

The Landscape and the Swampland

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In the past year I have worked with a number of colleagues on some ideas which aim to delineate what is the universality class of the space of effective theories that can be consistently coupled to quantum gravity [1, 2, 3].

It has become evident that there are many vacua in string theory, some might say too many. Clearly the attitude that just one single vacuum exists is mistaken. But this was known from the early days of string theory. Some people had the delusion that by restricting to, say, 4D with N=1 supersymmetry, there might be only one solution. But that is wishful thinking reflecting misguided theoretical questions. It is usual that physical systems have more than one solution, and string theory is no exception - even though it is quite a rigid structure. On the other hand, one might then think that for any apparently consistent effective theory one can produce a string vacuum compatible with it. This would imply that string theory has no predictive power, being demoted to a mere custodian of the UV completion of the effective Lagrangian. Of course, it would still take care of conceptual issues such as black holes, unitarity, etc., but would have no dealing with experimental and pragmatic questions.

So it seems we should reorient our questions to string theory, so we can benefit from what string theory can teach us. A natural question is: which theories can appear in string theory and which cannot? The point I wish to make is that not every consistent-looking garden variety of an effective Lagrangian appears in string theory. In fact, those that do appear in string theory, despite their variety, are a relatively rare subset. In some sense they have measure zero in the space of effective theories. They are very special, for reasons we still do not fully understand. I do not mean by special that they are anomaly-free, renormalizable, etc. (in fact, they need not be renormalizable because gravity might fix their divergencies). How they are special is still a mystery.

Given an effective matter Lagrangian, we should ask: can it be successfully coupled to quantum gravity? The theories that allow us to do that belong to the *Landscape*, those outside the Landscape belong to the *Swampland*. We have found three kinds of conjectural criteria distinguishing the Swampland from the Landscape. These criteria can be checked in the huge amount of examples provided by string theory, which is the only known consistent theory of quantum gravity. In all the cases it can be shown that the criteria are violated when gravity is decoupled. By this we mean that the volume of the internal manifold is taken to infinite, so that the Planck mass in the physical space becomes infinite.

1. Criteria on the number of fields and their types

For a given spacetime dimension, the number of massless fields is bounded. Also the gauge groups that can appear are restricted.

As an example, consider $U(N)$ $\mathcal{N} = 4$ SYM. From the point of view of effective field theory, there is no upper limit to N . Note that this is not in contradiction with the fact that for any N the theory can be realized in a stack of N D3 branes, since in this case gravity exists in 10 dimension, not in 4. As a second example, consider type II theory compactified to six dimensions. If we compactify to K3, we get gauge groups with finite rank. If the inner theory is an ALE space $\mathbb{C}^2/\mathbb{Z}_N$, we do obtain an $U(N)$ gauge theory with any N , but then the inner theory is non-compact, so gravity is decoupled. A way for mathematicians to disprove this conjecture (namely, that there are only a finite number of massless particles), would be to find a sequence of compact Calabi-Yau's with ever increasing Hodge numbers. But nobody has found such a sequence.

This conjecture is very strange from an effective theory point of view. A way to justify this criteria might come from an holographic bound. If we restrict to a finite volume, we could not possible have arbitrary many fields without violating the amount of information allowed to that volume.

2. Criteria on the strength of the coupling constants

Gravity is always the weakest force.

Given two equal massive objects charged under a $U(1)$, their electrical repulsion should be stronger than their gravitational attraction. The statement can be made sharper by saying that the particle with minimum relation between mass and charge satisfies

$$\left(\frac{m}{q}\right)_{min} \leq 1$$

in appropriate units. Note that had we had the opposite relation, the gravitational attraction would form bound states of charged objects with unbounded charge, which is strange. Now, for large q , we know that there are extremal black holes which saturate the bound. So we are

claiming that this bound is saturated from below as q increases. Examples of how this bound is indeed saturated from below can be obtained from any heterotic string compactification.

Moreover the conjecture states that in a $U(1)$ theory with coupling constant g , there is a new scale $\Lambda = gM_{Planck}$, beyond which the theory breaks down. These kind of scales are familiar from GUT models.

Note that when $M_{Planck} = \infty$, gravity is decoupled and there is no restriction. The existence of this scale is surprising, because in an effective theory, one expects that the theory behaves better as one makes the coupling g weaker. But here this has the effect of making Λ smaller, so the theory is defined for lower ranges of energy. Note that if this were not the case, one could make the $U(1)$ weaker than gravity by taking g small enough, thus contradicting the original criterion. This is a very predictive criterion. For example, it predicts that the LHC will not find $U(1)$ interactions with very weak coupling constants.

3. Criteria on the geometry of the field space

In this case we have the most detailed set of conjectures. Suppose in our Lagrangian we have a set of scalar fields ϕ^i ,

$$\mathcal{L}_{eff} = g(\phi^i)_{ij} \partial_\mu \phi^i \partial^\mu \phi^j + \dots$$

The fields ϕ^i live in a manifold \mathcal{M} with metric $g(\phi^i)_{ij}$. The conjectures about these fields and the manifold \mathcal{M} are:

- All the coupling constants of the theory come from expectation values of these scalar fields, and can be varied at a finite cost in energy.
- The volume of \mathcal{M} , when suitable defined, is finite.
In the string theory examples, this follows from U dualities.

- The possible distances between pairs of points in \mathcal{M} are unbounded. In other words, \mathcal{M} has infinite diameter.

This property taken together with the previous one implies that \mathcal{M} has directions of negative curvature.

- Let $p, p_o \in \mathcal{M}$, and call $d(p, p_o)$ their distance in \mathcal{M} . As $d(p, p_o) \gg 1$, there appear in the effective Lagrangian light states with masses

$$m_p \sim A e^{-Bd(p, p_o)}$$

as we keep p_o fixed and vary p . Since $d(p, p_o)$ can grow without bound, there are directions on \mathcal{M} where the theory is singular and is typically described by a dual description which takes over.

- $\Pi_1(\mathcal{M}) = 0$.

One can argue that this is related to the fact that duality symmetries are realized as discrete gauge symmetries in different points of \mathcal{M} . A field theory counter-example for this is an axion which lives in a circle. So when an axion is coupled to quantum gravity, it should have a radial direction where the circle shrinks.

Our examples are all for supersymmetric theories, but we believe the criteria will survive if stable non-supersymmetric vacua are found in string theory. Clearly there should be many more criteria and the above three are just the tip of the iceberg.

References

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- [3] H. Ooguri and C. Vafa, “On the geometry of the string landscape and the swampland,” arXiv:hep-th/0605264.