

Aspects of Hypermultiplet Moduli Spaces

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We review recent progress in understanding the hypermultiplet moduli space arising from compactifications of type II strings on Calabi-Yau threefolds.

The couplings of the massless modes of type II string theory compactified on a Calabi-Yau threefold (CY_3) can be encoded in low energy effective actions (LEEA) with $\mathcal{N} = 2$ supersymmetry. These LEEA generally receive α' and g_s quantum corrections, both perturbatively and non-perturbatively from instantons. The α' -corrections can be determined from mirror symmetry, while determining the g_s -corrections is still unresolved.

Besides the supergravity gravity multiplet these LEEA contain n_V vector- and n_H hypermultiplets. Supersymmetry implies that the total moduli space \mathcal{M} of these theories factorizes into a local product

$$\mathcal{M} = \mathcal{M}_{VM} \otimes \mathcal{M}_{HM}, \quad (1)$$

where \mathcal{M}_{VM} and \mathcal{M}_{HM} are parameterized by the scalars of the VM and HM, respectively. Supersymmetry further dictates that \mathcal{M}_{VM} be a special Kähler (SK) manifold, and \mathcal{M}_{HM} a quaternion-Kähler (QK) manifold. The factorization (1) has the profound consequence that only those subsectors which contain the dilaton receive g_s corrections.

For both IIA and IIB compactifications the dilaton sits in a hypermultiplet. Thus understanding the g_s corrections to the LEEA requires understanding the quaternion-Kähler manifolds \mathcal{M}_{HM} .

The classical result for \mathcal{M}_{HM} can be obtained by the (classical) c-map [1] which relates the VM sector of the type IIA (IIB) to the HM sector of the type IIB (IIA) string compactification on the same CY_3 .

When going beyond this classical result the following observations are of crucial importance. First one notes that, in the presence of a suitable number of Peccei-Quinn isometries, the hypermultiplets parameterizing \mathcal{M}_{HM} have a dual description in terms of $\mathcal{N} = 2$ tensor multiplets,

$$\eta^I = \frac{v^I}{\zeta} + x^I - \bar{v}^I \zeta, \quad (2)$$

where v and x denote a $\mathcal{N} = 1$ chiral and $\mathcal{N} = 1$ tensor superfield, respectively. The (rigid) couplings between an arbitrary number of tensor multiplets can then readily be obtained using projective superspace [2, 3], with Lagrangian density

$$\mathcal{L}_{\text{TM}} = \int d\theta^2 d\bar{\theta}^2 \oint \frac{d\zeta}{2\pi i \zeta} H(\eta(\zeta), \zeta). \quad (3)$$

In order to couple the resulting tensor multiplet action to supergravity, one imposes invariance under superconformal transformations. This leads to the additional conditions that $H(\eta, \zeta)$ has no explicit ζ -dependence and is homogeneous of degree one (after taking the contour) [4]. The resulting expression is then conformally coupled to the Weyl multiplet [5] which contains the degrees of freedom of conformal supergravity. Gauge-fixing the $SU(2)_R$ and dilatational symmetries of the conformal theory and dualizing the tensors to scalars gives the hypermultiplet manifold \mathcal{M}_{HM} .

The off-shell description of the classical c-map was then obtained in [6], see also [7], and it was shown that

$$H^{\text{cl}}(\eta^0, \eta^I) = \frac{F(\eta^I)}{\eta^0}. \quad (4)$$

Here F is the prepotential of the dual special Kähler geometry evaluated on the tensor superfields (2), η^0 can be thought of as the universal hypermultiplet containing the dilaton, and the contour appearing in (3) is taken around the origin. Imposing constraints from string perturbation theory, it was shown [8] that H^{cl} receives a one-loop correction

$$H^{1\text{-loop}}(\eta^0, \eta^I) = \pm \frac{i\chi_E}{3\pi} \eta^0 \ln \eta^0. \quad (5)$$

Here the upper (lower) sign correspond to compactifications of the IIB (IIA) string respectively and χ_E is the Euler character of the CY_3 . It was further argued in [8] that there are no higher-loop corrections to the hypermultiplet moduli space.

Further analysis [9] indicates that the tensor multiplet framework is also suitable to incorporate a certain class of membrane instanton corrections to \mathcal{M}_{HM} ,

$$H^{\text{inst}}(\eta^0, \eta^I) = \sum_{\vec{q}} c(\vec{q}) \eta^0 \exp\left(\frac{-iq_I \eta^I}{2\eta^0}\right), \quad (6)$$

where $c(\vec{q})$ are, hitherto undetermined, numerical coefficients. Upon taking the contour integral H^{inst} gives rise to correction terms proportional to $e^{iq_I A^I} K_0(2|q_I v^I|/x^0)$, where the $A^I \equiv x^I/2x^0$ is a RR-scalar and K_0 is the modified Bessel function of the second kind. Upon expanding for small values of x^0 (which plays the role of the dilaton), one finds precisely the correct g_s dependence expected of a membrane instanton [10], including the perturbative fluctuations around the instanton. What remains is to determine the numerical constants $c(\vec{q})$. They could be determined by requiring regularity of the hypermultiplet moduli space, along the lines of [11]. This is currently being investigated [9].

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