravity description is useful for studying properties of hadrons as well as qualitative features of QCD such as confinement and chiral symmetry breaking. In this talk I will discuss a particular model which describes chiral symmetry breaking (χSB). which stands for the dynamical breaking of the $U(N_f) \times U(N_f) \to U(N_f)$ symmetry acting on N_f right and left handed quarks of the high energy Lagrangian down to its diagonal subgroup $U(N_f)$.

0.1 Outline

1. The weak coupling description;

2. Gravity description (at zero temperature and finite temperature);

3. Thermodynamics and chiral symmetry restoration at high temperatures.

1 The system

The system [2] consists of N_f–$D8$, N_f–$\bar{D}8$ and N_c $D4$ branes. $D4$ branes are extended in the $0, \cdots, 4$ directions, while $D8$ and $\bar{D}8$ are separated by distance L in the x_4 direction and span the rest of the directions. Moreover we assume that $N_f << N_c$. The x_4 direction is compactified on a circle with non-supersymmetric boundary conditions for the fermions. For the most of this talk we will assume that the radius of compactification $r \to \infty$. This system has a $U(N_f) \times U(N_f)$ symmetry, where the first component acts on $D8$ branes, while the second component acts on $\bar{D}8$.

Particle content: The states normalizable in the 3+1 dimensions correspond to $4-8$ and $4-\bar{8}$ strings, which live on the intersection of the $D4 - D8$ and $D4 - \bar{D}8$. This system is not supersymmetric so one might be worried about the appearance of tachyons. But since the $\#ND = 6$ all the modes in the NS sector are actually massive. Hence the only massless excitations are fermionic and correspond to the left q_L $(4-8)$ and right moving q_R $(4-\bar{8})$ quarks.

The interaction is mediated by the exchange of five-dimensional gauge fields A_M. Integrating out the A_M in the single gluon approximation we get non-local 4-fermion interaction

$$V_{eff} \sim \frac{g_5^2}{4\pi^2} \int d^4x d^4y G(x-y,L)[q_L(x)^\dagger \cdot q_R(y)][q_R(y)^\dagger \cdot q_L(x)] \tag{1}$$

$$g_5^2 = (2\pi)^3 g_s l_s^2, \quad \lambda = \frac{g_5^2}{4\pi} N_c \tag{2}$$

This interaction is an example of Nambu-Jona-Lasinio construction, which was suggested as an early model for chiral symmetry breaking.

2 Gravity description

2.1 Zero temperature

Now we turn to the holographically dual description of our system. First we should determine the near horizon geometry of our system. Since the $N_c >> N_f$ we can work in the probe approximation in which we replace the $D4$ branes by the geometry they create, while treating the $D8$ branes as probes. At zero temperature the near horizon geometry of $D8$ branes is given by[1]:

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} \left(\eta_{\mu\nu} dx^\mu dx^\nu + d\tau^2\right) + \left(\frac{R}{U}\right)^{3/2} \left(dU^2 + U^2 d\Omega_4^2\right) \tag{3}$$

$$e^\phi = g_s \left(\frac{U}{R}\right)^{3/4}, \qquad F_4 = \frac{2\pi N_c}{V_4}\epsilon_4 \, , \tag{4}$$

$$R^3 = \pi g_s N_c l_s^3 = \pi \lambda l_s^2 \tag{5}$$

The dynamics of the $D8$ branes is described by the DBI action on this background. In particular we will be interested in solutions which wrap $R^3 \times S^4$ and form a curve $U = U(\tau)$ in (U, x^4) plane,

[1]For simplicity we relabeled $\tau \equiv x^4$

which satisfies the following boundary conditions

$$U(\pm L/2) = \infty \ . \tag{6}$$

In this case the DBI action takes the form

$$S_{D8} \sim \int d\tau U^4 \sqrt{1 + \left(\frac{R}{U}\right)^3 U'^2} \ . \tag{7}$$

The equations of motion which follow from this action allow two types of solutions: $\tau = \pm L/2$ and

$$\frac{U^4}{\sqrt{1 + \left(\frac{R}{U}\right)^3 U'^2}} = U_0^4 \tag{8}$$

The latter solution is U shaped–it starts at $(\tau = -L/2, U = \infty)$, approaches minimum for U at $(\tau = 0, U = U_0)$ and ends at $(\tau = L/2, U = \infty)$. The first of these two solutions has the $U(N_f) \times U(N_f)$ symmetry, while the second breaks it down to a single $U(N_f)$. One can check [3] that the solution which breaks the chiral symmetry has lower energy and hence it is preferred. So we conclude that the chiral symmetry is dynamically broken in this system. Our main purpose here is to generalize the above analysis to non-zero temperature. We will observe that at sufficiently low temperature the phase with broken symmetry dominates, while as we increase the temperature further the phase with unbroken symmetry will come to dominate.

2.2 Finite temperature

As usual to study the finite temperature case we euclideanise and compactify the time coordinate $t \sim t + 1/T$. The near horizon geometry of N_c $D4$ branes at finite temperature is that of a euclidean black hole:

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} \left(-f(U)dt^2 + \delta_{ij}dx^i dx^j + d\tau^2\right) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right) \tag{9}$$

$$e^\phi = g_s \left(\frac{U}{R}\right)^{3/4}, \qquad F_4 = \frac{2\pi N_c}{V_4}\epsilon_4, \qquad f(U) = 1 - \frac{U_T^3}{U^3} \ , \tag{10}$$

$$R^3 = \pi g_s N_c l_s^3 = \pi \lambda l_s^2 \tag{11}$$

U_T is related to the temperature as follows

$$U_T = \left(\frac{4\pi T}{3}\right)^2 R^3 \ . \tag{12}$$

From the point of view of a Minkowskian black hole $U = U_T$ corresponds to the position of the horizon. Now we can again use the probe approximation to study the dynamics of the $D8 - \bar{D}8$

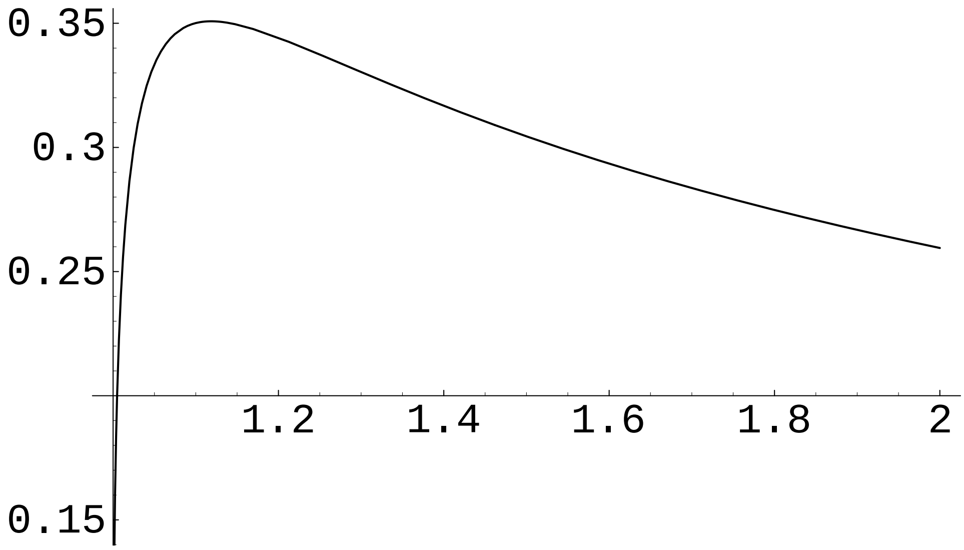

Figure 1: The $L/2$ (in the units of $3/4\pi T$)as a function of U_0/U_T.

branes in this background. The DBI action takes the from

$$S \sim \int d^3x \int dU U^{5/2} \sqrt{1 + \left(\frac{U}{R}\right)^3 f(U)(\partial\tau)^2} \ . \tag{13}$$

The action does not depend on τ explicitly hence

$$0 = \frac{d}{dU}\frac{\delta L}{\delta\partial\tau} = \frac{d}{dU}\left[\frac{U^{11/2}\tau' f(U)}{\sqrt{1 + \left(\frac{U}{R}\right)^3 f(U)(\partial\tau)^2}}\right] \ . \tag{14}$$

Solving this equation we again find two types of solutions: $\tau = \pm L/2$ and

$$\tau = \pm\sqrt{f(U_0)}U_0^4 R^{3/2} \int_{U_0}^{U} \frac{dU}{\sqrt{U^3 f(U)(f(U)U^8 - f(U_0)U_0^8)}} \tag{15}$$

Actually it turns out that for a given asymptotic separation L there are 2 solutions of the second type (see Fig.1). We will see below that for a given L the solution with largest U_0 will dominate. We also conclude from this graph that there is a limiting temperature above which only the phase with unbroken symmetry exists.

In order to determine the phase diagram we should compute the free energies corresponding to these configurations. Since the free energy diverges for all 3 solutions, we will compute the differences in

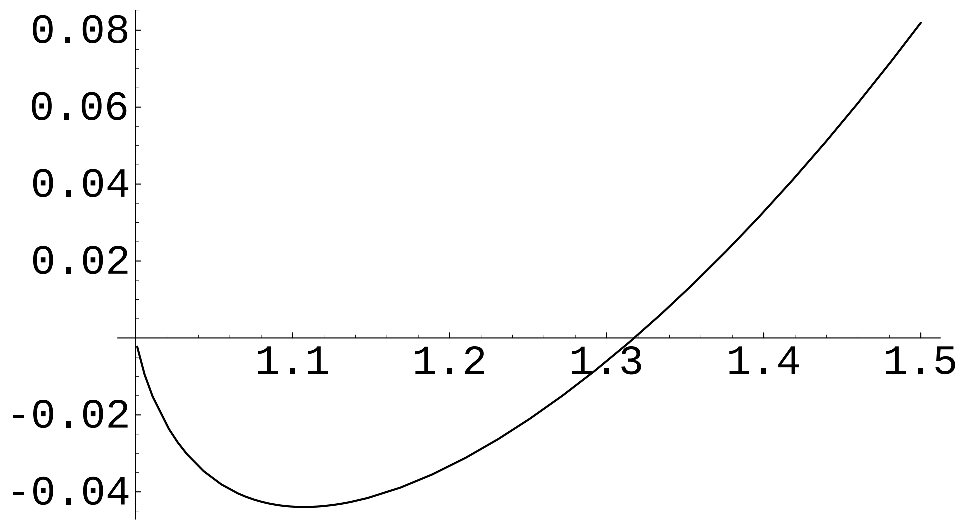

Figure 2: The $\delta\mathcal{F}/T$ (in appropriate units)as a function of U_0/U_T.

the free energy.

$$\frac{1}{T}(\mathcal{F}_{straight} - \mathcal{F}_{curved}) \sim (2/7)[U_0^{7/2} - U_T^{7/2}] + \int_{U_0}^{\infty} dU U^{5/2} \left[1 - \sqrt{1 + \frac{f(U_0)U_0^8}{f(U)U^8 - f(U_0)U_0^8}}\right] \quad (16)$$

The result is depicted on (Fig.2). Using the (16) and (Fig.1), we find that the chiral phase transition happens at $T \simeq 0.15L^{-1}$. The transition is of first order, which can be confirmed by computing the latent heat.

References

[1] A. Parnachev and D. A. Sahakyan, "Chiral phase transition from string theory," arXiv:hep-th/0604173.

[2] T. Sakai and S. Sugimoto, "Low energy hadron physics in holographic QCD," Prog. Theor. Phys. **113**, 843 (2005) [arXiv:hep-th/0412141].

[3] E. Antonyan, J. A. Harvey, S. Jensen and D. Kutasov, "NJL and QCD from string theory," arXiv:hep-th/0604017.