F-theory, GUTs and Chiral Matter

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1. Motivation

This is a write-up of a two hour talk on [1],[2]. The discussion is aimed at non-experts and may be useful for people new to the subject. Related work can be found in [3, 4, 5].[1]

There are many ways to engineer Standard Model-like theories in string theory. The class of models in this talk are based on two principles, namely (1) local model building and (2) built-in unification. Let us discuss these two principles in more detail.

(1) *Local model building.* Gauge groups are usually localized on branes in string theory. Charged chiral matter arises from intersections of these branes. This means that the matter we observe at particle accelerators may all be localized in the extra dimensions, and we may try to construct 'local models' in which we only study a small neighbourhood of the brane. What happens in the rest of the extra dimensions is described by some unknown parameters in the Lagrangian, whose values are set by the UV dynamics which we have not yet included in our local description. We can state this more precisely by saying that we would like the existence of a decoupling limit $M_{pl,4}/\mu \to \infty$ while keeping $g^2_{YM}(\mu)$ fixed.[2] This does not happen in generic models with branes. If we scale up

[1] Many interesting papers discussing phenomenological aspects of F-theory have recently appeared; see [6, 7, 8, 9, 10, 11, 12].

[2] This is actually too strong a requirement and has to be relaxed a bit, since $U(1)$ couplings are not asymptotically free and may have an interesting dependence on the compactification. An interesting compactification effect on the $U(1)$'s was discussed in [13]. Rather we will assume such decoupling for all the couplings that are asymptotically free.

dim	*stringy realization*
10d	$E_8 \times E_8$ heterotic string
9d	strongly coupled type I'
8d	F-theory on ALE
7d	M-theory on ALE
6d	IIa on ALE/IIb with NS5

Table 1: *Branes with exceptional gauge symmetry in string theory.*

the volume of the internal space, we are typically also forced to scale up the cycles on which the branes are wrapped, turning off the gauge couplings.

Focussing on such local scenarios allows us to address questions of particle physics without knowing the complicated dynamics of the whole internal Calabi-Yau. It also alleviates the landscape problem in that it should lead to much fewer and more predictive models. However unfortunately it is not enough; in local models we can engineer the MSSM with arbitrary parameters from open strings, as well as any similar quiver model [14], and we will likely never be able to rule out that these models cannot be compactified. In order to get something more interesting, we need some additional top-down input beyond requiring that the MSSM can be realized in such a scenario. This is the purpose of the second principle.

(2) *Built-in unification.* This means we would like to construct GUT models with an $SU(5)$, $SO(10)$, or E_6 gauge symmetry. One may even extend this list to E_7 and E_8 gauge symmetry provided the symmetry is realized in higher dimensions and broken in four dimensions. However such models can not be realized using perturbative open strings:

- gauge groups are always $U(n)$,$SO(n)$ or $Sp(n)$, ruling out E_6;
- for $SO(10)$, quarks and leptons sit in the **16** of $SO(10)$ which is a spinor representation, which again cannot be realized with perturbative open strings;
- for $SU(5)$ the problem is that the top quark Yukawa coupling uses the epsilon tensor of $SU(5)$, and so can only be realized non-perturbatively, whereas the bottom quark Yukawa's are realized perturbatively. However in Nature the top quark Yukawa is of order one, and the bottom Yukawa is hierarchically smaller.

On the other hand, in the old E_8 heterotic string there was no problem with any of these issues. This is for purely group theoretic reasons: the gauge indices of charged matter live in the coset

E_8/G, where G is the GUT group, and the algebra of E_8 (and also the other exceptional groups) allows for the spinor representation of $SO(10)$ or the epsilon tensor in the up type Yukawa coupling for $SU(5)$. So the upshot is we do not necessarily need the heterotic string, but we do need branes with exceptional gauge symmetry rather than conventional D-branes. These actually appear in settings other than the heterotic string. In the table we gave a list of branes with exceptional gauge symmetry in string theory. However several requirements cut down this list further. Five-branes are too much localized, they can always be separated in the extra dimensions and thus have no charged chiral matter.[3] The requirement of local model building rules out the heterotic string and type I', and even if we were interested in global models the type I' set-up doesn't have any good constructive tools. A similar problem plagues the M-theory models, and in addition the M-theory models have phenomenological problems: they are still too much localized in that even though they yield chiral matter, their Yukawa couplings have to be generated through membrane instantons. Thus our two principles lead us to consider the entry in the middle of the table: F-theory. Fortunately, there are some powerful constructive techniques available here from algebraic geometry, which we will briefly review later.

So why were such F-theory models not pursued previously? After all, brane models were constructed in IIb which is a close cousin, and F-theory had also appeared in discussions on moduli stabilization and Randall-Sundrum type scenarios. Essentially there were two problems. The first is that no one had previously shown how to engineer charged chiral matter in F-theory. There was also a second serious problem, in that there seemed to be no good mechanism for breaking the GUT group in F-theory. We will explain below how these problems are addressed.

2. Overview of model building with F-theory

2.1. Elliptic fibrations

F-theory [15] is basically a book-keeping device to describe vacua of IIb string theory with a varying axio-dilaton. The $Sl(2,Z)$ duality group of IIb string theory acts as fractional modular transformation on

$$\tau = a + ie^{-\phi} \tag{2.1}$$

so that τ may be identified with the modular parameter of an auxiliary torus (an 'elliptic curve.') Thus we can formally attach this torus at each point in the IIb space-time and speak of twelve-dimensional compactifications of F-theory. Essentially this is a clever change of variable: instead of specifying τ directly, which can get rather complicated due to the monodromies acting on τ, we specify the torus directly. The T^2 is typically written in Weierstrass form, i.e. described as an

[3] This argument does not exclude getting chiral matter from general 5d or 6d theories; eg. a 6d theory with both vector and hypermultiplets would work.

equation of the form

$$y^2 = x^3 + fx + g \tag{2.2}$$

which can be done globally on the IIb space-time. Instead of specifying τ, we specify f and g. The area of the torus has no meaning in F-theory and should be taken zero. In these variables, supergravity solutions with 7-branes (which have varying dilaton) are much easier to describe, as we review next.

Let us label the one-cycles of the elliptic fiber as a and b, with $a \cap b = 1, a \cap a = b \cap b = 0$. On a subset of real codimension 2 on the IIb space-time (namely $\Delta = 4f^3 + 27g^2 = 0$) , the elliptic fiber pinches due to a 1-cycle $\gamma = pa + qb$ shrinking to zero. As we go around this locus, which is called the discriminant locus, the one-cycles undergo a monodromy following the Picard-Lefschetz formula:

$$\delta \to \delta + (\delta \cap \gamma)\gamma \tag{2.3}$$

Let us denote the holomorphic one-form on the T^2 by Ω. Then we can express τ as $\int_b \Omega / \int_a \Omega = \tau$. With a little algebra, we see that the monodromy acts on τ as

$$\tau \to K_{[p,q]}\tau, \qquad K_{[p,q]} = \begin{pmatrix} 1+pq & p^2 \\ -q^2 & 1-pq \end{pmatrix} \tag{2.4}$$

We claim this identifies the locus with a (p, q) 7-brane, i.e. a type of 7-brane on which a (p, q) string can end. To see this, consider the case of a $(1, 0)$ brane. In this case we have $\tau \to \tau + 1$ as we go around a 7-brane, i.e. $a \to a + 1$ and $e^{-\phi}$ invariant. This is precisely the right monodromy for a single $D7$-brane (it means that the 7-brane sources one unit of RR flux). By applying $Sl(2, Z)$ duality transformations, we recover the other cases.

Two (p, q) 7-branes are said to be mutually local if

$$\begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix} = 0 \tag{2.5}$$

Otherwise they are said the be mutually non-local. In the latter case, the degrees of freedom on the branes are not independent of each other and the combined system is generically strongly coupled. In the former case, the degrees of freedom are independent and we can always make the dilaton small and get a weakly coupled system.

There is another view on F-theory from M-theory, as follows. M-theory on T^2, in the limit that the area goes to zero, is equivalent to type IIb on a circle of radius $R = 1/A$, with axio-dilaton given by the modular parameter of the T^2. By fibering and taking $A \to 0$ we can get general F-theory compactifications with 3+1 dimensional Poincaré invariance.

2.2. Abelian gauge fields

As usual in type II settings, abelian tensor fields on the brane arise as zero modes of the ten-dimensional tensor fields localized on the soliton. Since the gauge symmetry exists already in eight dimensions, they must come form zero modes of B_{NS} and B_{RR}. However B_{NS} and B_{RR} are not invariant under the monodromies; they form a doublet under the $Sl(2,Z)$ duality group. In keeping with the philosophy of F-theory, we want to reformulate this in terms of an object that can be specified globally, and is not subject to monodromies. This can be done by encoding the two-form fields in a three-form field:

$$C_{(3)} \sim (B_{RR} - \tau B_{NS}) \wedge (dx - \tau dy) + c.c. \tag{2.6}$$

where x and y are the two coordinates on the T^2. Note this has two indices on the IIb space-time and one index on the elliptic fiber. Three-form fields with different numbers of indices in the base and the fiber do not exist in F-theory. This three-form field $C_{(3)}$ is $Sl(2,Z)$ invariant and can be defined globally. By compactifying on S^1 and going to M-theory, this corresponds to the usual $C_{(3)}$ field of eleven-dimensional supergravity, except that some components are turned off in the F-theory limit. The four-form flux of this tensor field is conventionally called the G-flux.

Now to get a 7-brane gauge field, we need to expand $C_{(3)}$ in terms of real harmonic two-forms, with one index on the base and one index on the fiber (so that the gauge field index lives in the IIb space-time):

$$C_{(3)} = A_I \wedge \omega^I \tag{2.7}$$

Consider for instance F-theory compactified on an elliptically fibered $K3$-surface. There are 22 harmonic 2-forms, but one of these has two indices on the base and one has two indices on the fiber. Thus there are 20 harmonic forms we can expand in (which we can further subdivide as $20 = 18 + 2$), which yields the same number of $U(1)$ gauge fields as expected from the heterotic string on T^2. Note that these harmonic forms cannot be normalizable in the local supergravity solution for a 7-brane: if so we would get an independent gauge field for each singular fibre, but there are 24 singular fibers in an elliptically fibered $K3$ and only 20 gauge field from the 7-branes.

Later we will specialize to vacua with 3+1 dimensional Poincaré invariance and $N = 1$ supersymmetry. For this F-theory needs to be compactified on an elliptically fibered Calabi-Yau four-fold and the G-flux needs to satisfy certain conditions, which were derived by Becker and Becker:

$$\begin{aligned} G &\in H^{2,2}(CY_4) \qquad && (\mathrm{F-term\ equation}) \\ J \wedge G &= 0 && (\mathrm{D-term\ equation}) \end{aligned} \tag{2.8}$$

Here J is the Kähler form on the Calabi-Yau. Note that these conditions are similar to the ASD equations on the internal worldvolume of a 7-brane, $F^{2,0} = 0 = J \wedge F^{1,1}$, and reduces to them in a weak coupling limit.

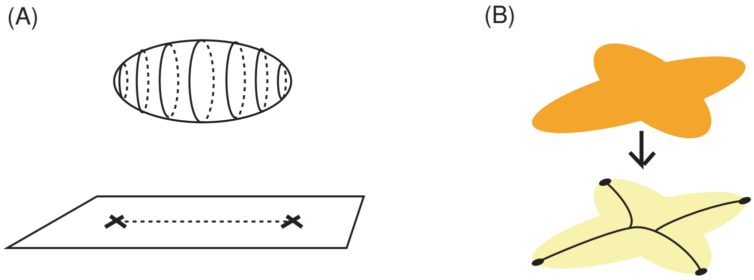

Figure 1: *An open fundamental string in type IIb lifts to a membrane wrapping an exceptional cycle in F-theory. Some W-bosons may correspond to ground states of multi-pronged (p,q) strings .*

We further want to specialize to local scenarios. This can be expressed by saying that the normal bundle to the cycle S wrapped by the gauge 7-branes should be negative, but let us instead use a slightly weaker requirement. Deformations of the 7-branes correspond to the parameters in f and g in (2.2), i.e they are complex structure deformations of the Calabi-Yau four-fold. These can be equivalently parametrized by harmonic $(3,1)$ forms, and decomposing

$$\delta\Omega^{3,1} = \Phi_I \wedge \omega^I \tag{2.9}$$

shows that they correspond to harmonic $(2,0)$ forms on S, the four-cycle wrapped by the 7-brane. In order to eliminate such massless adjoint fields, we need $h^{2,0}=0$, which essentially means that S should be a Del Pezzo surface. (Hirzebruch surfaces and the Enriques surface are also allowed by this argument, but less phenomenologically interesting). A del Pezzo surface dP_k is a complex surface which can be constructed as a blow-up at $k \leq 8$ points on $\mathbf{P}^2$.

2.3. Non-abelian gauge fields

The non-abelian gauge bosons, as usual in type II settings, arise from BPS states that can not be seen in supergravity and have to be added by claiming some knowledge of the UV completion.

Let us consider two parallel D7-branes. The non-abelian gauge bosons that enhance the symmetry to $SU(2)$ come from open strings stretched between the two 7-branes. How is this seen in F-theory? Consider the path associated to an open string stretching between the branes. On top of each point of this path we can associate a 1-cycle of the T^2 fiber, which we take to be the $(1,0)$ cycle. On the left and right ends of the path, this $(1,0)$ cycle shrinks to zero. Altogether then we reconstruct a topological S^2. Using the M-theory perspective, we can wrap an $M2$-brane on this S^2, which turns into the fundamental open string as we go to F-theory. As the we let the

ord(f)	*ord (g)*	*ord(Δ)*	*fiber type*	*singularity type*
≥ 0	≥ 0	0	smooth	$-$
0	0	n	I_n	A_{n-1}
≥ 1	1	2	II	$-$
1	≥ 2	3	III	A_1
≥ 2	2	4	IV	A_2
2	≥ 3	$n+6$	I_n^*	D_{n+4}
≥ 2	3	$n+6$	I_n^*	D_{n+4}
≥ 3	4	8	IV^*	E_6
3	≥ 5	9	III^*	E_7
≥ 4	5	10	II^*	E_8

Table 2: *Kodaira classification of singularities of elliptic fibrations, indicating the order of vanishing of* Δ, f *and* g.

7-branes approach each other, the S^2 shrinks to zero and the Calabi-Yau fourfold develops an A_1 singularity. (The IIb space-time is still perfectly smooth, only when we add the elliptic fibration do we see the singularity). As the S^2 shrinks to zero, the ground states of the wrapped $M2$ brane or open fundamental string become massless, yielding the off-diagonal components of an $SU(2)$ vector multiplet. We get both W^+ and W^- by reversing the orientation of the membrane.

The type of singularities in an elliptic fibration were classified by Kodaira (see table 2). The elliptic fibration may develop an ADE singularity by letting various branes approach each other, and one would naturally expect that by wrapping $M2$ branes on the vanishing cycles one gets an enhanced ADE gauge symmetry. How do we understand these more general situations from the IIb space-time? It turns out that the exceptional cycles of an ALE do not necessarily project to open strings with two ends, but may yield so-called multi-pronged strings with multiple ends. This is the key to getting the exceptional groups and can happen when the dilaton cannot be taken small. All the ADE Lie algebras have been reproduced from such (generally multi-pronged) strings. For instance the roots of E_8 can be recovered from a configuration of seven A-branes, one B brane, and two C-branes, where $A = (1, 0), B = (1, -1), C = (1, 1)$, see figure 2. Configurations for the type D Lie algebras are similar except that they just use one C brane instead of two. A B-brane and C-brane can be combined into an orientifold plane and yield weak coupling limits, but this is not possible for the exceptional cases.

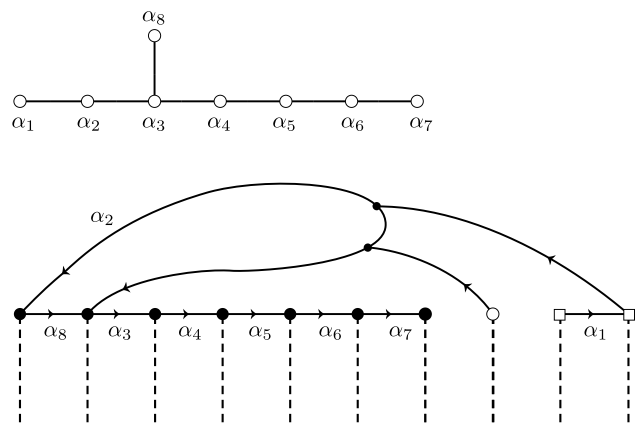

Figure 2: *Representation of the fundamental roots of E_8 from [16]. Here a cross denotes an A-brane, a circle denotes a B-brane, and a box denotes a C-brane.*

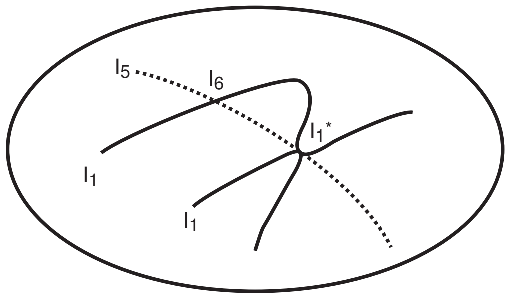

Figure 3: *An I_5 locus in the IIb space-time, intersecting an I_1 locus over an I_6 degeneration and an I_1^* degeneration.*

2.4. Charged matter

Now we'd like to understand how to get charged matter. This should arise form the intersections of 7-branes. When the discriminant locus self-intersects, the order of vanishing of Δ increases and the singularity type of the fibration will be enhanced, i.e. there are extra vanishing cycles sitting over the intersection of the 7-branes. By wrapping membranes on them and quantising, we get six-dimensional hypermultiplets living on the intersection, basically because that's the only possibility given the symmetries. The hypermultiplets naturally sit in a kind of generalized bifundamental representation of the gauge groups on the 7-branes.

Let us consider some examples that are relevant for GUT model building. We take an I_1 locus (which has a single pinched cycle on the T^2) with an I_5 locus (which has an A_4 singularity, hence an $SU(5)$ gauge group; the T^2 has degenerated to five S^2's intersecting according to the affine A_4 Dynkin diagram). Over the intersection the singularity can get enhanced in two ways: either to I_6, which has a $SU(6)$ Lie algebra worth of vanishing cycles, or an I_1^* singularity, which has a $SO(10)$ Lie algebra worth of vanishing cycles. Decomposing

$$Ad(SU(6)) = Ad(SU(5)) + \mathbf{5} + \bar{\mathbf{5}} + \mathbf{1} \tag{2.10}$$

we see that the membranes wrapped on the extra vanishing cycles naturally yield a hypermultiplet in the **5**, as is familiar from perturbative D-branes. Similarly decomposing

$$Ad(SO(10)) = Ad(SU(5)) + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{1} \tag{2.11}$$

we see that we naturally get a hypermultiplet in the **10** on the intersection. As another example, consider the intersection of an I_1 and an I_1^* singularity, which yields an enhancement to E_6. Decomposing

$$Ad(E_6) = Ad(SO(10)) + \mathbf{16} + \overline{\mathbf{16}} + \mathbf{1} \tag{2.12}$$

we see that the 'bifundamental' is in this case a hypermultiplet in the spinor representation, which we could not get in perturbative IIb. It should be noted that intersections of branes which are not mutually local are highly non-transversal.

All this was known in the nineties. On the other hand, before [1] it was not understood how to get chiral matter in F-theory. Of course this is a crucial issue for model building because quarks and leptons are chiral.

To understand how to get chiral matter, one may ask how we get chiral matter in type IIb string theory. This was also already understood in the nineties, but somehow did not make its way into the F-theory literature. The intersection of the 7-branes is six dimensional, so we have two dimensions left to make the hypermultiplet chiral. We compactify on a Riemann surface and write the six-dimensional hypermultiplet spinors as $\psi \otimes \chi$ where χ are four-dimensional spinors and ψ are spinors on the Riemann surface. The Dirac action splits in a four-dimensional piece and a piece for the fermions on the Riemann surface:

$$\int_\Sigma \psi_+^{\bar{a}} \bar{\partial}_{A_1-A_2} \psi_+^a + \psi_-^{\bar{a}} \partial_{A_1-A_2} \psi_-^a \tag{2.13}$$

Here a corresponds to the $(N_1, \bar{N}_2)$ and $\bar{a}$ corresponds to the $(\bar{N}_1, N_2)$. When no flux is turned on, we have two fermionic zero modes $\psi_+^a, \psi_+^{\bar{a}}$ and their complex conjugates. The spinors of a six-dimensional hypermultiplet may be decomposed as

$$\Psi^a = \chi_\alpha \otimes \psi_+^a + \bar{\chi}_{\dot{\alpha}} \otimes \psi_-^a, \qquad \Psi^{\bar{a}} = \bar{\chi}_{\dot{\alpha}} \otimes \psi_-^{\bar{a}} + \chi_\alpha \otimes \psi_+^{\bar{a}} \tag{2.14}$$

Here χ_α is a four-dimensional Weyl spinor, ψ are the complex spinors on Σ. Thus to get chiral matter we want to have zero modes for $\psi_+^a, \psi_-^{\bar{a}}$, but not for $\psi_-^a, \psi_+^{\bar{a}}$. The way to do this is familiar from the Schwinger model of two-dimensional electro-dynamics: since the fermions do not transform in a real representation of the gauge group, we can introduce an asymmetry by turning on a background flux $F = F_1 - F_2$ through Σ, and the net number of chiral fermion zero modes will then be given by

$$N_{gen} = \frac{1}{2\pi}\int_\Sigma F \tag{2.15}$$

(One can actually do better in situations with $N = 1$ SUSY and compute the absolute number, rather than just the net number, but we will not discuss that here). Clearly this must also be true in F-theory. Thus our main task consists of two things: (1) we need to formulate this in F-theory language, by using the G-flux; and (2) we need to generalize it to branes which are not mutually local. And finally of course we need to implement it in explicit models.

As for the first, let us consider two stacks of intersecting D-branes. The overall $U(1)$'s of these two stacks are encoded as

$$G \sim F_1 \wedge \omega_1 + F_2 \wedge \omega_2 \tag{2.16}$$

Now let's consider a fundamental string stretching between these two stacks, and lift it to a membrane wrapped on a vanishing cycle α. This is one of the extra vanishing cycles which sits over the intersection, and corresponds to a root of $SU(N_1 + N_2)$ which is not in $SU(N_1) \times SU(N_2)$. Then

$$\int_\alpha \omega_1 = +1, \qquad \int_\alpha \omega_2 = -1 \tag{2.17}$$

since the integral of ω over α gives the charge of the wrapped membrane under the $U(1)$ associated to ω. Now we can construct a four-cycle $\Sigma \times \alpha$, and we can write the net number as

$$N \; = \; \frac{1}{2\pi}\int_\Sigma F_1 - F_2 \; = \; \frac{1}{2\pi}\int_{\Sigma\times\alpha} G \tag{2.18}$$

The last expression can be used generally, even when there exist no harmonic forms in the class of ω_1, ω_2 or even $\omega_1 - \omega_2$. (It is in fact crucial in GUT models that such ω do not exist because we don't want extra massless $U(1)$'s in the low energy spectrum, but we do want their fluxes in order to get chiral matter).

For the second, we can use the same procedure of integrating over suitable vanishing cycles, even though the charges of the extra states may not be ± 1. Eg. in the case of an I_5 and I_1 locus intersecting over I_1^*, we get

$$N \; = \; \frac{1}{2\pi}\int_\Sigma 2F \; = \; \frac{1}{2\pi}\int_{\Sigma\times\alpha} G \tag{2.19}$$

because the extra roots (similar to the root α_2 in figure 2) carry charge ± 2 under the overall $U(1)$'s associated to the intersecting 7-branes.

It is interesting to compare this with the heterotic string. Here we have a Calabi-Yau three-fold Z and a holomorphic bundle V which breaks the E_8 gauge group in ten dimensions. There is a famous formula for the number of generations in that context:

$$N_{gen} = \frac{1}{2}\int_Z c_3(V) \tag{2.20}$$

As shown in [1], under F-theory/heterotic duality this turns precisely into the F-theory expression for the net number.

2.5. Explicit models

To conclude, we still need a method for constructing models. Constructing local Calabi-Yau four-folds was explained by Freedman-Morgan-Witten, and turns out to be remarkably easy. Our four-fold will be an ALE fibration over a del Pezzo surface S, which we get to choose. We write the equation of a deformed E_8 singularity as

$$y^2 = x^3 + a_0 z^5 + a_2 x z^3 + a_3 y z^2 + a_4 x^2 z + a_5 x y \tag{2.21}$$

This can easily be embedded in an elliptic fibration (2.2) by adding some extra terms which are subleading at the singularity. Here the a_i are certain polynomials on the del Pezzo surface S, i.e they are sections of line bundles on S. Note that when all $a_i = 0$ except for a_0, we have an E_8 singularity, but when $a_5 \neq 0$ we generically have an A_4 singularity, so this corresponds to an E_8 gauge theory which is broken to an $SU(5)$ GUT model through compactification. To specify the ALE fibration, we need to specify a class $\eta \in H^2(S)$, and we need to choose five polynomials with Chern classes

$$a_i \sim \eta - i\, c_1(TS) \tag{2.22}$$

This completely specifies the local geometry, i.e. the 7-branes and their intersections. Of course η should be sufficiently positive so that the sections a_i exist. In addition we need to specify a flux in order to get chiral matter. These fluxes are specified by a (half)-integer λ. Due to a formula of Curio's, the net amount of chiral matter is given by

$$N_{gen} = \lambda\, \eta \cdot_{dP} (\eta - 5c_1) = \lambda \int_{\Sigma_{\mathbf{10}}} \eta \tag{2.23}$$

where $\Sigma_{\mathbf{10}} = \{a_5 = 0\}$ is the matter curve on which the hypermultiplet in the **10** is localized. For the case of $SU(5)$ models, λ should be of the form $\frac{1}{2}$+integer. It's an easy exercise to make your own three-generation model using these formulae. Some examples may be found in [1].[4]

[4] In v1 of [1] it is incorrectly stated that this flux is not primitive. An error in the expression for the flux was pointed out to us by Taizan Watari.

3. Breaking the GUT group

We succeeded in constructing four-dimensional GUT gauge groups and chiral matter. However this is not quite what we want: at low energies we only observe the Standard Model gauge group, $SU(3) \times SU(2) \times U(1)$. We still need a mechanism for breaking the GUT group, but in such a way that we don't screw up the nice predictions like gauge coupling unification. There are basically three ideas for doing this:

1. *Adjoint Higgses.* This is a priori allowed in F-theory but yields a conventional four-dimensional GUT model with all the associated problems. In addition, this would not be a local model because the existence of an adjoint means that the 7-brane is allowed to explore the whole internal space.
2. *Discrete Wilson lines.* This is the primary method for breaking the GUT group in the heterotic string, or on manifolds of G_2 holonomy in M-theory. However in local models in F-theory, the seven-brane is necessarily wrapped on a four-cycle with trivial fundamental group, so this method is not available for us.
3. *$U(1)$ fluxes with a hypercharge component.* This method is a priori also available in the heterotic string, but turns out to spoil unification. However, due to a mechanism discovered in [13], it turns out this option is available in F-theory.

Let us elaborate on the last mechanism. If we turn on an internal flux for hypercharge then the gauge group will break to the commutant of Y in $SU(5)$. This is precisely the Standard Model gauge group $SU(3)\times SU(2)\times U(1)$, and to leading order the $SU(5)$ relations between the gauge couplings will be preserved, so at first sight this looks promising. So why is this not usually claimed to be the solution to breaking the GUT group in the heterotic string? This is because in the heterotic string there is a coupling

$$\int d^{10}x \, (dB + A \wedge F)^2 \tag{3.1}$$

As a result of this coupling, if we turn on an internal hypercharge flux, the four-dimensional hypercharge field will swallow an axion (a zero mode of the B-field) and pick up a mass. Analogously in F-theory there is a Chern-Simons coupling to RR forms:

$$\mathscr{L} \sim \int C_4 \wedge G \wedge G \tag{3.2}$$

Now we expand

$$G = F_Y \wedge \omega^Y + G_{int}, \qquad C_4 = C_2^M \wedge \beta_M \tag{3.3}$$

where β^M is a basis for $H^2(B_3)$, B_3 is the base of the Calabi-Yau fourfold, and G_{int} is the internal G-flux which has all its indices on the four-fold. This leads to

$$\mathscr{L} \sim \Pi^Y_M \int d^4x \, F_Y \wedge C_2^M, \qquad \Pi^Y_M = \int_{CY_4} \beta_M \wedge \omega^Y \wedge G_{int} \tag{3.4}$$

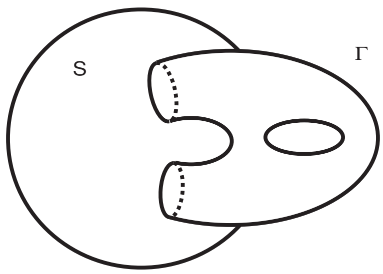

Figure 4: *If we thread fluxes through cycles which are trivializable in the bulk, then the overlap with zero modes of bulk axions vanishes.*

This is a Stückelberg coupling for the hypercharge gauge field. So we are in danger of generating a mass for hypercharge, which is certainly not what we want. If we turn on an internal hypercharge flux, then we have

$$\Pi_M^Y \sim \int_S i^* \beta_M \wedge F_Y \tag{3.5}$$

Now in F-theory we have a possibility that we didn't have in the heterotic string. In F-theory, $\Pi_M^Y = 0$ for all M does not imply that the internal hypercharge flux must be zero. Instead it is equivalent to the statement that the Poincaré dual two-cycle Ξ of F_Y in S becomes a boundary when embedded in B_3, see figure 4. I.e. it is a topological condition on the compactification of the local model. This happens quite generically, but not in F-theory duals of the heterotic string. Essentially the same mechanism for getting a massless hypercharge in models with branes at singularities was first discovered in [13].

This mechanism for breaking the GUT group also does not lead to a doublet-triplet splitting problem. If we arrange the fluxes correctly, there are basically no four-dimensional $SU(5)$ partners of the Higgs doublets. In this aspect it is similar to breaking by discrete Wilson lines.

Now a pure hypercharge flux actually turns out to give the wrong spectrum. However there are additional (massive) $U(1)$ symmetries available from the flavour branes, which commute with $SU(5)_{GUT}$. So instead one has to turn on a flux for a massive $U(1)$ symmetry, which is not hypercharge but has a hypercharge component. The fact that we may turn on such a flux may sound strange, but one may check [2] that the equations of motion (which require the G-flux to be harmonic, primitive and of type $(2,2)$) can in fact be satisfied. We refer to [2] for further details.

4. Phenomenological signatures

I'd like to briefly discuss three signatures of the models we have described. They are: monopoles, threshold corrections to unification, and proton decay.

4.1. GUT monopoles

One of the classic predictions of conventional unification models is the existence of monopoles carrying hypercharge. Let us recall the argument. We have the long exact homotopy sequence

$$\ldots \to \pi_2(G) \to \pi_2(G/H) \to \pi_1(H) \to \pi_1(G) \to \ldots \tag{4.1}$$

Here G is the GUT group and H is what's left after breaking. If we use an adjoint Higgs to break $SU(5)$, then we have

$$H = [SU(3) \times SU(2) \times U(1)]/Z_6 \tag{4.2}$$

Monopoles are classified by $\pi_2(G/H)$. Since $\pi_2(G) = \pi_1(G) = 0$, we see that they can be equivalently classified by $\pi_1(H)$ (which corresponds to the monodromy around a Dirac string). Thus these monopoles carry hypercharge, as well as an additional Z_3 colour and Z_2 electro-weak charge.

How do we find such monopoles in our GUT models? Recall that the Poincaré dual of the GUT breaking flux on the del Pezzo surface is a class Ξ which is the boundary of a 3-chain Γ in the IIb space-time. One can wrap a $D3$-brane on Γ and check that when we turn on an internal flux for hypercharge, the resulting four-dimensional particle carries magnetic hypercharge.

In addition there are other solitons: strings from $D3$ branes wrapped on 2-cycles of the del Pezzo, domain walls interpolating between vacua with and without G-flux. It might be interesting to explore their phenomenological consequences

4.2. Threshold corrections

I've argued that there is a mechanism for breaking the GUT group that preserves the $SU(5)$ relations between the gauge couplings. However there are many heavy states charged under $SU(5)$ and integrating them out will give small corrections to the couplings at the GUT scale. There are basically two types of corrections:

1. loop corrections from KK modes of the eight-dimensional gauge theory. These can be expressed as Ray-Singer torsion of the compactification manifold and matter curves;
2. contributions from the massive excitations of open strings that were not included in the eight-dimensional gauge theory (analogous to 'gravitational smearing' in 4D GUT models). Fortunately the leading contributions are constrained by non-renormalization conjectures.

The leading corrections can be computed/estimated. They are of order a few percent and come with varying signs, and so can be consistent with the known values of the couplings at low energies. See [2] for details.

4.3. Proton decay

Generic GUT models have catastrophic proton decay. The basic problem is that the leptons and Higgses have the same quantum numbers, so that if we have down type couplings

$$\mathbf{10}_m \cdot \bar{\mathbf{5}}_m \cdot \bar{\mathbf{5}}_h \tag{4.3}$$

then we also expect R-parity violating couplings

$$\mathbf{10}_m \cdot \bar{\mathbf{5}}_m \cdot \bar{\mathbf{5}}_m \tag{4.4}$$

which lead to proton decay. Even if these are absent by R-parity, exchange of higgsino colour triplets leads to baryon number violating dimension five operators $d^2\theta QQQL$ and $d^2\theta UDUE$. The classic signature for this type of decay is $p \to K^+\bar{\nu}$. With our mechanism for breaking the GUT group, there are no four-dimensional colour $SU(5)$ partners of the Higgsinos, however there is a tower of Kaluza-Klein modes with the same quantum numbers.

The basic idea for eliminating dimensions four and five operators is to use localization of the wave functions in the extra dimensions. Such ideas appeared in the phenomenology literature around '99 and in the context of F-theory unification models in [17]. (In this paper the precise F-theory description of chiral matter and couplings was not yet understood, but the qualitative picture was deduced using F-theory/heterotic duality.) The ideas of [17] can be generalized slightly (and a general prescription for $E_8 \to SU(5)_{GUT}$ models in (2.21) is given in [2]) but for the purpose of this talk, let me assume that the dimension four and five operators are eliminated and move on to the dimension six operators.

The dimension six proton decay operators are mediated by massive gauge bosons in the representation $(2,3)_{-5/6}$, leading to the decay $p \to \pi^0 e^+$. These are KK modes of the eight-dimensional gauge fields, i.e. they are bulk modes and so we cannot appeal to localization of the wave functions to reduce their effect. Of course such operators are suppressed by $1/M_{GUT}^2$ so we would not necessarily expect them to lead to any problems. The amplitude is hard to calculate exactly because it depends on the profile of the zero modes as well as the Green's function for the Laplacian on S. However one may get the parametric dependence. The leading term in the limit $\alpha_{GUT} \to 0$ turns out to be[5]

$$\mathscr{M} \sim \alpha_{GUT} \log \alpha_{GUT}^{-1} \frac{J_\mu \tilde{J}^\mu(0)}{M_{GUT}^2} \tag{4.5}$$

where $J^\mu = \bar{\psi}\gamma^\mu\psi$ and ψ corresponds to the $\mathbf{10}$ or $\bar{\mathbf{5}}$. The analogous amplitude in four-dimensional GUT models is

$$\mathscr{M} \sim \alpha_{GUT} \frac{J_\mu \tilde{J}^\mu(0)}{M_{GUT}^2} \tag{4.6}$$

[5]This differs from the estimate given in v1 of [2], which was a bit too naive.

so there is a mild parametric enhancement of proton decay in F-theory. In practice with $\alpha_{GUT} \sim 1/25$ this is not so big however, and it is not clear how the numerical coefficient compares with that of four-dimensional models, so the best guess is that there is only a minor enhancement and such decay won't be seen any time soon.

5. Outlook

The F-theory models we described here seem to be the first new class of models since the heterotic string that can successfully incorporate unification. Two intriguing aspects are the option of constructing local models (which allows for scenarios not available in the heterotic string, like gauge mediation), and the method for breaking the GUT group using fluxes. Despite some crucial differences, there are also many similarities with the heterotic models and also with models based on G_2 manifolds (assuming they exist, which has not yet been shown). We expect that exploring the relations between these models will lead to many new insights. Further, if supersymmetric unification still holds up after the LHC then with the state of knowledge today the F-theory models look like some of the most viable phenomenological candidates.

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