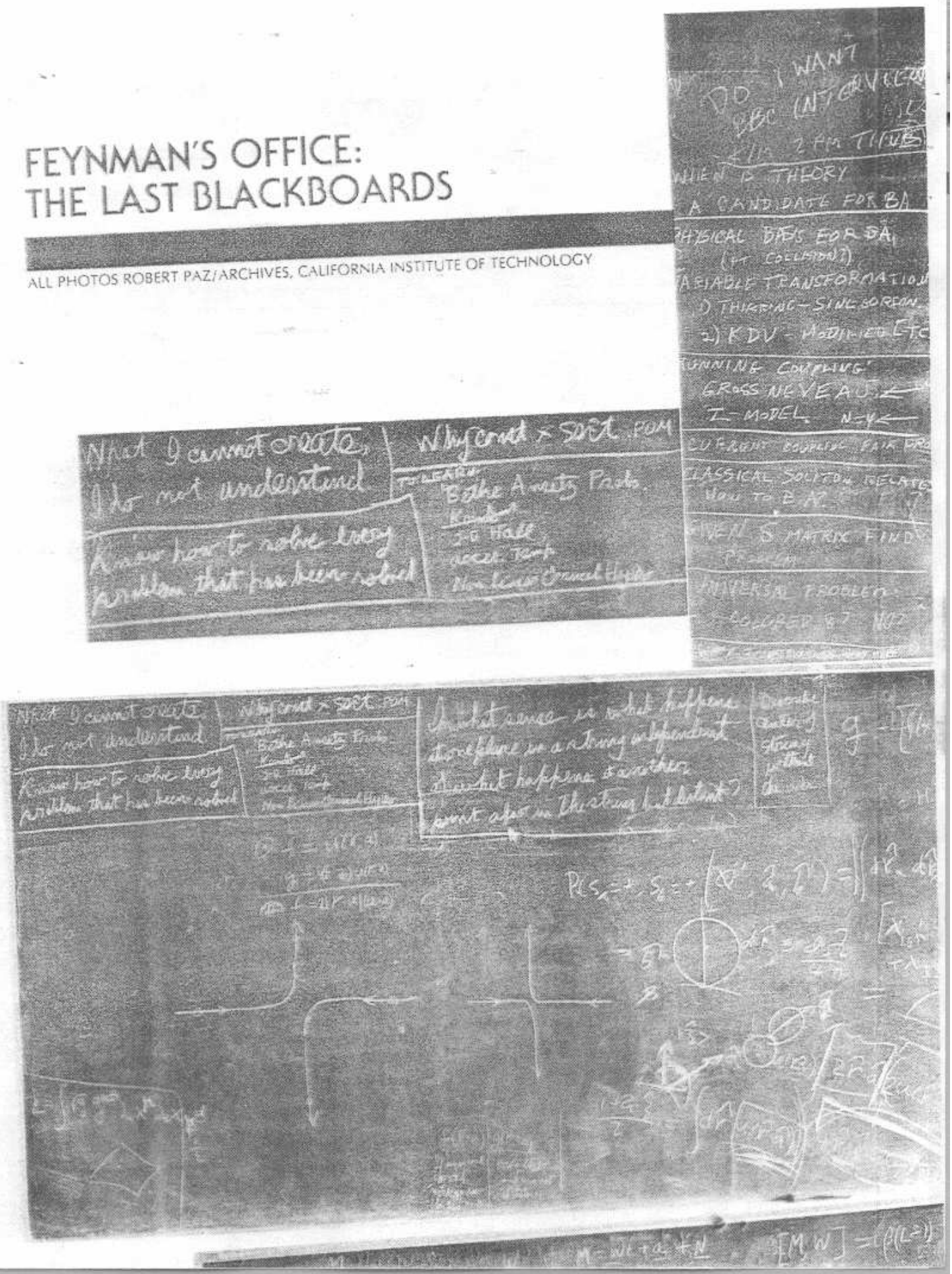
The Story of Fractional Charge

in the Fractional Quantum Hall Effect

Uncovering the True Nature
of the FQHE quasiparticle.

FEYNMAN'S OFFICE: THE LAST BLACKBOARDS

ALL PHOTOS ROBERT PAZ/ARCHIVES, CALIFORNIA INSTITUTE OF TECHNOLOGY



• **1998 Nobel Prize to R.B. Laughlin, H.L. Stormer, and D.C. Tsui** *"for discovery of a new form of quantum fluid with fractionally charged excitations"*

• "If we attribute [the quantized Hall resistance] to the presence of a gap at E_F when 1/3 of the lowest Landau level is occupied, [Laughlin's] argument will lead to quasiparticles with fractional electronic charge of 1/3 ..." (**Tsui, Stormer, Gossard, 1982**)

• "The ground state is a new state of matter, a quantum fluid the elementary excitations of which, the quasielectrons and quasiholes, are fractionally charged." (**Laughlin, 1983**)

"CHARACTERIZATION OF FQHE QUASIPARTICLES"

Goldhaber + JKJ 1995

Plan

IQHE

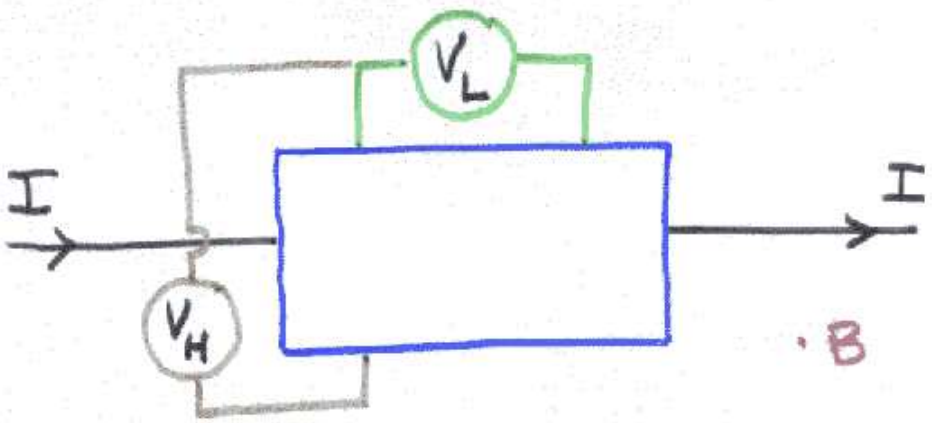
FQHE hierarchical description

Composite fermion

Fractional charge

Controversy ?!

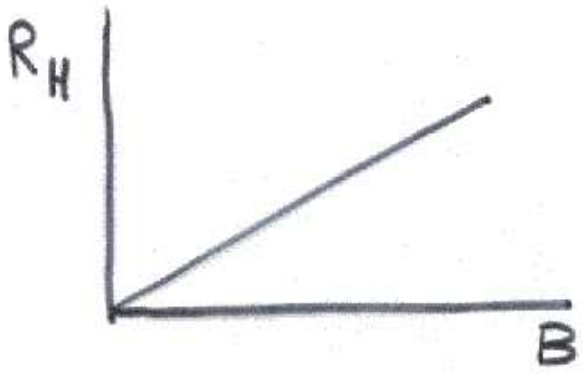
CLASSICAL HALL EFFECT (Hall, 1880)



$$R_L = R_{xx} = \frac{V_L}{I}$$

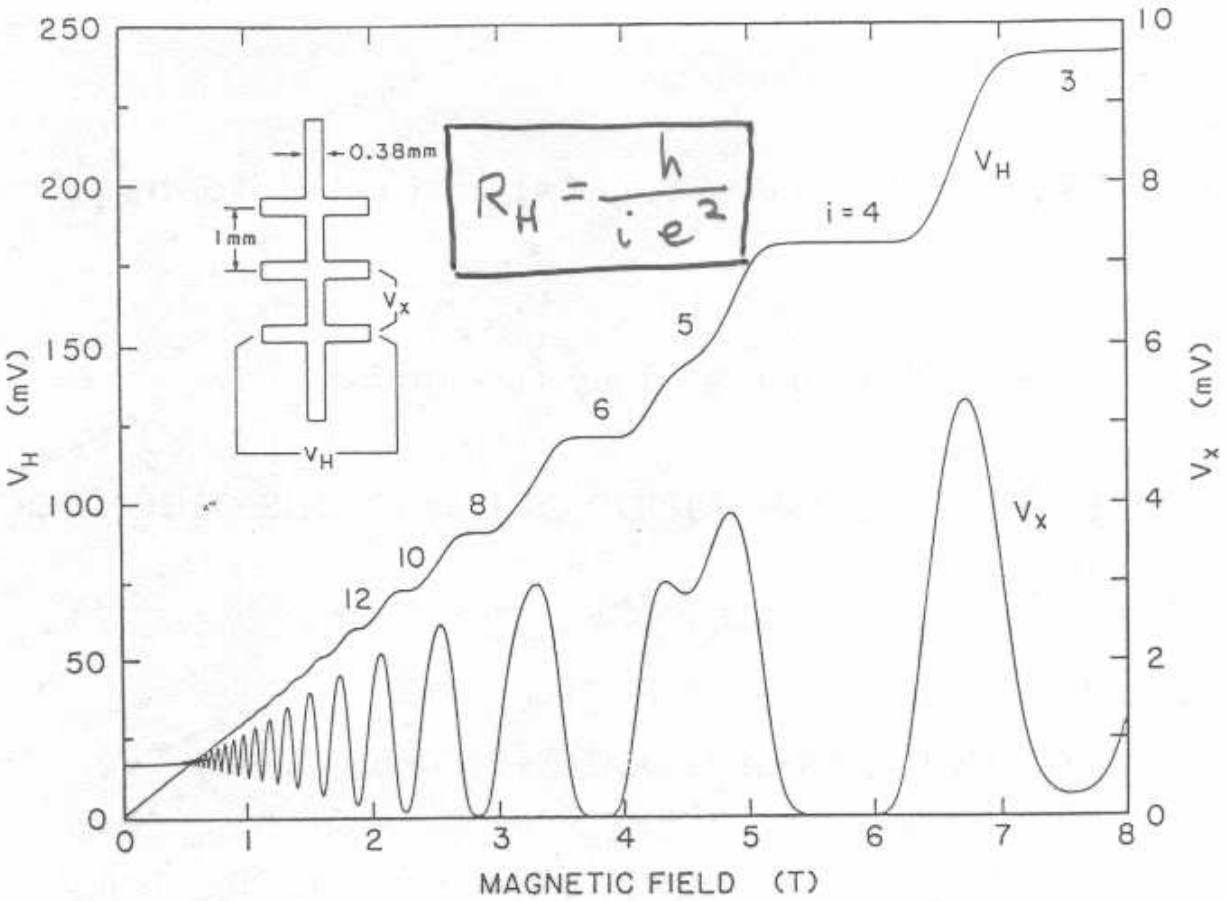
$$R_H = R_{xy} = \frac{V_H}{I} = \frac{B}{\rho e c} \quad (\rho = \text{2D density})$$

$$(v = \frac{cE}{B})$$



Hall effect is routinely used to measure the density of charge carriers.

IQHE



von Klitzing '80

- Universal effect. Independent of sample type, geometry or disorder.
- Accuracy of quantization ~ 3 parts in 10^{10}.
- One of the most accurate measurements of the fine structure constant $\alpha = \frac{e^2}{\varepsilon \hbar c}$.

- Quantum mechanical origin.

Single electron in a magnetic field (Landau)

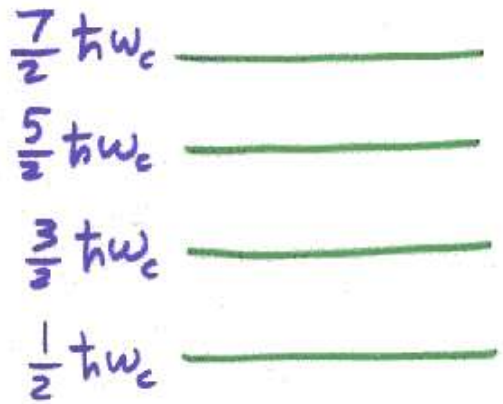
Darwin + [redacted]

$$H = \frac{1}{2m}(p + \frac{e}{c}A)^2$$

$$E = (n + \frac{1}{2})\hbar\omega_c \qquad \omega_c = \frac{eB}{m}$$

$n = 0, 1, 2, \ldots =$ Landau level index

- Degeneracy of each LL /area $= \frac{B}{\phi_0}$ $(\phi_0 = \frac{h}{e})$

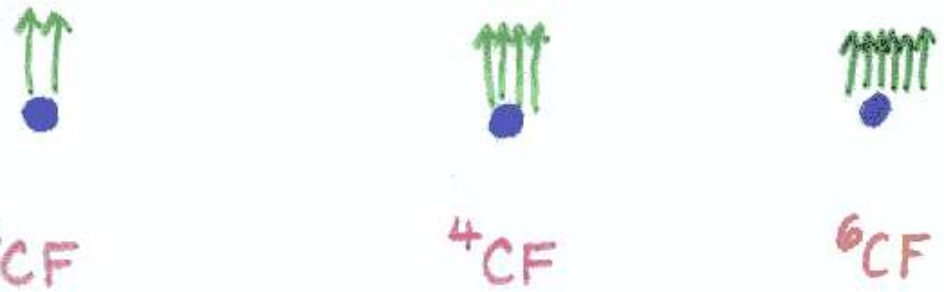


WHAT IS A COMPOSITE FERMION?

CF = electron + 2p vortices

Since a vortex ≈ a flux quantum, it is intuitively useful to view composite fermion as:

CF = electron + 2p flux quanta



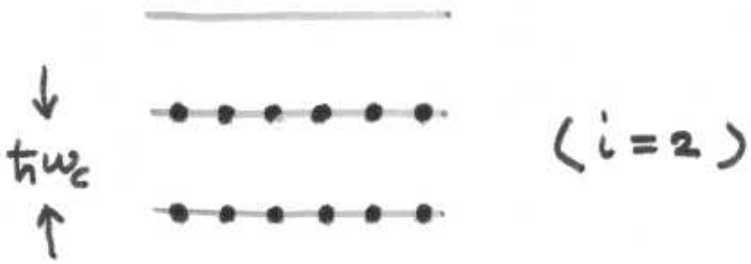
Only ~~two~~ three flavors of composite fermions are relevant to experiment.

Many non-interacting electrons: IQHE

- Filling factor ν = number of occupied LLs

$$\nu = \frac{\rho\phi_0}{B} \quad (\rho = \text{2D electron density})$$

- A unique ground state with a gap ($\hbar\omega_c$) is obtained when $\nu = i$.

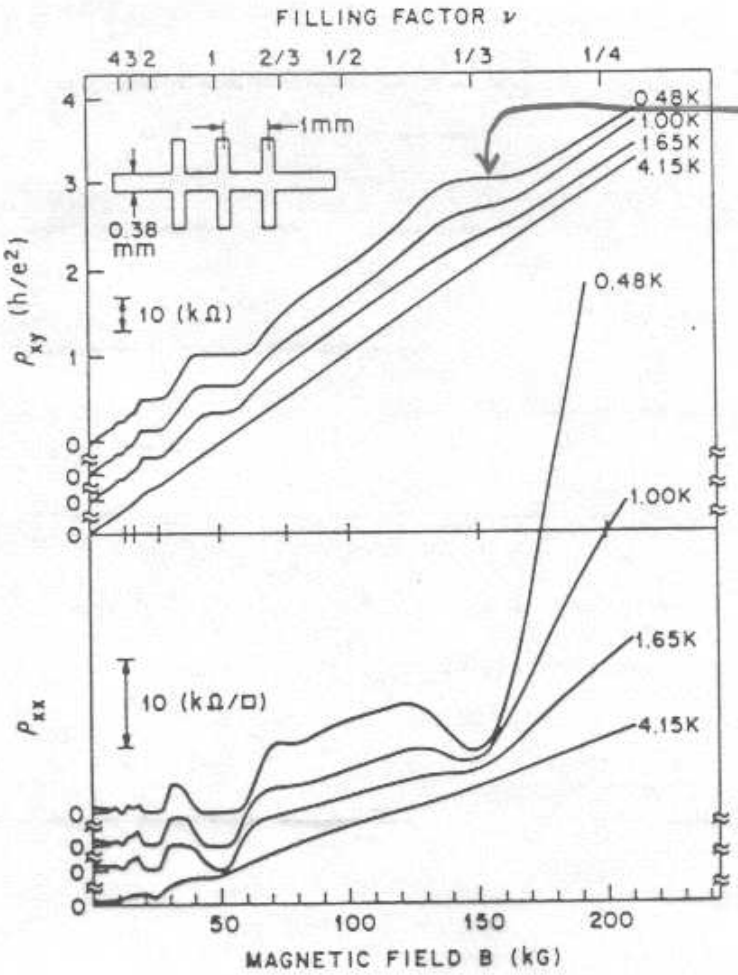


- This leads to the IQHE with $R_H = \frac{h}{ie^2}$.

- The physics of the IQHE lies in the formation of quantized Landau levels.

- A model of non-interacting electrons produces the IQHE. There is no other quantization in this model.

FQHE



$R_H = \frac{h}{\frac{1}{3}e^2}$

Tsui, Stormer, Gossard 1982

• Conditions for QHE at $R_H = h/fe^2$:

(i) a gap at $\nu = f$ in a disorder-free system;

(ii) weak disorder

• For noninteracting electrons, a gap occurs only at $\nu =$ integer;
for $\nu \neq$ integer, the ground state has an enormous degeneracy $\binom{\nu^{-1}N}{N} \sim \exp[()N]$.

• The interelectron interaction must somehow lift this degeneracy to produce a unique ground state with a gap !!

• Nonperturbative effect.

FQHE: Statement of the problem

- $H\Psi = E\Psi$

$$H = \frac{1}{2m}\Sigma_i\,(p_i + \frac{e}{c}A)^2 + \frac{1}{2}\Sigma_{j\neq k}\,\frac{e^2}{r_{jk}} + E_Z$$

$\sim B$ $\quad$ $\sim\sqrt{B}$ $\quad$ $\sim B$

- Consider $B \to \infty$, $\nu = \rho\phi_0/B =$ fixed

- Fully polarized electrons in the lowest LL

$$\boxed{H = \frac{1}{2}\Sigma_{j\neq k}\,\frac{e^2}{r_{jk}}}$$

+ lowest Landau level constraint

- No kinetic energy.
- No mass.
- No small parameter. $r_s = \infty$
- No conventional perturbation theory.
- No exact solution.

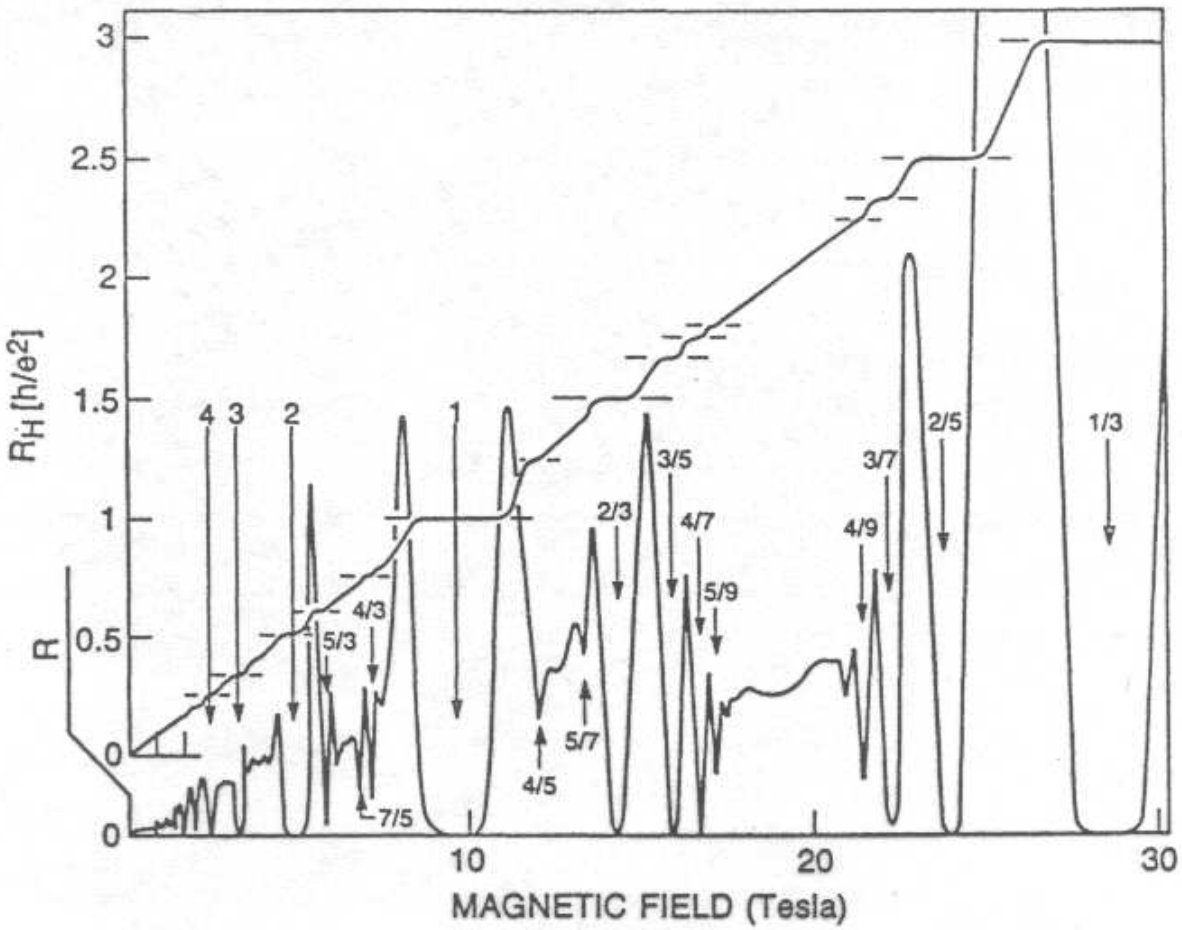
Theory of 1/3 (Laughlin, 1983)

$$\Psi_{1/3} = \prod_{j<k} (z_j - z_k)^3 \exp[-\frac{1}{4}\sum_i |z_i|^2]$$

$$z_j = x_j - iy_j, \;\; l = \sqrt{\hbar c/eB} = 1$$

• $\Psi_{1/3}$ provides a good representation for the ground state at $\nu = 1/3$.

• It may be argued that
(i) there is a gap to excitations, and
(ii) the excitations have a fractional charge of magnitude $e/3$.
$\Rightarrow$ FQHE at 1/3.


3
2.5
2
1.5
1
0.5
0
R
R_H [h/e²]
4
3
2
1
5/3
4/3
7/5
4/5
5/7
2/3
3/5
4/7
5/9
4/9
3/7
2/5
1/3
10
20
30
MAGNETIC FIELD (Tesla)

EXPERIMENTAL FACTS

- $R_H = \frac{h}{fe^2}$

- Evidence now exists for more than 50 fractions in the lowest Landau level.

- Prominent fractions belong to the sequences:

$$f = \frac{n}{2pn \pm 1}$$

$$f = \frac{n}{2n+1} = \frac{1}{3}, \frac{2}{5}, \cdots \frac{11}{23}$$

$$f = \frac{n}{4n+1} = \frac{1}{5}, \frac{2}{9}, \cdots \frac{6}{25}$$

- The denominator is an odd integer.

- $R_{xx} \sim \exp[-\Delta/kT]$ is activated, implying a gap in the activation spectrum (that is, an incompressible ground state).

Laughlin revisited

$$\Psi_{1/m} = \prod_{j<k} (z_j - z_k)^m e^{-\frac{1}{4}\sum_i |z_i|^2}$$

- The exponent m must be an odd integer.

$\Rightarrow \frac{1}{3}, \frac{1}{5}$ (also $\frac{2}{3}, \frac{4}{5}$)

- Laughlin's theory is incomplete. It is a part of a vast, deeper conceptual structure, which is yet to be discovered.

The FQHE puzzle

- What principle describes the physics of the problem? What is the nature of correlations? Why do they manifest themselves in such a rich, yet stunningly simple fashion?

- Why do the fractions appear in certain sequences?

- Why is the denominator odd?

- What is the nature of the state at even denominator fractions, e.g. $\nu = 1/2$? Why is there no FQHE there? (Why is $\nu = 5/2$ an exception?)

- What are the excitations?

- What is the quantitative theory?

-

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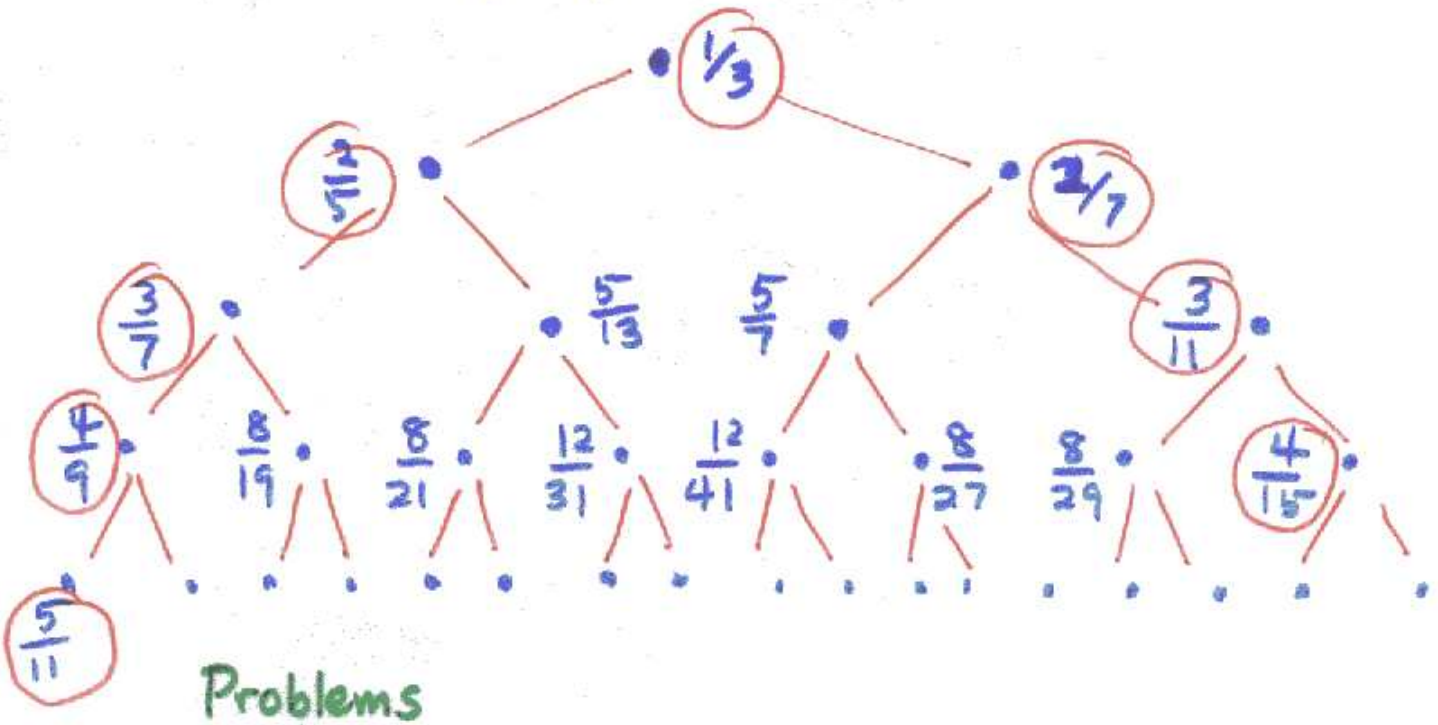
Hierarchy scenario (Haldane, Halperin, Laughlin '84)

New states occur because of the FQHE of the quasiparticles.



Problems

- Inconsistent with the observation of such a large number of fractions.
- A large number of quasiparticles are required at each step. (E.g. $\frac{1}{3} + \frac{N}{2}$ qp's $\Rightarrow \frac{2}{5}$.) At such large densities the quasiparticles are strongly overlapping, and description in terms of independent quasiparticles is implausible.

$$\frac{1}{3} \to \frac{2}{5} \to \frac{3}{7} \to \cdots \to \frac{10}{21} \; !$$

- No natural explanation for observed fractions.
- No microscopic theory. No confirmation.
- Too many quasiparticles. Too complicated.

• **AIM ??**

given particles + strong interactions
= new particles + weak interactions

(Duality)

• The new particles are the true particles. Only weakly interacting objects have a sufficiently well defined identity to deserve the title "particle."

• The strongly interacting system of one kind of particles reorganizes itself into a weakly interacting system of a new kind of particles. The physics of strong correlations is often described by the emergence of new particles.

• The original particles were the particles of the problem, the new particles are the particles of the solution.

• Sometimes the true particles are simple bound states of the input particles. At other times they are fantastically complicated collective objects (solitons) when viewed in terms of the old particles.

• When the true particles are qualitatively different from the initial ones, it is a clear sign of non-perturbative physics.

• Once the true particles are identified, phenomena that seemed mysterious or impossible to understand in terms of the old particles become straightforwardly comprehensible as properties of nearly free particles.

• AIM: Identify the true particles !

Input particles

True particles

Coupled vibrating atoms

phonons

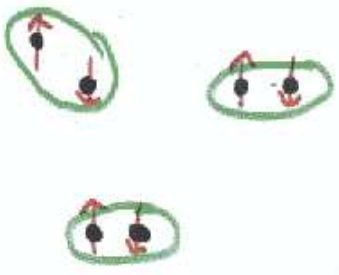
Coupled spins

magnons (spin waves)

Interacting electrons
(normal metal)

Landau quasiparticles

Interacting electrons
(Superconductor)

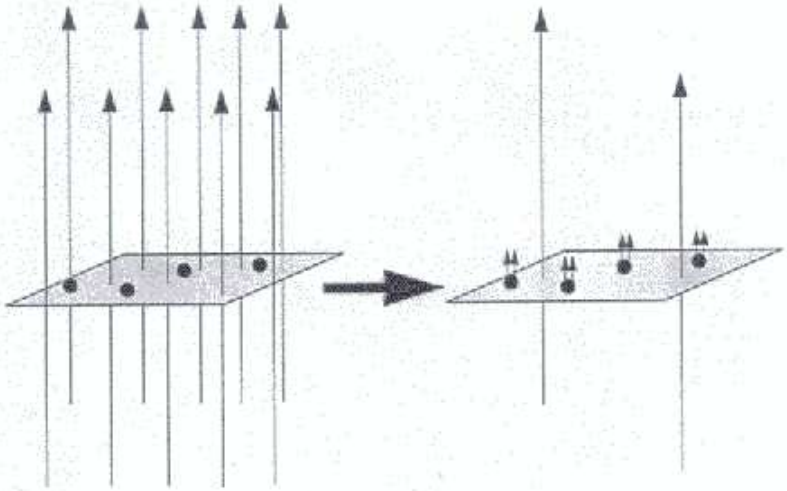


Cooper pairs

Q. What are the true particles of the FQHE?

(Not electrons.)

Q. How can we unify the FQHE and the IQHE?



Electrons transform into composite fermions by capturing 2p "flux quanta".

Composite fermions experience an effective magnetic field.

- $B^* = B - 2p\rho\phi_0$
- $\nu = \dfrac{\nu^*}{2p\nu^* \pm 1}$ $\quad \left(\begin{array}{l} \nu = \rho\phi_0 / B \\ \nu^* = \rho\phi_0 / |B^*| \end{array} \right)$
- $\Psi_\nu = \prod_{j<k} (z_j - z_k)^{2p} \Phi_{\nu^*}$

Interacting electrons at ν

Non-interacting electrons at ν^*

Gives wave functions for ground and excited states at arbitrary fillings.

magnitude $2p \times 2\pi$. By defi-
d a unit vortex generates a
uct in equation 4 attaches $2p$
n the noninteracting Slater-

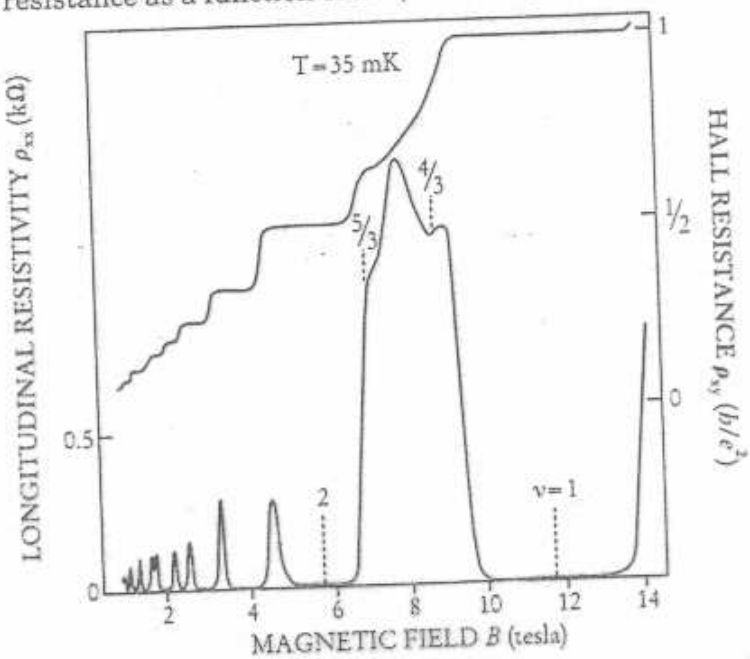
quanta of our "Bohr theory"
vortices of the many-particle
posite fermion is actually the
nd $2p$ quantum vortices. A flux
imilar to a vortex; it also pro-
phase of 2π for a closed path
s often useful to model the vor-
vision the composite fermion as
x quanta.
cancel part of the external B
which one particle executes a
losing an area A, with all the
d. Equating the sum of the
BA/ϕ_0 and the phase $-2\pi 2p\rho A$
led vortices to an *effective*
AB^*/ϕ_0, we get the new field B^*
agnetometer will still measure
omposite fermion is concerned,
hall see.
function Ψ provides an insight
interaction between electrons
em. The wavefunction is very
ectrons apart. The probability
thin a distance r of each other
ast that with the r^2 vanishing
fying the Pauli principle. In
ansmute into composite fermi-
s because that is how they best
mb interaction. The interaction
as is weak because most of the
een screened out—or used up—

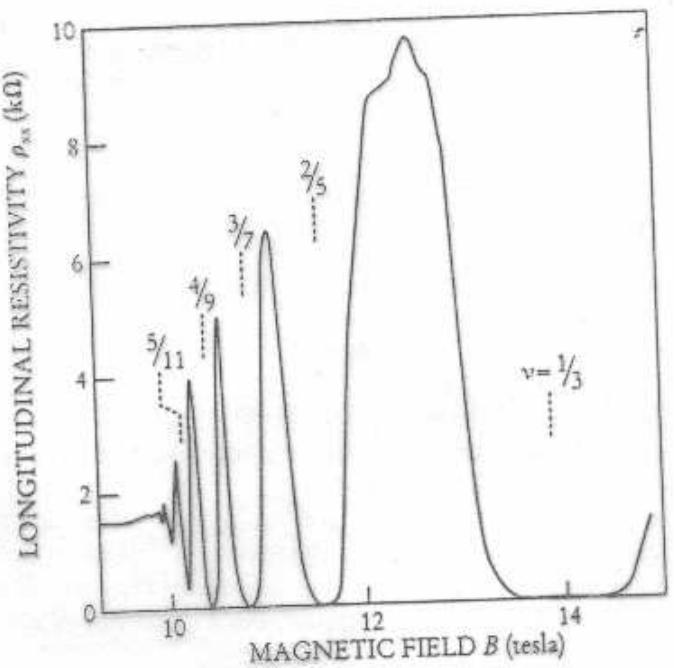
4 are the master equations
id of composite fermions. Since
nd can be derived from the third,
as from a single equation. The
mposite fermion follow straight-
ion that each one is produced by

Seeing composite fermions

The crucial, non-perturbative respect in which composite fermions distinguish themselves from electrons is that they experience an effective magnetic field, B^*, that is drastically different from the external magnetic field. The effective magnetic field is so central, direct, and dramatic a consequence of the formation of composite fermions that its observation is tantamount to an observation of the composite fermion itself.

At filling factors ν less than 1, the experiments clearly show us composite fermions subject to the magnetic field B^* rather than electrons subject to B. The most compelling experimental evidence for the composite fermion comes simply from plotting the high-field magnetoresistance as a function of $1/\nu^*$, which is proportional to B^*.





NG INTEGRAL AND FRACTIONAL
nd bottom panels, respectively.[16,3]
he top panel) is the Hall resistance
d curves are ordinary longitudinal
by its electron filling factor ν, for
tor $\nu = n/(2n + 1)$ corresponds to
ng factor $\nu^* = n$. Thus the two red
ifferent electron filling factors, are
remarkably similar.

Explanation of FQHE

IQHE of composite fermion occurs at $\nu^* = n$. Here,

$$\nu = \frac{\nu^*}{2p\nu^* \pm 1} = \frac{n}{2pn \pm 1}$$

The FQHE of electrons is a manifestation of the IQHE of composite fermions.

- Simple intuitive understanding of FQHE.
- All fractions explained in a single step, and on the same footing.
- Principal sequences of fractions explained. These are manifestations of the integer sequence of composite fermions.
- FQHE and IQHE unified.
- Only two flavors of composite fermions needed – those carrying two or four vortices.

BEYOND FQHE

Composite fermions also explain the physics of the gapless states where no FQHE is seen.

$\nu = 1/2$

$$B = 2\rho\phi_0$$

$$B^* = B - 2\rho\phi_0 = 0$$

Composite fermions form a Fermi sea at $\nu = 1/2$. (Halperin, Lee, Read, 93; Kalmeyer and Zhang, 92)

• Cyclotron radius of composite fermions:

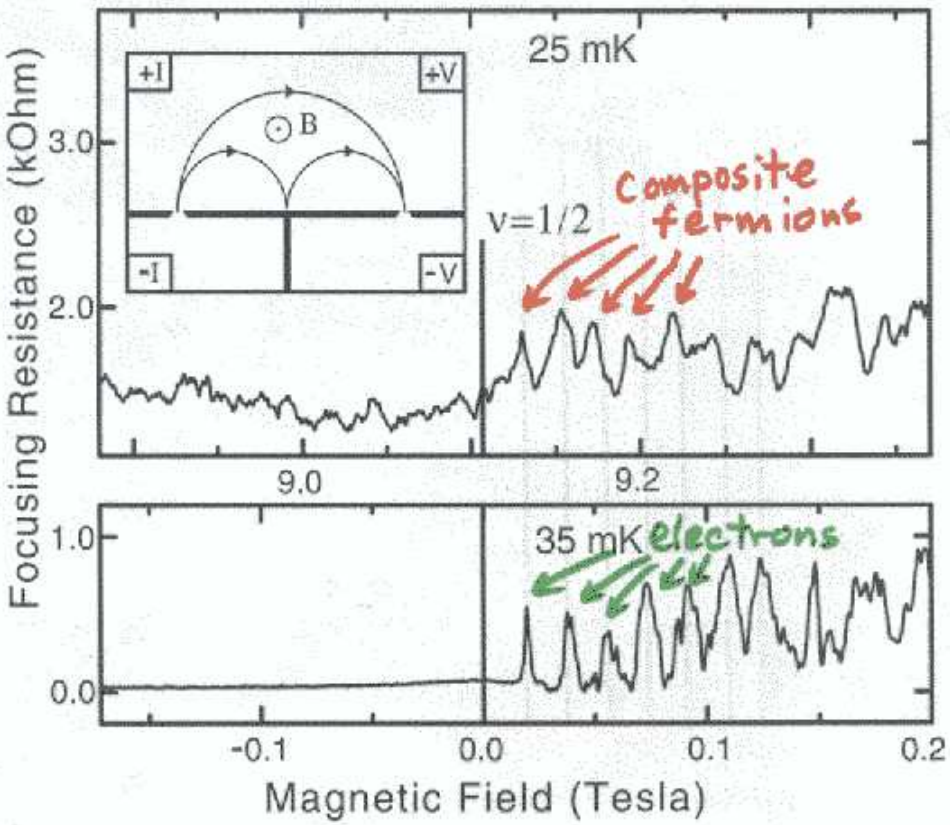
$$R = \frac{\hbar k_F^*}{eB^*} \quad (k_F^* = \sqrt{4\pi\rho} = \sqrt{2}\, k_F)$$

Measured in several experiments.
- Antidot resonances
(Kang, Stormer, Tsui, 93; Smet, von Klitzing 96)
- Magnetic focusing
(Goldman 94, Smet, von Klitzing, 97)
- Surface acoustic wave absorption
(Willet 93)

• Shubnikov-de Haas oscillations of composite fermions.
(Leadley, 94; Du, Stormer, Tsui, 94)

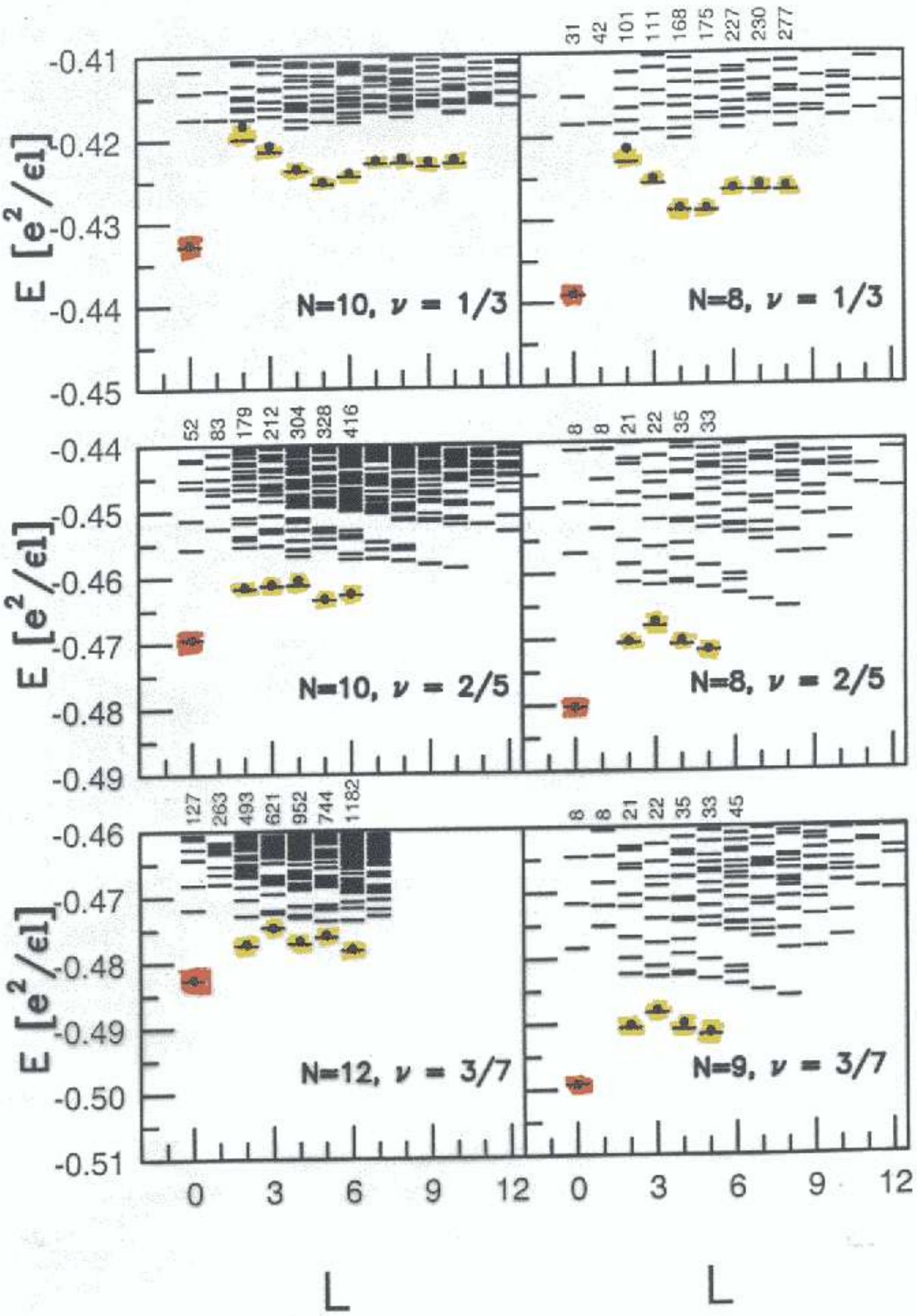
• Thermopower of composite fermions.
(Maan, 93; Shayegan 94)

Composite fermions provide a unified description of all manifestations of the lowest-LL liquid.



Goldman 1994

Magnetic focusing of composite fermions


E [e²/εl]
-0.41
-0.42
-0.43
-0.44
-0.45
31 42 101 111 168 175 227 230 277
N=10, ν = 1/3
N=8, ν = 1/3
52 83 179 212 304 328 416
8 8 21 22 35 33
N=10, ν = 2/5
N=8, ν = 2/5
-0.44
-0.45
-0.46
-0.47
-0.48
-0.49
127 263 493 621 952 744 1182
8 8 21 22 35 33 45
N=12, ν = 3/7
N=9, ν = 3/7
-0.46
-0.47
-0.48
-0.49
-0.50
-0.51
0 3 6 9 12
0 3 6 9 12
L
L

ν	N	**overlap**
$\frac{1}{3}$	7	0.9964
	8	0.9954
	9	0.9941
$\frac{2}{5}$	6	0.9998
	8	0.9996
$\frac{3}{7}$	9	0.9994
$\frac{2}{3}$	6	0.9965
	8	0.9982
	10	0.9940

Wu, Dev, Jain
1993

ν	N	**ground state**	
		composite fermion	exact
$\frac{1}{3}$	8	-0.4389	-0.4391
	10	-0.4326	-0.4328
$\frac{2}{5}$	8	-0.4802	-0.4802
	10	-0.4693	-0.4694
$\frac{3}{7}$	9	-0.4991	-0.4992
	12	-0.4825	-0.4826

Energies accurate to better than 0.05%.

No adjustable parameter in CF theory.

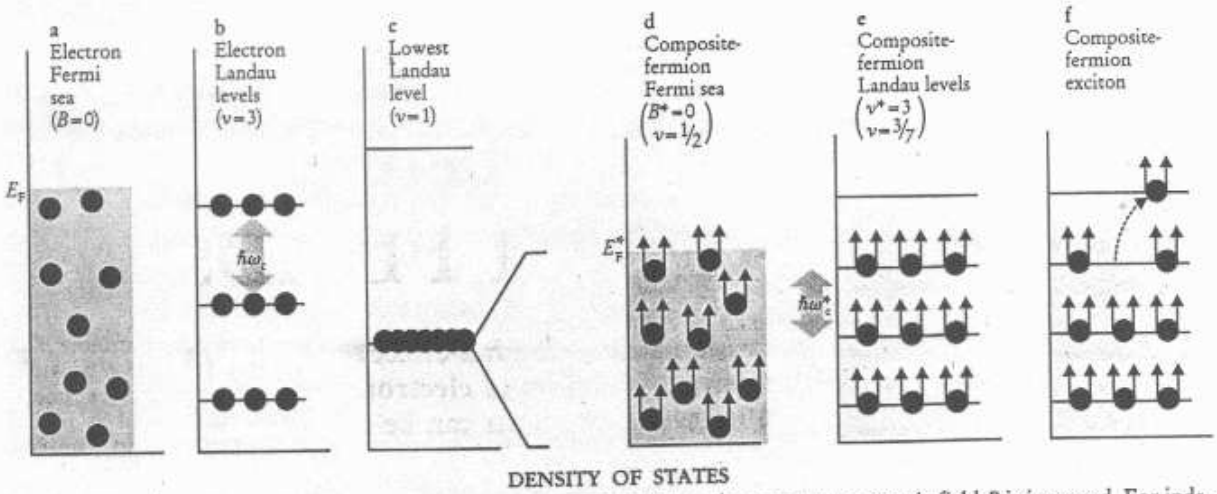


FIGURE 1. EVOLUTION OF A TWO-DIMENSIONAL ELECTRON SYSTEM as the transverse magnetic field B is increased. For independent electrons, the Fermi sea (a) (filled to Fermi energy E_F) at $B = 0$, splits into Landau levels (b) separated by the cyclotron energy. The lowest Landau level (c) is split by interactions into energy levels of composite fermions with attached flux quanta, which fill a composite-fermion Fermi sea (d) at $\nu = 1/2$ and occupy composite-fermion Landau levels (e) at other filling factors. jump out of such a level (f) creates an exciton, a neutral particle–hole excitation. At still higher fields, this scenario (d—f) repeats itself, but now with composite fermions carrying four or more flux quanta.

explain the fractional quantum Hall effect, discovered in 1982 by Daniel Tsui, Horst Stormer, and Arthur Gossard at simple fractional values of ν. (See PHYSICS TODAY, December 1998, page 17.) But subsequent work has shown that it describes a superstructure that encompasses other phenomena as well.

The quickest way to introduce the composite fermion is through the following series of steps, which I call the "Bohr theory" of composite fermions because it obtains some of the essential results with the help of an oversimpified but useful picture. The outcome is that strongly interacting electrons in a strong magnetic field B transform into weakly interacting composite fermions in a weaker effective magnetic field B^*, given by

$$B^* = B - 2p\rho\phi_0 , \qquad (2)$$

where $2p$ is an even integer. Equivalently, one can say that electrons at filling factor ν convert into composite fermions with filling factor $\nu^* = \rho\phi_0/|B^*|$, given by

$$\nu = \frac{\nu^*}{2p\nu^* \pm 1} . \qquad (3)$$

The minus sign corresponds to situations when B^* points antiparallel to B.

Start by considering interacting electrons in the transverse magnetic field B. Now attach to each electron an infinitely thin, massless magnetic solenoid carrying $2p$ flux quanta pointing antiparallel to B, turning it into a composite fermion. Such a conversion preserves the minus sign associated with an exchange of two fermions, because the bound state of an electron and an even number of flux quanta is itself a fermion. Hence the name. It also leaves the Aharonov–Bohm phase factors associated with all closed paths unchanged, because the additional phase factor due to a flux $\phi = 2p\phi_0$ is $\exp\{2\pi i\phi/\phi_0\} = 1$. In other words, the attached flux is unobservable, and the new problem, formulated in terms of composite fermions, is identical to the one with which we began.

So, what have we gained? Well, a "mean-field approximation" now suggests itself, in which the new attached magnetic field is smeared to produce an additional uniform magnetic field $-2p\rho\phi_0$. With that addition, we g the net magnetic field B^* of equation 2. The net effect, a sense, is that each electron has absorbed $2p$ flux quan from the external field to become a composite fermion th experiences only the residual magnetic field B^*. (See figure

The crucial point is that the many-particle grou state of electrons at $\nu < 1$ was highly degenerate in t absence of interaction, with *all* lowest Landau level cc figurations having the same energy. But now, the dege eracy of the composite-fermion ground state at the cor sponding $\nu^* > 1$ is drastically smaller, even when the int action between composite fermions is switched off. F integral values of ν^*, in fact, one gets a *non-degenera* ground state.

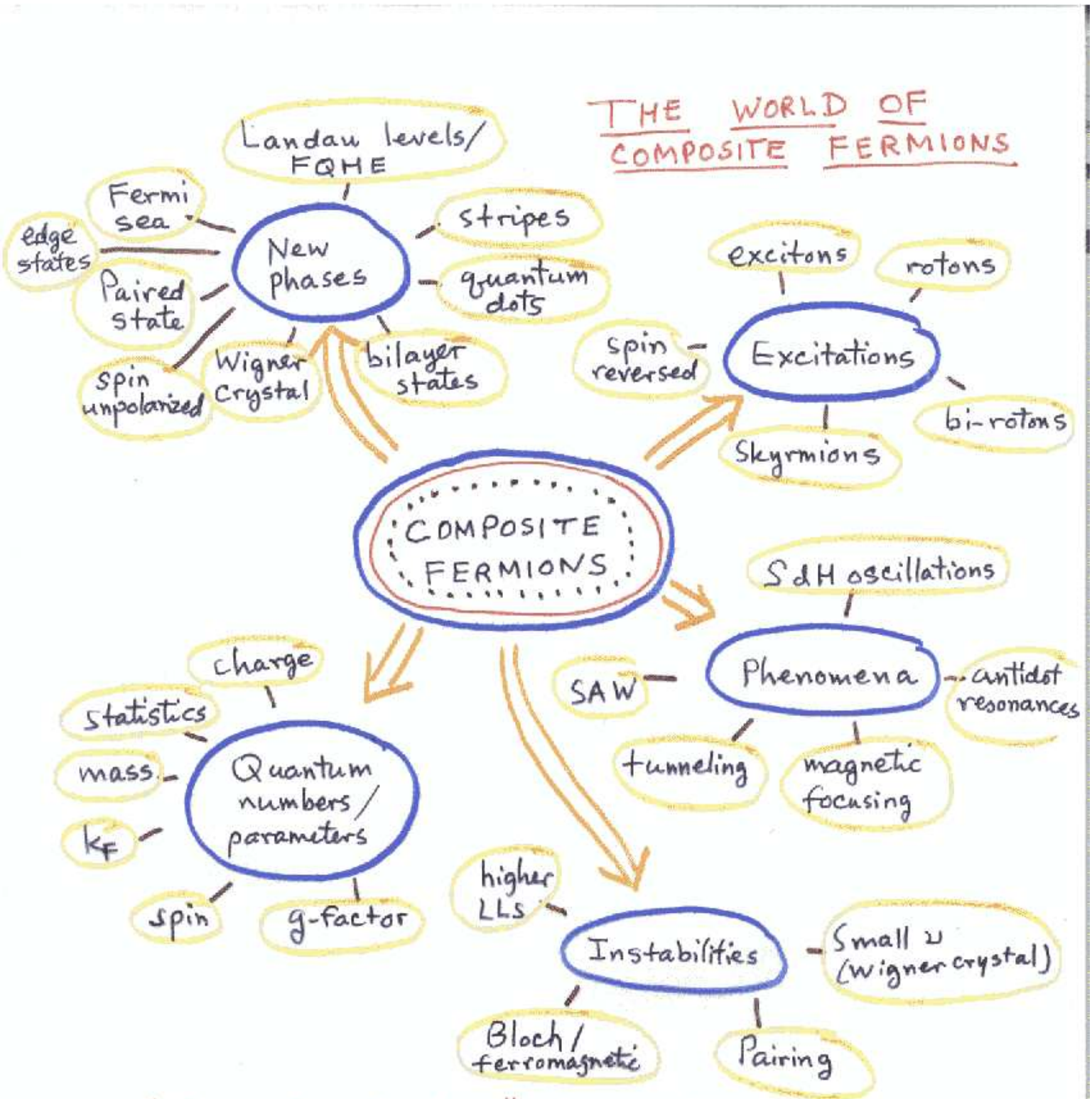
The reduced degeneracy suggests that one mig start by treating the composite fermions as independe In that approximation, the composite fermions fill a Fer sea of their own whenever B^* vanishes ($1/\nu = 2p$), and fo composite-fermion Landau levels when it does not. All this action, of course, takes place inside the lowest el tronic Landau level, as shown in figure 1.

Having identified interacting electrons at filling f tor ν with independent composite fermions at ν^*, we wr the (unnormalized) microscopic wavefunctions for int acting electrons at a given ν as:

$$\Psi_\nu = \Phi_{\nu^*} \prod_{j<k} (z_j - z_k)^{2p} ,$$

where $z_j = x_j - iy_j$ denotes the position of the jth electron a complex number, and Φ_{ν^*} are the known Slater-determ nant wavefunctions for *non-interacting* electrons at t corresponding ν^*. For simplicity, we have assumed th the electron population is fully polarized and we have su pressed the spin part of the wavefunction.

The wavefunctions Ψ_ν, which turn out to be extrem ly accurate approximations of the actual electron eige states, give a precise meaning to the intuitive physics our composite-fermion discussion. The factors in the pr uct over all electrons in equation 4 tell us that every el tron sees $2p$ vortices at every other electron. That is say, as the jth electron executes a closed path around t



"Composite fermions" ed. by O. Heinonen

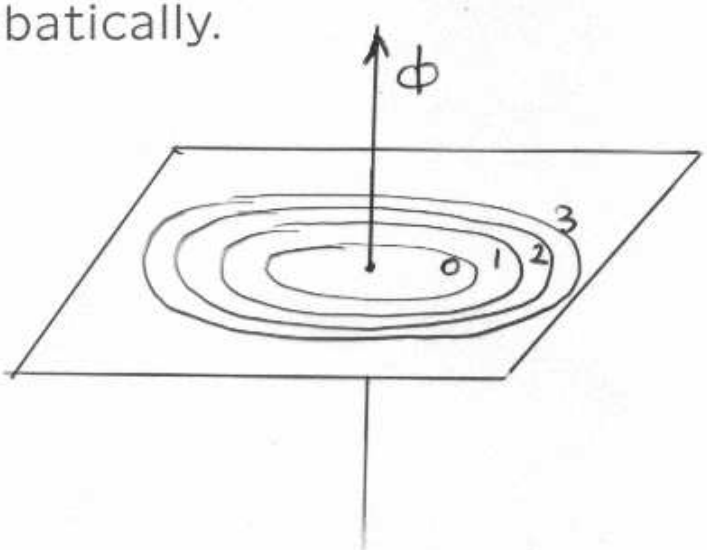
"Fractional Quantum Hall Effect" ed. Das Sarma and Pinczuk

"Composite Fermion: A Quantum Particle and Its Quantum Phases" JKJ Physics Today 2000.

What about the fractional charge?

Fractional charge

• Consider a FQHE state at $\nu = n/(2pn \pm 1)$. Change a test flux piercing through the origin adiabatically.



• As the test flux ϕ varies from 0 to $\phi_0 = hc/e$, the single particle eigenstates move by one unit, $m \to m \pm 1$.

• The charge access or deficiency at the origin is precisely equal to the amount of charge in a single state, i.e., νe. It is localized near the origin.

If the charge of the most fundamental quasi-particle is e^*, then we have

$$\frac{n}{2pn \pm 1} e = i\, e^*$$

$$e = j\, e^*$$

$$\frac{n}{2pn \pm 1} = \frac{i}{j}$$

Simplest choice: $i = n,\ j = 2pn \pm 1$

$$e^* = \frac{e}{2pn \pm 1}$$

- The fractional charge is a consequence of a gap at a fractional filling factor, not the cause of it. The existence of fractional charge does not tell us anything about the physics leading to the gap.

- No "fractionalization" of electron !! The fractional charge is made up of whole electrons.

BUT WHAT ARE THESE FRACTIONALLY CHARGED OBJECTS ??

Puzzle: How does one reconcile the composite fermion theory with the general result regarding the existence of fractional charge?

- Composite fermions have the same charge as electrons, because

# of composite fermions = # of electrons

- Where is the fractional charge in the composite fermion theory?

Goldhaber: One ought to define two kinds of charges for the composite fermion.

- The intrinsic / AB charge is e.

- The local / screened charge, e^* may be fractional.

• Analogy An external electron in a dielectric.

The intrinsic / AB charge is e. This is the charge that would be measured in an AB experiment.

The local / screened charge is $e^* = \frac{e}{\epsilon}$. This is the charge inside a gaussian box around the electron, as measured from the electric field emanating from the charge: $E = \frac{e}{\epsilon r^2}$

• The local charge can be calculated by asking what happens when an *electron* is added to the system containing an integral number of filled composite fermion Landau levels.

• the fractionally charged quasiparticle
= a composite fermion in an otherwise empty composite fermion Landau level

It is simply an electron dressed by an even number of vortices into a composite fermion. The

dressing, however, is of a non-perturbative nature, because the vorticity is quantized to be in integer.

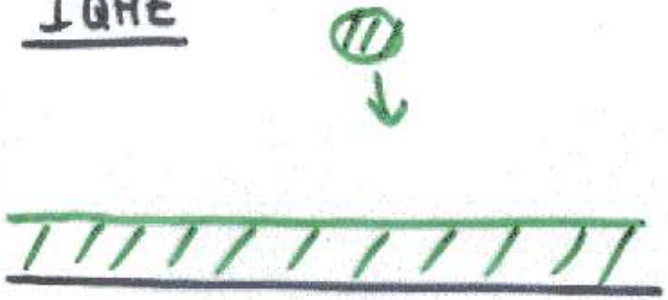
- The quasiparticle has the same intrinsic charge, spin, and exchange statistics as an electron, independent of the background state. This is very much in the spirit of quasiparticle description in other interacting electron systems and unifies the view of quasiparticle dynamics in and even beyond the FQHE regime.

- The local / screened charge depends on the background state, and can be shown to be quantized.

- The local charge is sharply defined only when there is a gap to excitations. The intrisic charge, on the other hand, is always well defined.

- In particular, the intrinsic charge is a sharp observable at $\nu = \frac{1}{2}$, where there is no gap and no FQHE. The observed cyclotron radius is consistent with the existence of a Fermi surface of a charge e fermions.

IQHE



$e^* = e$

FQHE

$e^* = \frac{e}{3}$

At $\nu = \frac{n}{2pn \pm 1}$, as the added electron is dressed into a CF, other CF's are excited out of the "vacuum."

$$e^* = \frac{e}{2pn \pm 1}$$