Some new aspects of Heterotic - F-theory duality

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Introduction

In my talk I discussed about the results of the paper [1]. For a description of our real world, we need both up-type quark Yukawa couplings

$$\Delta W = \mathbf{10}^{ab}\ \mathbf{10}^{cd}\ H(\mathbf{5})^{e}\ \epsilon_{abcde} \tag{1}$$

and down-type quark (and charged lepton) Yukawa couplings

$$\Delta W = \bar{\mathbf{5}}_{a}\ \mathbf{10}^{ab}\ \bar{H}(\bar{\mathbf{5}})_{b}. \tag{2}$$

Here, we used a notation of effective theory with $SU(5)_{GUT}$ symmetry and $\mathcal{N}=1$ supersymmetry. Simple perturbative Yang–Mills interactions of open strings of Type IIA / IIB string theory may be able to give rise to the latter, but the up-type Yukawa couplings with $SU(5)_{GUT}$ indices contracted by an epsilon tensor are hard to be generated. Heterotic $E_8 \times E_8'$ string theory, G_2-holonomy compactification of 11-dimensional supergravity and F-theory compactification, however, are capable of generating both types of Yukawa couplings.

The matter chiral multiplets in supersymmetric compactification are identified with independent elements of bundle-valued cohomology groups in the Heterotic, Type I and Type IIB string theory. Net chirality of matter multiplets in chiral representations is expressed in terms of topological numbers such as Euler characteristics of vector bundles or pairing of D-brane charges in K-theory. In Type IIA string compactification on a Calabi–Yau orientifold, we know that one chiral multiplet is localized at each D6–D6 intersection, and it is straightforward to extend this picture to compactification of 11-dimensional supergravity on a G_2-holonomy "manifold" with A–D–E singularity. We have answers to the question above in a satisfactory level in all those theories. Surprisingly, though, such an effort to identify quarks and leptons in F-theory language went only halfway in 1990's, and almost came to a halt (at least to our knowledge), until the recent results of [2, 3]. Elementary degrees of freedom in F-theory can be described by (p, q) strings or $M2$-branes of 11-dimensional supergravity, and it may be possible, in principle, to identify chiral matter multiplets on 3+1 dimensions with some of their fluctuation modes. In practise, however, it is extremely difficult to disentangle complicated geometry of triple intersection of (p, q) 7-branes, or to maintain distinction between left-handed and right-handed fermions in Calabi–Yau 4-fold compactification of 11-dimensional supergravity down to 2+1 dimensions. Instead, the duality between the Heterotic string and F-theory [4, 5, 6, 7, 8] will be the most powerful tool in studying F-theory.

This article is along the line of this approach; the Heterotic string theory and the Heterotic–F-theory duality are used to study F-theory. The Heterotic string theory compactified on an elliptically fibered Calabi–Yau 3-fold $\pi_Z : Z \to B_2$ is dual to F-theory compactified on an elliptically fibered Calabi–Yau 4-fold $\pi_X : X \to B_3$ whose base 3-fold B_3 is a ¶^{1} fibration over B_2. Various matter multiplets in low-energy effective theory are identified with $H^1(Z; \rho(V))$ in Heterotic string description, where $\rho(V)$ is a vector bundle V in representation ρ. Cohomology groups on a fibered space can be calculated first on the fiber geometry, and later on the base geometry; except for certain cases

$$H^1(Z; \rho(V)) \simeq H^0(B_2; R^1\pi_Z{*}\rho(V)), \tag{3}$$

and the direct images $R^1\pi_{Z*}\rho(V)$ have their support only on curves in B_2. In the Heterotic–F duality, these support curves correspond to 7-brane intersections, and the sheaves on the curves should be those on the 7-brane intersection curves. Chiral matter multiplets are identified with global holomorphic sections of such sheaves (except for certain cases). Thus, by using the Heterotic–F duality, we can obtain the sheaves whose sections are identified with quarks and leptons. Direct images $R^1\pi_{Z*}\rho(V)$, therefore, are the information we would like to obtain from the Heterotic string theory.

Direct images of bundles in the fundamental representation $\rho(V) = V$ were obtained in 1990's [9, 10]. Those of bundles in the anti-symmetric representation $\rho(V) = \wedge^2 V$ have not been clearly described as sheaves so far in the last decade, apart from some developments in [11, 12] in the context of Heterotic theory compactification. Calculation of the direct images of $\wedge^2 V$, therefore, is one of the central themes of our work. This is by no means a minor problem. Both $\bar{\mathbf{5}}$ and $\bar{H}(\bar{\mathbf{5}})$ multiplets arise from $\wedge^2 V$ of an $SU(5)$ bundle V, and $H(\mathbf{5})$ from $\wedge^2 V^\times$, where $V^\times$ is the dual bundle of V. Without understanding the geometry associated with $R^1\pi_{Z*} \wedge^2 V$ and $R^1\pi_{Z*} \wedge^2 V^\times$, there is no way to understand the Yukawa couplings of quarks and leptons in F-theory.

We introduce a new notion,[1] covering matter curve, in order to deal with singularities that appear along matter curves. The direct image $R^1\pi_{Z*} \wedge^2 V$ is represented as a pushforward of a locally free rank-1 sheaf $\widetilde{\mathcal{F}}_{\wedge^2 V}$ on the covering matter curve for all the cases we have study i.e., rank $V = 3, 4, 5, 6$. (We should also note here that a minor assumption is made on structure of $R^1\pi_{Z*} \wedge^2 V$

[1] Essentially the same object was already introduced in [12].

around a particular type of singularity for the rank $V = 4$ case.) Divisors determining the locally free rank-1 sheaves are determined in terms of data defining spectral surfaces.

The description of the sheaves $R^1\pi_{Z*}\rho(V)$ are translated into language of F-theory. Dictionary for translation between the Heterotic and F-theory was almost established in 1990's, but we find that the dictionary has to be refined in some aspects. Improvements include

- precise map between the moduli of spectral surface in Heterotic theory description and complex structure moduli in F-theory, and
- refinement of the correspondence between the discrete twisting data of vector bundles in Heterotic theory description and four-form fluxes in F-theory description; we are basically along the line of the idea laid out in [13], but our statements differ in some aspects from a recent paper [2].

Using the precise map between the two moduli space, we find that most of the components of the divisors determining the sheaves $\mathcal{F}_{\rho(V)}$ (and all that we identified) correspond to codimension-3 singularities in B_3.

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