Limits on the Rest Mass of the Photon: from

to Vulcan-Solar Probe

Alfred S. Goldhaber and MMN
PRL 21, 567 (1968)
26, 1390 (1971)
RMP 43, 227 (1971)
PRD 9, 1119 (1974)
Sci. Am., May (1976)
with L. Davis, Jr. PRL 35, 1402 (1975)

New work with V. A. Kostelecky, PLB 317, 223 (1993).

1) Maxwell Eqs. $\rightarrow$ Proca Eq.

($\mu \neq 0$)

2) Consequences of $\mu \neq 0$

3) Limits on μ from

a) c = Const.

b) Coulomb's Law

c) Earth's magnetic field

d) Jupiter's magnetic field

THIS CAN ALL BE DONE

RIGOROUSLY (See RMP)

$$\left.\begin{array}{l}\textbf{Special relativity}\\ \textbf{Fields Linear in Currents}\\ \textbf{Energy Conservation}\end{array}\right\} \Rightarrow \textbf{PROCA}$$

PROCA Eq. [Maxwell $\mu \neq 0$]

$$\partial^\lambda F_{\lambda\nu} + \mu^2 A_\nu = \left(\frac{4\pi}{c}\right) J_\nu$$

$$F_{\lambda\nu} = \partial_\lambda A_\nu - \partial_\nu A_\lambda$$

$$(\Box + \mu^2) A_\nu = \frac{4\pi}{c} J_\nu$$

[CHARGE CONSERVED]

$$\partial^\lambda A_\lambda = 0$$

[LORENTZ GAUGE
no gauge invariance]

<u>new "Maxwell's Eq"</u>

$$\left\{\begin{aligned} \nabla \cdot \underset{\sim}{E} &= 4\pi\rho - \mu^2 V \\ \nabla \times \underset{\sim}{E} &= -\frac{1}{c}\frac{\partial \underset{\sim}{H}}{\partial t} \\ \nabla \cdot \underset{\sim}{H} &= 0 \\ \nabla \times \underset{\sim}{H} &= \frac{1}{c}\frac{\partial \underset{\sim}{E}}{\partial t} + \frac{4\pi}{c}\underset{\sim}{J} - \mu^2 \underset{\sim}{A} \end{aligned}\right\}$$

STILL

$$\left\{\begin{aligned} \underset{\sim}{H} &= \nabla \times \underset{\sim}{A} \\ \underset{\sim}{E} &= -\nabla V - \frac{1}{c}\frac{\partial \underset{\sim}{A}}{\partial t} \end{aligned}\right\}$$

b) $F \not\propto 1/r^2$

$$(\nabla^2 - \mu^2) A_\lambda = -4\pi J_\lambda / c$$

$$V = \frac{\exp(-\mu r)}{r}$$

<u>YUKAWA POTENTIAL</u>

$$F = -\nabla V$$

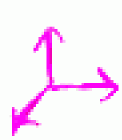
$$= \frac{e^{-\mu r}}{r^2}\left[1 + \mu r\right]$$

$$\simeq \frac{1}{r^2}\left[1 + \frac{\mu^2 r^2}{2} + \cdots\right.$$

THM

$\Rightarrow$ Need High Prec. or Big Apparatus

c) 3rd Polarization ---



SIDELIGHT

μ's

① BIG APPARATUS
(ASG/MN)

~ CLUSTERS
CDM ?

② SMALL APPARATUS
+ HIGH PRECISION

<merc> — D. Groom

(3) (b) "Coulomb's Law"

I) PRIESTLY
(1766-1767)
"The History + Present State of Electricity"
FROM
FRANKLIN'S
1755 EXPT

XV. Experiments with an ELECTRIFIED CUP.

I shall cloſe the account of my experiments with a ſmall ſet, in which, as well as in the laſt, I have little to boaſt beſides the honour of following the inſtructions of Dr. Franklin. He informed me, that he had found cork balls to be wholly unaffected by the electricity of a metal cup, within which they were held; and he deſired me to repeat and aſcertain the fact, giving me leave to make it public.

May we not infer from this experiment, that the attraction of electricity is ſubject to the ſame laws with that of gravitation, and is therefore according to the ſquares of the diſtances; ſince it is eaſily demonſtrated, that were the earth in the form of a ſhell, a body in the inſide of it would not be attracted to one ſide more than another.

II) Franklin. (single fluid ⇒ ±)

Franz Æpinus (1759 .. later in St. Petersburg) speculates r^{-2}

JOHN ROBISON (classicist 1769) published in <u>1803</u> (Ency. Brit. supp.)

$$F \propto \frac{e_1 e_2}{r^{2+q}} \qquad q = 0.06$$

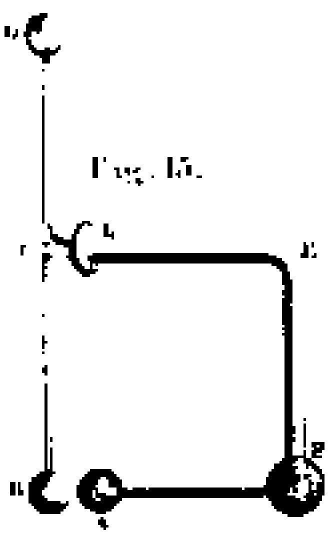
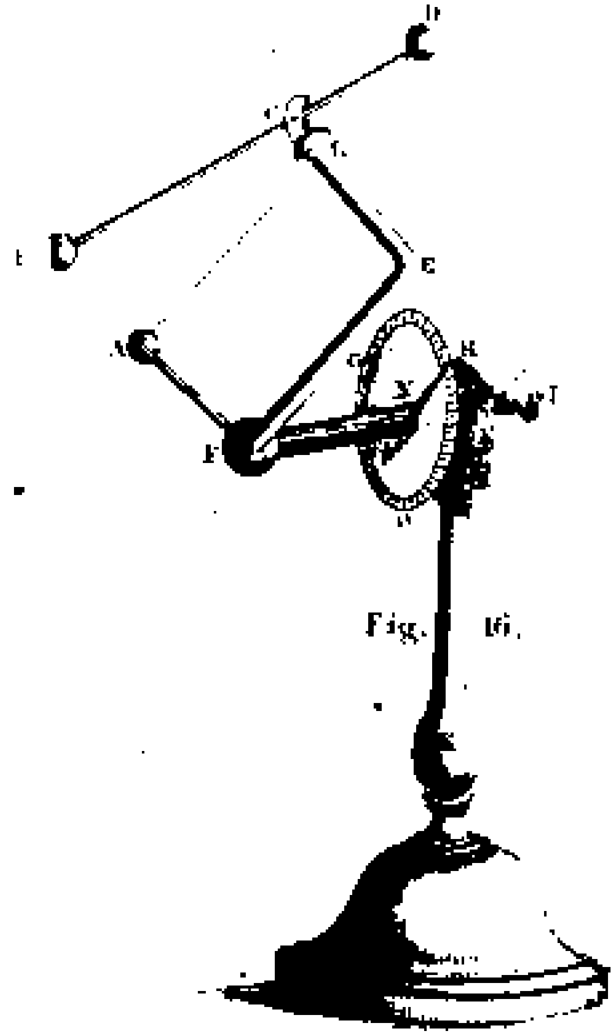


Fig. 15.



Figg. 16.

III) CAVENDISH (1773)

Published by Maxwell in 1879

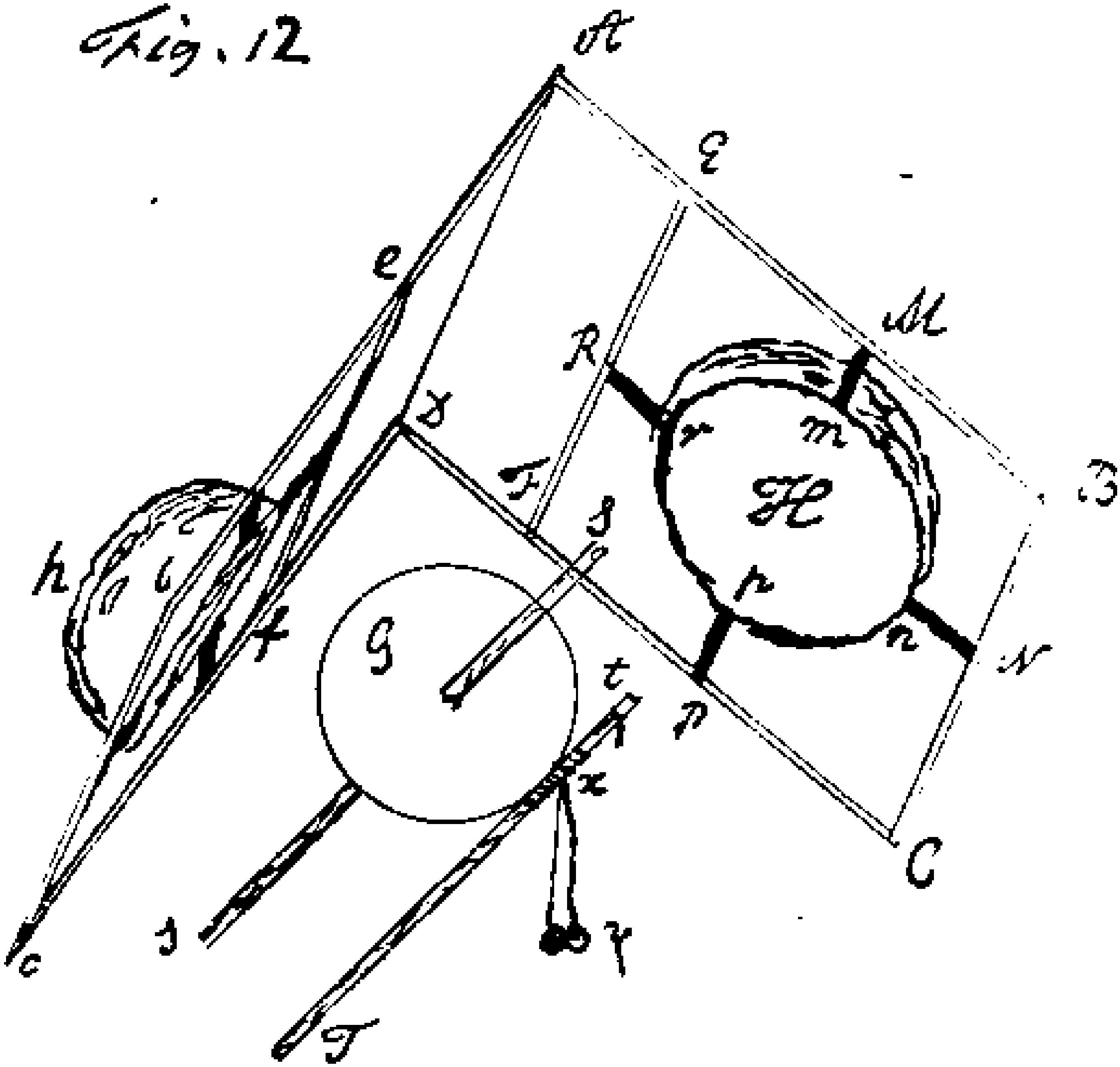
$|q| < \frac{1}{50} = 0.02$

① Shells closed with contact. Charge placed on outer sphere.

② Break connection

③ Open outer sphere + see if any charge on inner sphere.

Fig. 12



IV) <u>COULOMB</u> (1785 - pub. 1789)

$q = 0.1$

Torsion balance at various distances --- Both repulsion + attraction.

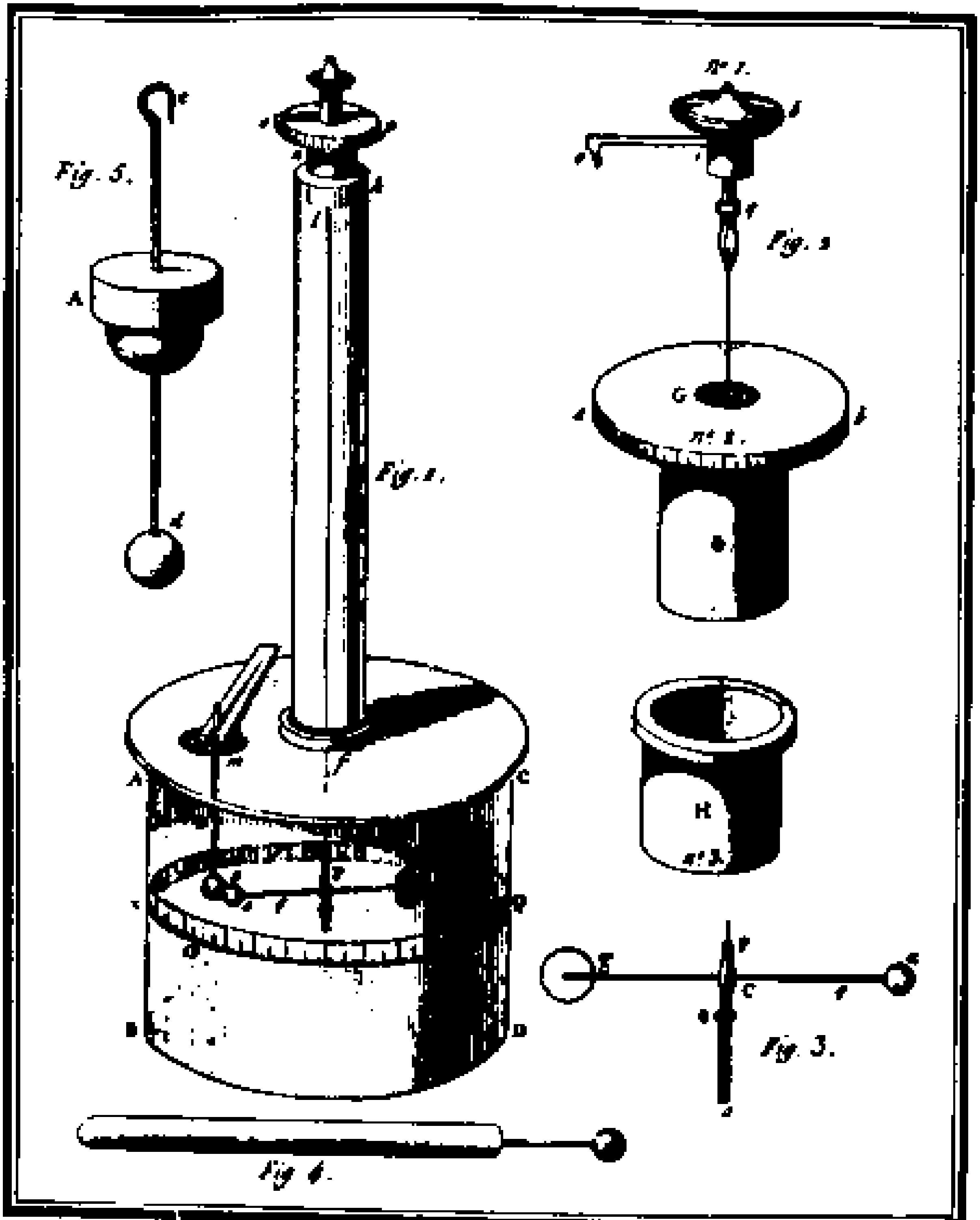


Fig. 5.
Fig. 1.
Fig. 2
Fig. 3.
Fig. 4.
N° 1.
N° 2.
N° 3.
G
H
A
B
C
D

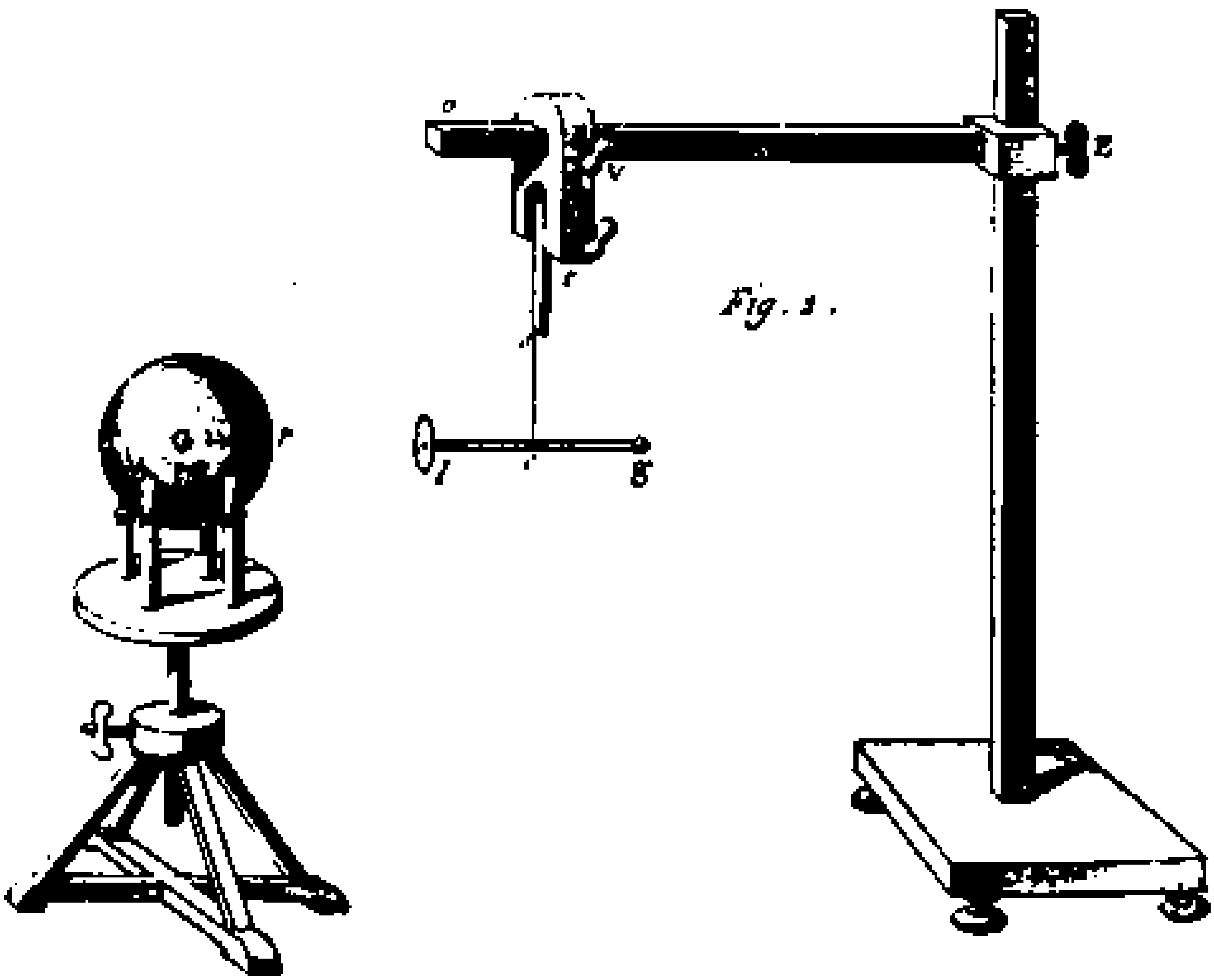
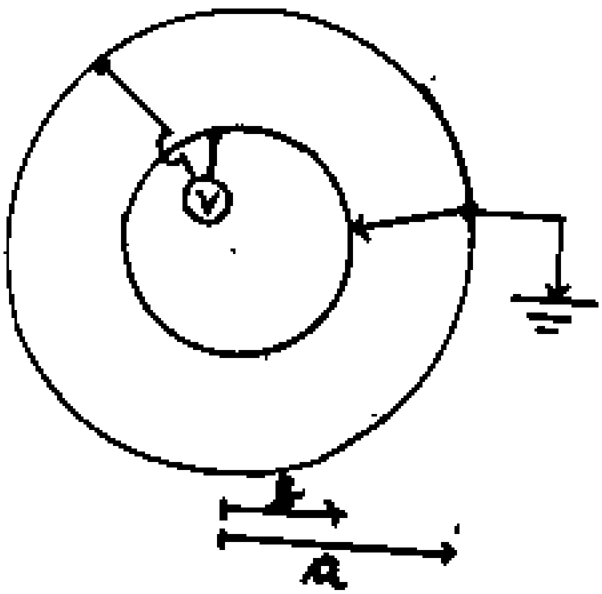
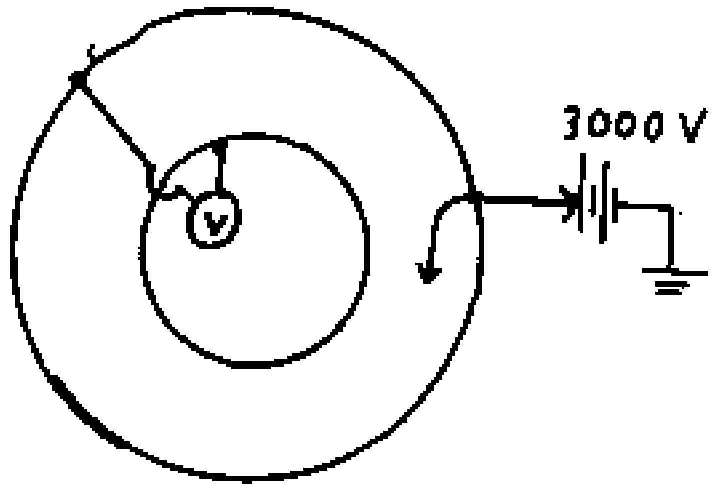


Fig. 1.

VI Plimpton & Lawton (1936)





$b = 2$ ft
$a = 2\frac{1}{2}$ ft
$\Delta V = 10^{-6}$ V

$$q = F(a, b)\,\frac{\Delta V}{V}$$
$$\simeq 2 \times 10^{-9}$$

NOW, in our Language

$$(\nabla^2 - \mu^2) V(r) = 0$$

$$\phi(r) = K \left[\frac{e^{\mu r} - e^{-\mu r}}{2 \mu r} \right]$$

$a \le r \le b$

BECAUSE $\rightarrow V = \text{Const} \Big|_{\mu = 0}$

$a \le r \le b$

$$V = \phi(a) \qquad \Delta V = \phi(a) - \phi(b)$$

$$\frac{\Delta V}{V} = \frac{1}{6} \mu^2 (a^2 - b^2) + O\left[(\mu a)^4 \right]$$

$$\mu \le 10^{-6}\,\text{cm}^{-1} = 2 \times 10^{-11}\,\text{eV} = 4 \times 10^{-44}\,g$$

VII Williams, Faller, Hill (1971)

In 1960's IMPROVE

1. Lock in Detectors
2. $\omega \gg 0$, NOISE DOWN
3. 1, 2, 3 orders better

$$\mu \leq 2 \times 10^{-47}\ g$$

$$= 5 \times 10^{-10}\ cm^{-1}$$

$$= 9 \times 10^{-15}\ eV$$

BEST LAB LIMIT

② CONSEQUENCES, $\mu \neq 0$

a) $v_g \neq c$

$$(\Box - \mu^2) A_\lambda = 0$$

$$\left(\frac{\omega}{c}\right)^2 - \underset{\sim}{k}^2 = \mu^2$$

$$v_g = \frac{d\omega}{dk} = \frac{c^2 k}{\omega} = ck/(k^2+\mu^2)^{1/2}$$

DISPERSION OF LIGHT

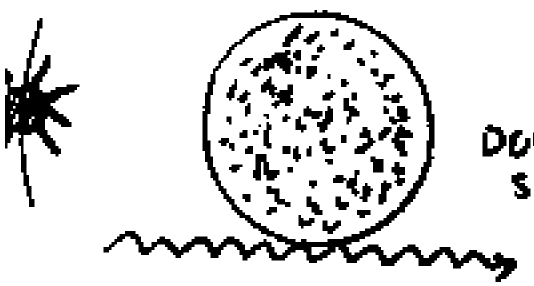
PHOTONS ARE REL. PARTICLES

③ a) dispersion of light signals

$$\delta t = \int dl \left[\frac{1}{v_R} - \frac{1}{v_B} \right] \sim \frac{L}{c^2}\left(v_B - v_R \right)$$

$$= \frac{\mu^2 L}{8\pi^2 c} \left(\lambda_B^2 - \lambda_R^2 \right) \left(\frac{c}{\hbar} \right)^2$$

De Broglie (1940)



DOUBLE STAR

$$\left. \begin{array}{l} \lambda = 4000\,\text{Å} \\ 8000\,\text{Å} \\ L = 10^3 \text{ lt yr.} \\ \delta t < 10^{-3} \text{ sec} \end{array} \right\} \begin{array}{l} \mu < 0.78 \times 10^{-39} \text{ gm} \\ \neq 10^{-44} \text{ gm.} \end{array}$$

Pulsars

Feinberg (1969) – Drake et al (1968)

Problem is plasma dispersion

$$k^2 = \frac{\omega^2}{c^2}\left[1 - \frac{\omega_p^2}{\omega^2 \pm \omega\,\omega_B}\right]$$

$$\omega_p^2 = \frac{4\pi n e^2}{m} \qquad \omega_B = \frac{eB}{mc}\cos\theta$$

IGNORE

$$v(\omega) = \frac{d\omega}{dk} = c\sqrt{1 - \frac{\omega_p^2}{\omega^2}} \simeq c\left[1 - \frac{1}{2}\frac{\omega_p^2}{\omega^2}\right]$$

SAME AS Eq FOR v_g

COMPARE

$$\left(\frac{\mu c^2}{\hbar}\right) = 8.2 \times 10^{47}\,[\mu(\mathrm{gm})]\ \mathrm{sec}^{-1}$$

$$\left(\frac{4\pi n e^2}{m}\right)^{1/2} = 5.6 \times 10^4\,[n(\mathrm{cm}^{-3})]^{1/2}\ \mathrm{sec}^{-1}$$

$\Rightarrow$

$n = 0.028$ elect/cm^3

OR

$\mu \lesssim 10^{-44}$ gm

$= 3 \times 10^7\ \mathrm{cm}^{-1}$

$= 6 \times 10^{-12}$ eV

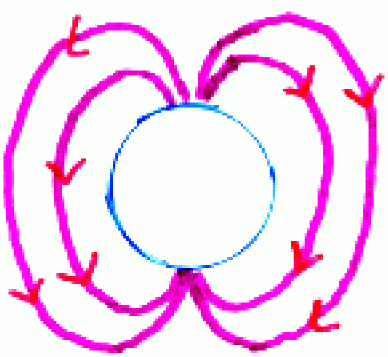
CRAB PULSAR – NP0532

3-c) Earth's Magnetic Field

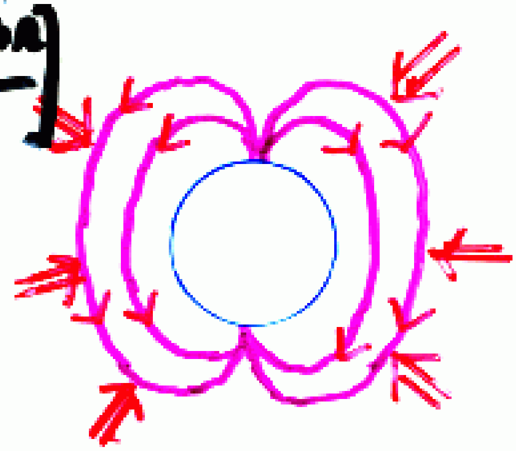
Schrödinger wanted a massive photon for his unified theory. (See Moore)

In 1943, inspired by McConnell's idea,

1. $\mu \neq 0 \Rightarrow$
$\vec{A} = \vec{\nabla} \times \vec{m} \left[\frac{e^{-\mu r}}{r} \right]$



$\mu = 0$



$\mu \neq 0$

2. BUT
$\vec{\nabla} \cdot \vec{H} = 0$
SO FLUX CONSERVED

3. So, lines closer in mostly at Equator

$$D\hat{z} = \underset{\sim}{D} = -\frac{1}{2}\int d^3r\, \underset{\sim}{J} \times \underset{\sim}{r} \left(\frac{4\pi}{c}\right)$$

$$(-\nabla^2 + \mu^2)\, \underset{\sim}{A} = J\left(\frac{4\pi}{c}\right)$$

$$\Rightarrow \quad \underset{\sim}{A} = \nabla \times \underset{\sim}{D}\left[e^{-\mu r}/r\right]$$

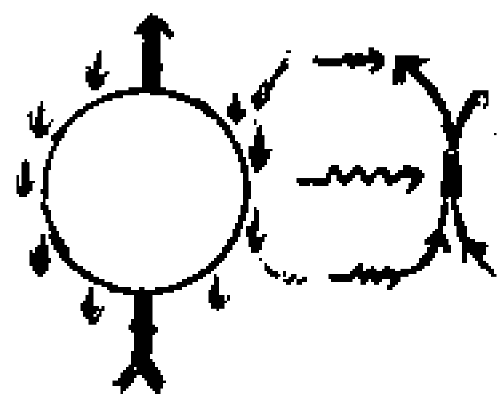
BUT $\underset{\sim}{H} = \nabla \times \underset{\sim}{A}$

$$\underset{\sim}{H} = \left[D e^{-\mu r}/r^3\right]\left[(1 + \mu r + \tfrac{1}{3}\mu^2 r^2)(3\hat{z}\cdot\hat{r}\,\hat{r} - \hat{z}) - \tfrac{2}{3}\mu^2 r^2 \hat{z}\right]$$

$$\underset{\sim}{H}_D = \left[D/r^3\right]\left[3\hat{z}\cdot\hat{r}\,\hat{r} - \hat{z}\right]$$

DIFFERENCES $\longrightarrow$ $\underline{H}_{\text{EXTERNAL}}$ (on sphere anti-∥ to $\underline{D}$ at equator but)

$$\frac{H_{ext}}{H_{D.E.}} = \frac{\frac{2}{3}\mu^2 R^2}{1 + \mu R + \frac{1}{3}(\mu R)^2}$$



I) Schrödinger had Ad Schmidt's {1885, 1922 Surveys}

$$\frac{H_{ext}}{H_{DE}} \sim \frac{539\,\gamma}{31089\,\gamma}$$

(Eq above) + (Factor of 2 later) $\Rightarrow$ $\mu \sim 10^{-47}$ gm

---

II) ASG + MMN had NASA's data (Cain et al)

From satellite + earthbd. observations,
FIT to ~ 100 spherical Harmonics.

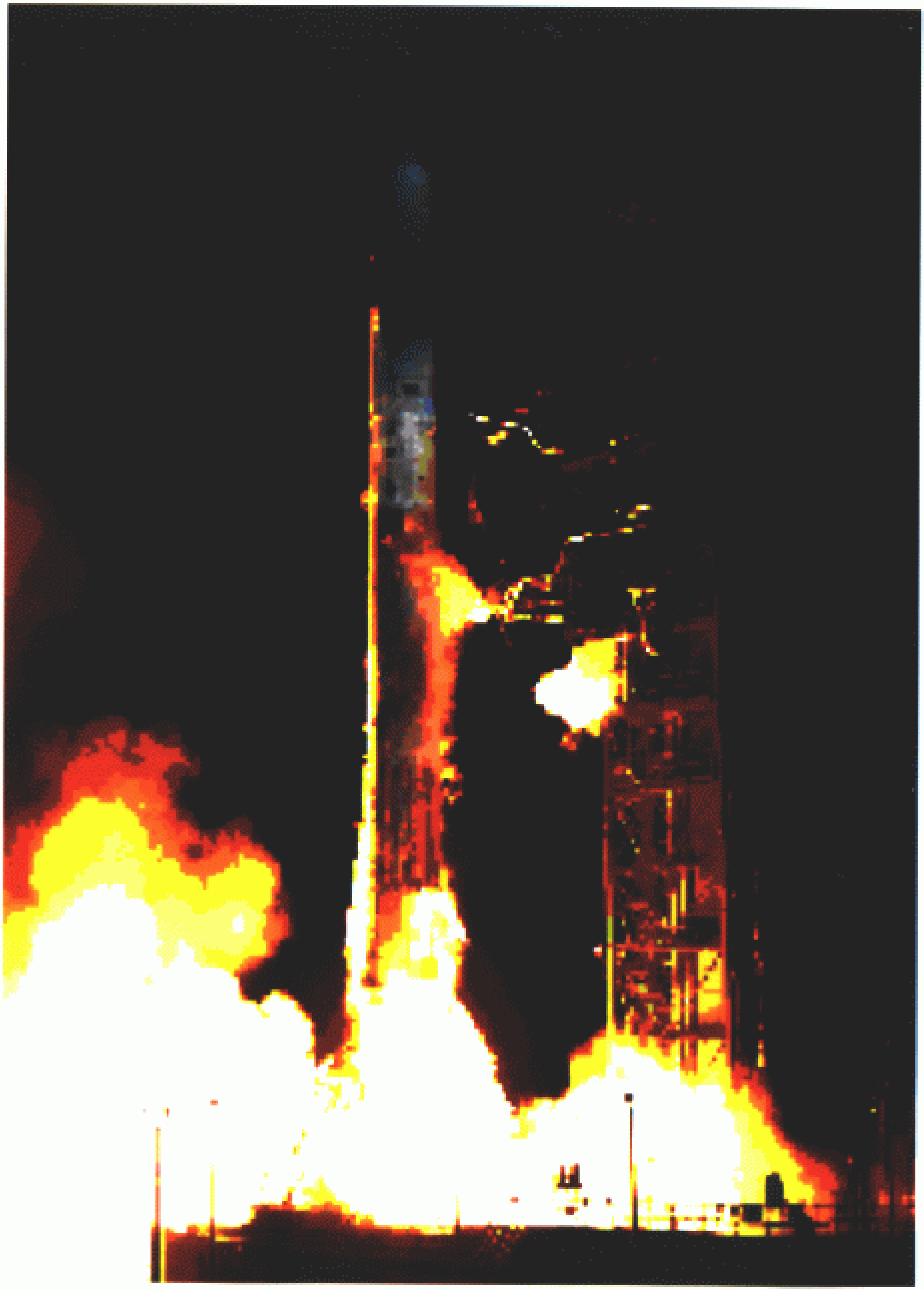
ERA (1960.0)

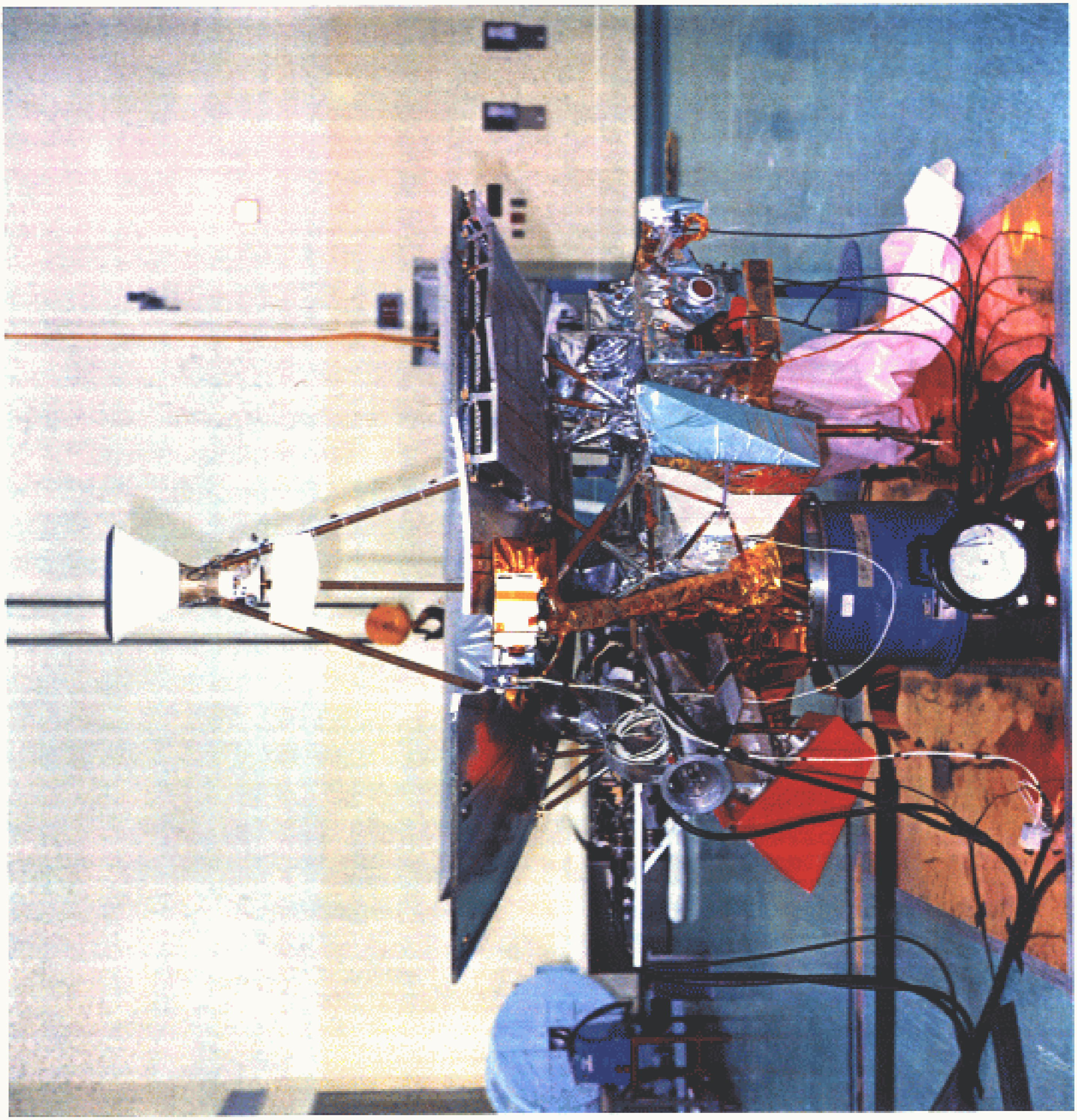
$$D = 31044\,\gamma\,R^3$$

$$\underline{H}_{ext} \cdot \underline{D} = (21 \pm 5)\,\gamma$$

$$\underline{H}_{ext} \cdot [\hat{S} \times \hat{D} / |\hat{S} \times \hat{D}|] = \underline{H}_{ext} \cdot \hat{\eta} = (14 \pm 5)\,\gamma$$

$$\underline{H}_{ext} \cdot \hat{\eta} \times \hat{D} = (8 \pm 5)\,\gamma$$



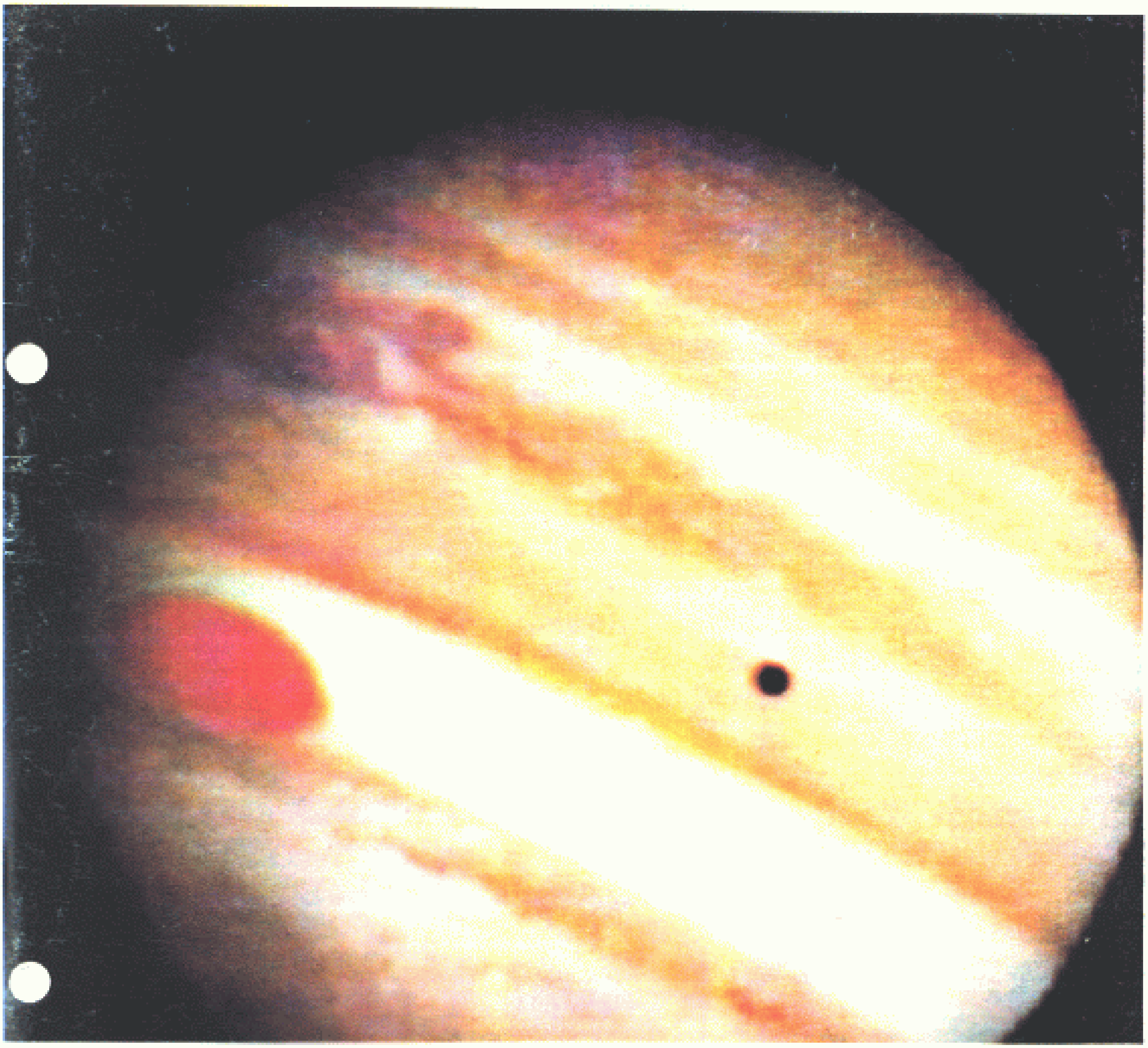


SCIENCE

25 January 1974

Vol. 183, No. 4122

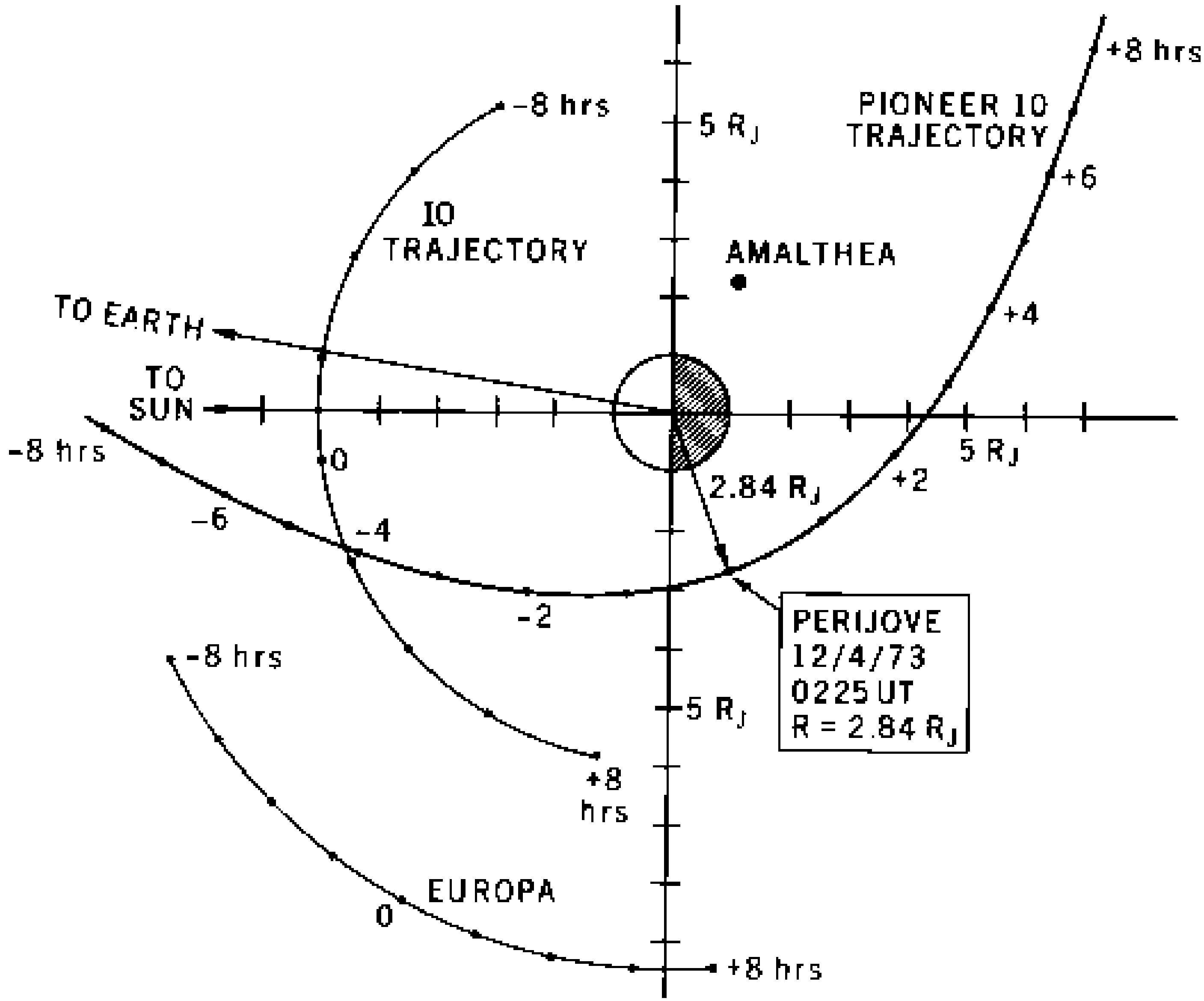
FOR THE ADVANCEMENT OF



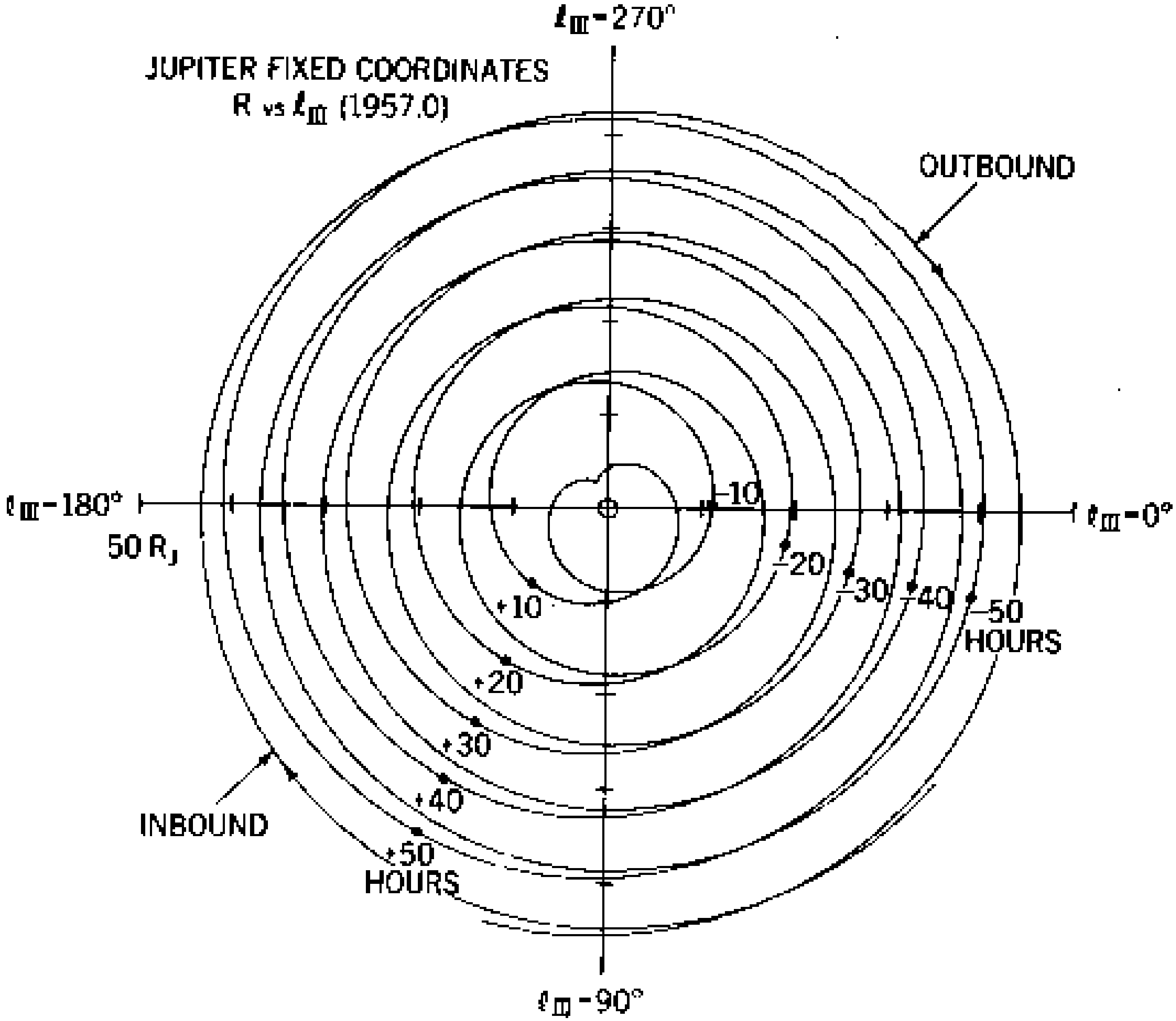
3-d) Jupiter's Magnetic Field

[L. Davis, Jr., ASG, MMN, Phys. Rev. Lett. 35, 1402, 1975.]

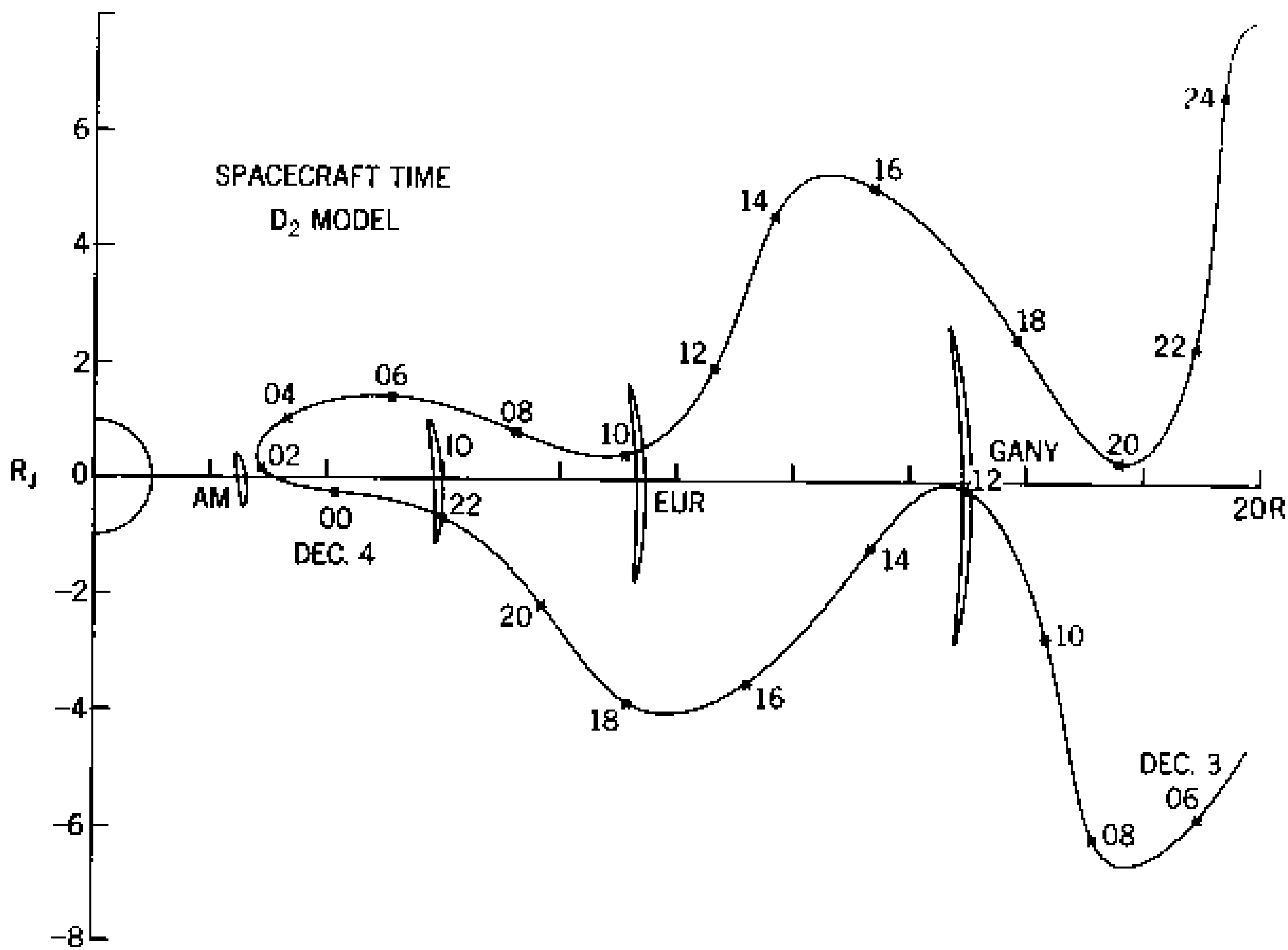
Fit to μ, internal & external dipoles and quadrapoles, taking out ring currents, etc.



PIONEER 10 TRAJECTORY
+8 hrs
+6
+4
+2
-2
-4
-6
-8 hrs
IO TRAJECTORY
-8 hrs
0
+8 hrs
AMALTHEA
5 R_J
5 R_J
5 R_J
2.84 R_J
PERIJOVE
12/4/73
0225 UT
R = 2.84 R_J
TO EARTH
TO SUN
EUROPA
-8 hrs
0
+8 hrs



JUPITER FIXED COORDINATES
R vs ℓ_{III} (1957.0)
ℓ_{III} = 270°
ℓ_{III} = 180°
ℓ_{III} = 0°
ℓ_{III} = 90°
50 R_J
OUTBOUND
INBOUND
-10
-20
-30
-40
-50
HOURS
+10
+20
+30
+40
+50
HOURS


SPACECRAFT TIME
D2 MODEL
R J
6
4
2
0
−2
−4
−6
−8
AM
IO
EUR
GANY
20R J
DEC. 4
DEC. 3
00
02
04
06
08
10
12
14
16
18
20
22
24

<u>VERY</u> conservative fit obtained

$$\mu \leq 8 \times 10^{-49}\ g$$

$$= 6 \times 10^{-16}\ eV$$

$$= 2 \times 10^{-11}\ cm^{-1}$$

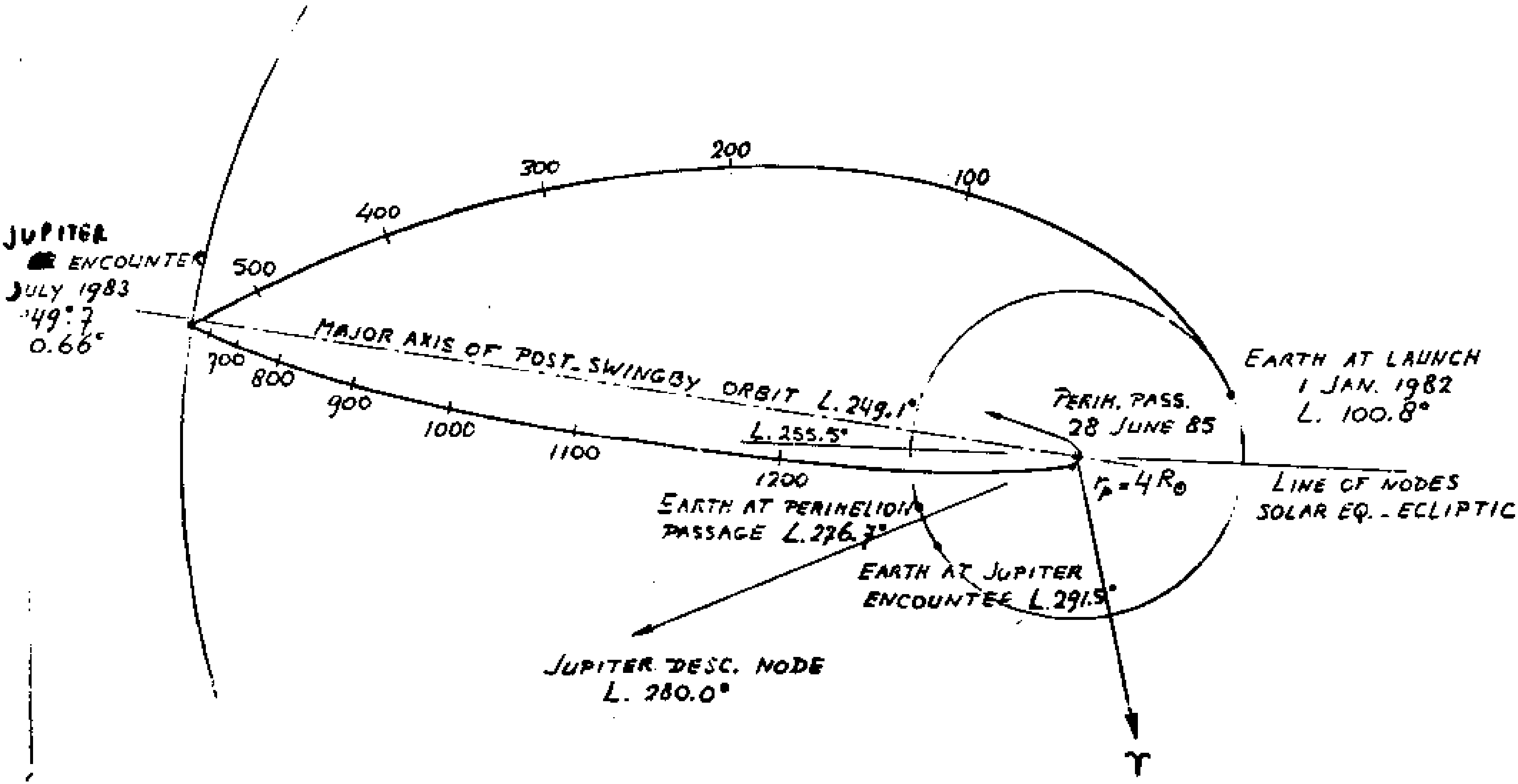
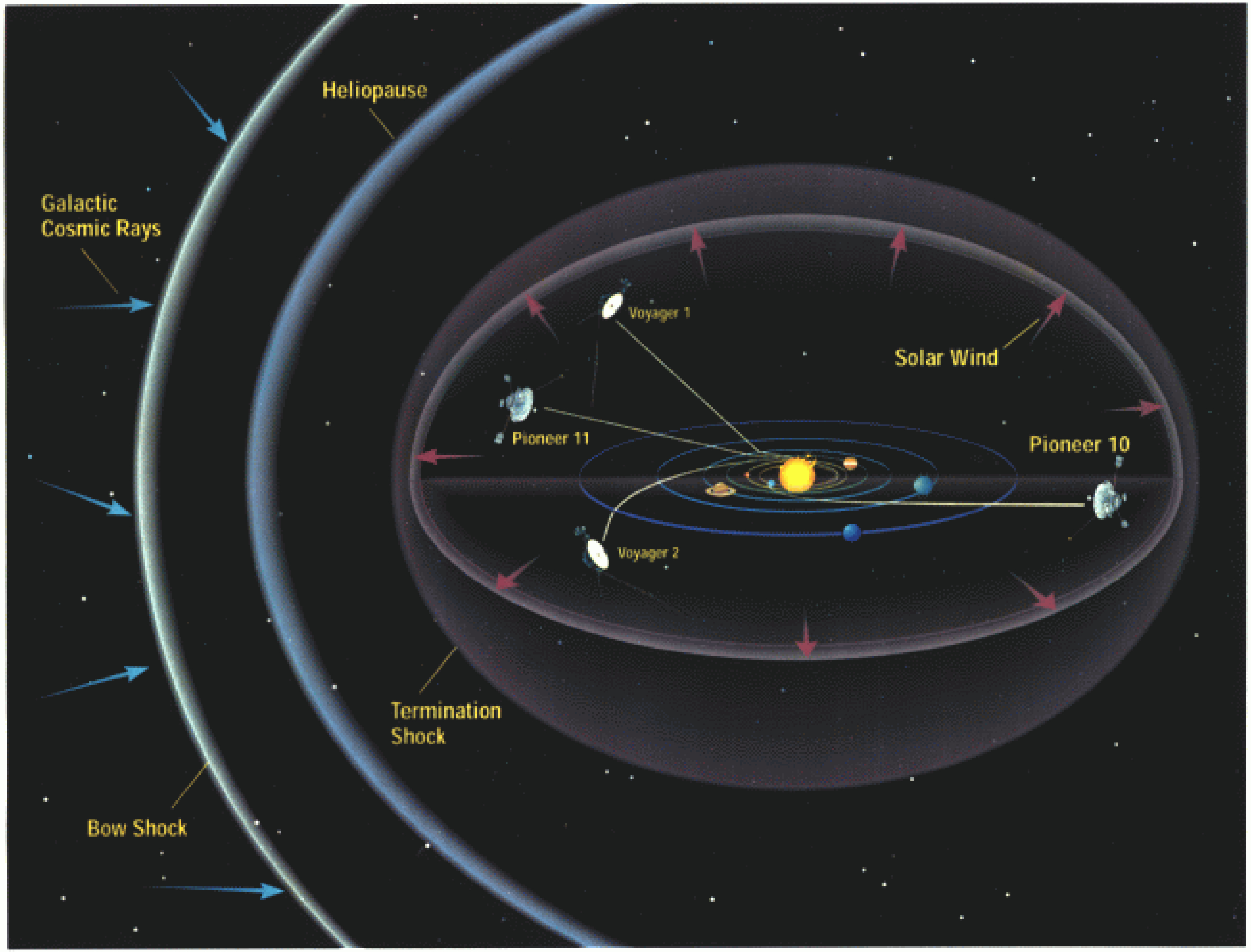


Fig 3.3 Trajectory geometry for in-ecliptic SPM with perihelion of 4 Solar radii


Heliopause
Galactic
Cosmic Rays
Voyager 1
Solar Wind
Pioneer 11
Pioneer 10
Voyager 2
Termination
Shock
Bow Shock

What don't we

understand about gravity?

☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆☆

Actually, we don't

understand gravity and

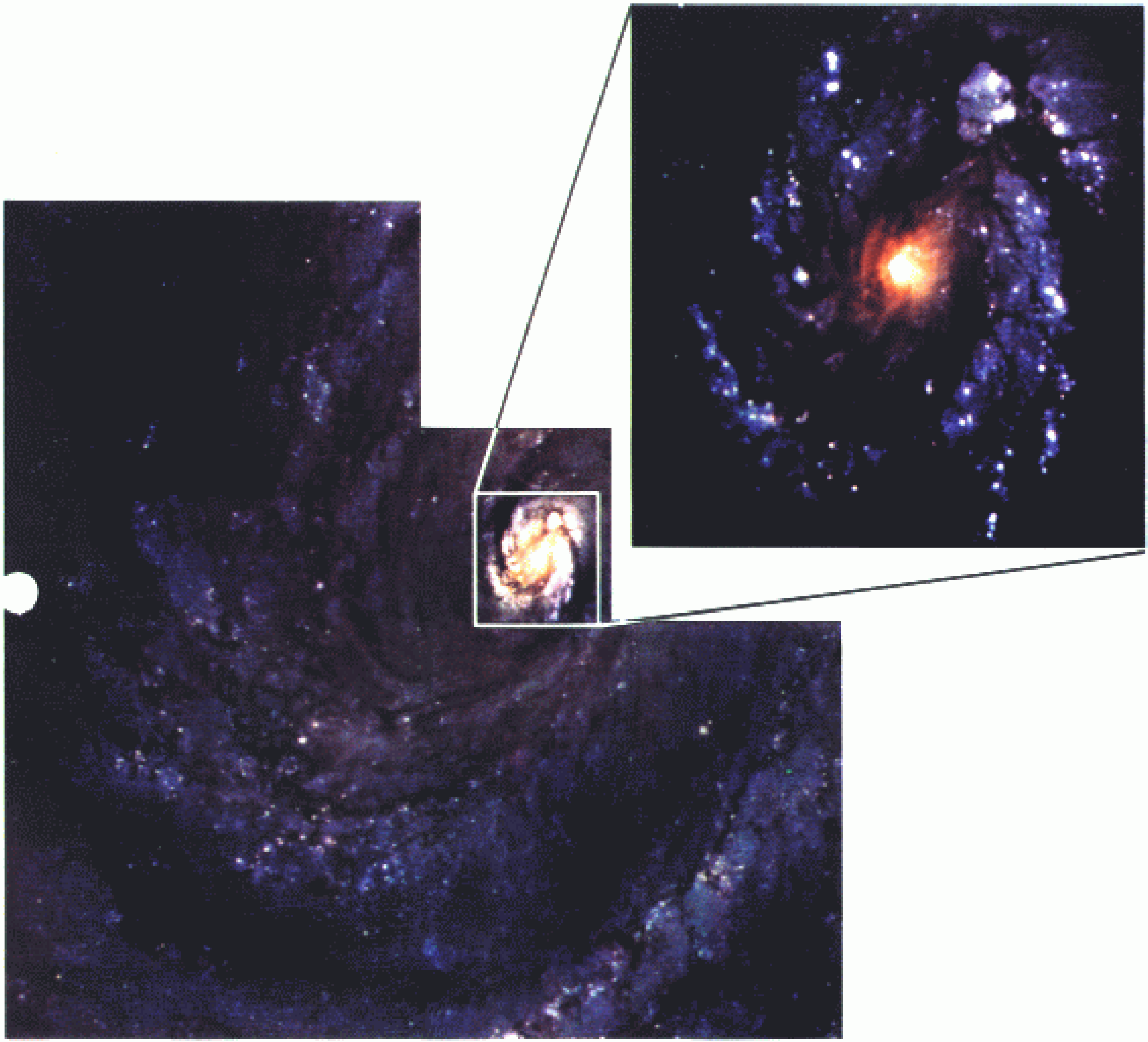
matter

(let alone antimatter)

for almost

all of the universe !!

DMP



By the way, the biggest systematic in our acceleration residuals is a bias of

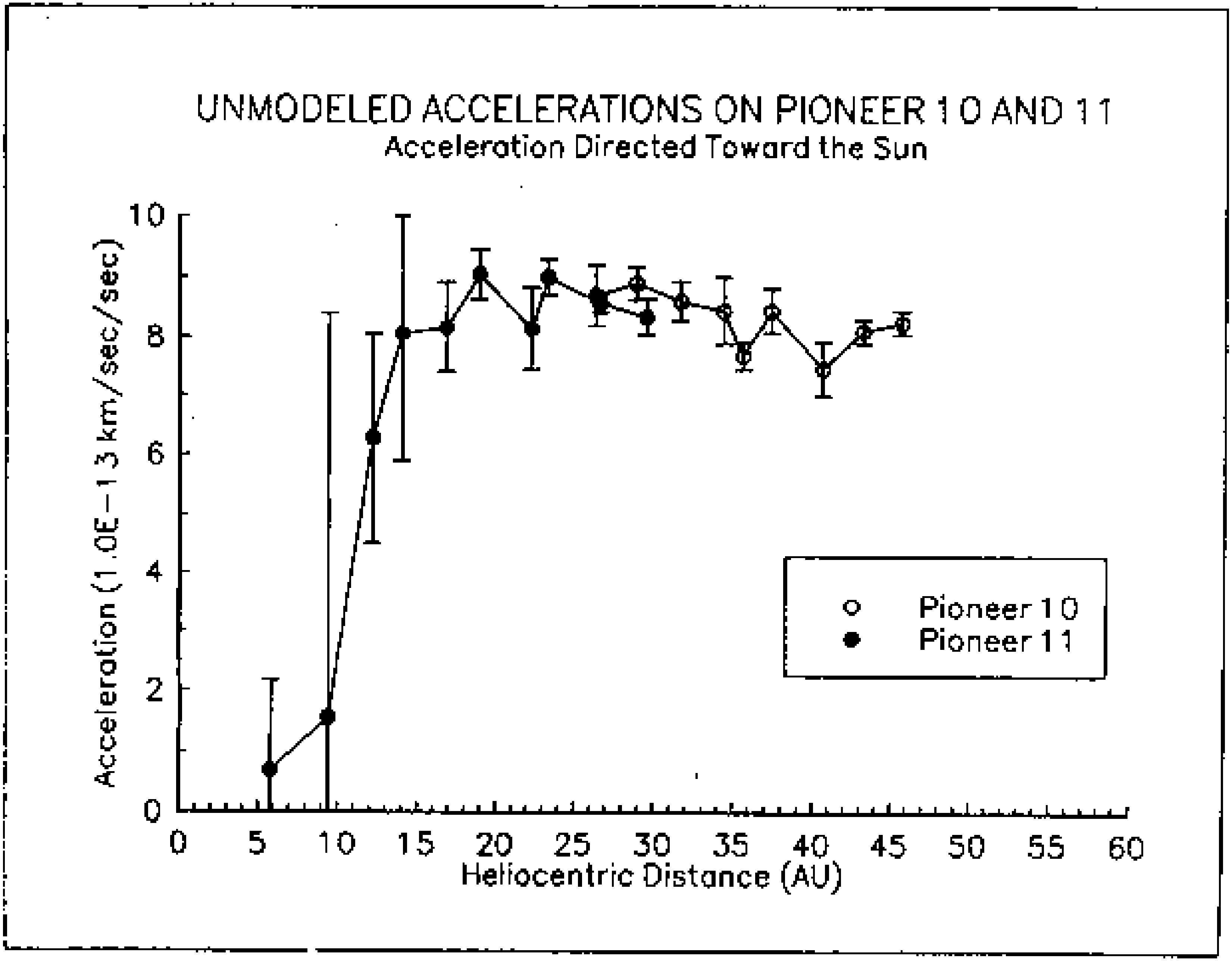
8 X 10^-13 km/s^2

directed toward the Sun.

1994
LEAP

$a_N = 5.93 \times 10^{-6} \frac{km}{s^2}$

Figure 1


UNMODELED ACCELERATIONS ON PIONEER 10 AND 11
Acceleration Directed Toward the Sun
Acceleration (1.0E−13 km/sec/sec)
Heliocentric Distance (AU)
Pioneer 10
Pioneer 11
0
2
4
6
8
10
0
5
10
15
20
25
30
35
40
45
50
55
60

JOHN D. ANDERSON (JPL)

PHILIP A. LAING (AEROSPACE)

EUNICE L. LAU (JPL)

ANTHONY S. LIU (ASTRODY. SCI.)

MMN (LANL)

SLAVA G. TURYSHEV (JPL)

AFTER 2 CODES ('98 PRL)

PIONEER 10 (ODP) ['87 – '94]

$$(8.09 \pm 0.20) \times 10^{-8} \text{ cm/sec}^2$$

PIONEER 11 (ODP) ['87 – '90]

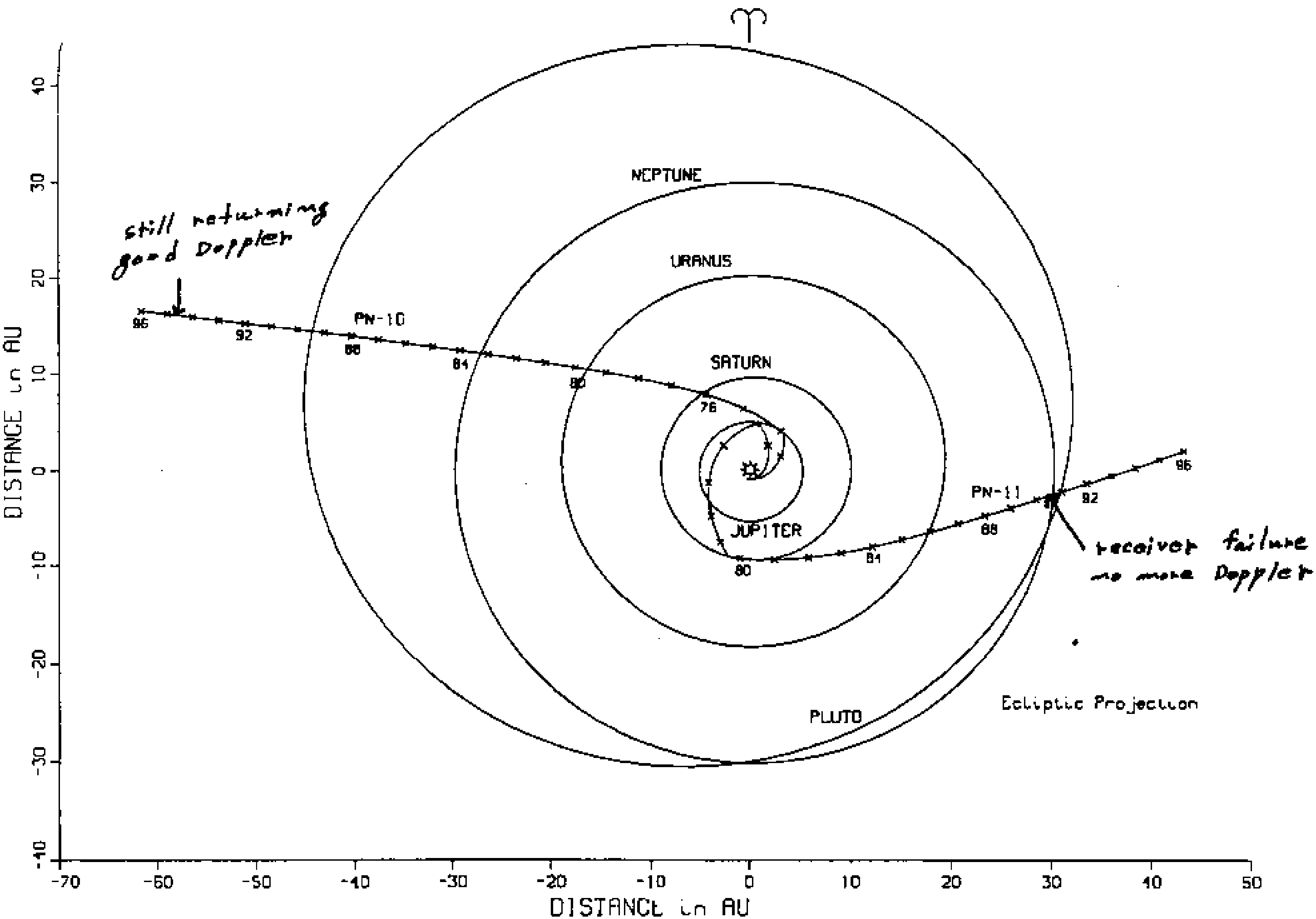
$$(8.56 \pm 0.15) \times 10^{-8} \text{ cm/sec}^2$$

PIONEER 10 (CHASMP)

$$(8.65 \pm 0.03)$$

NOW NEED SYSTEMATICS

\+ PIO 10 DATA → 1998



NASA/AMES
PPO/88-1.1-2
D' 3/2/94

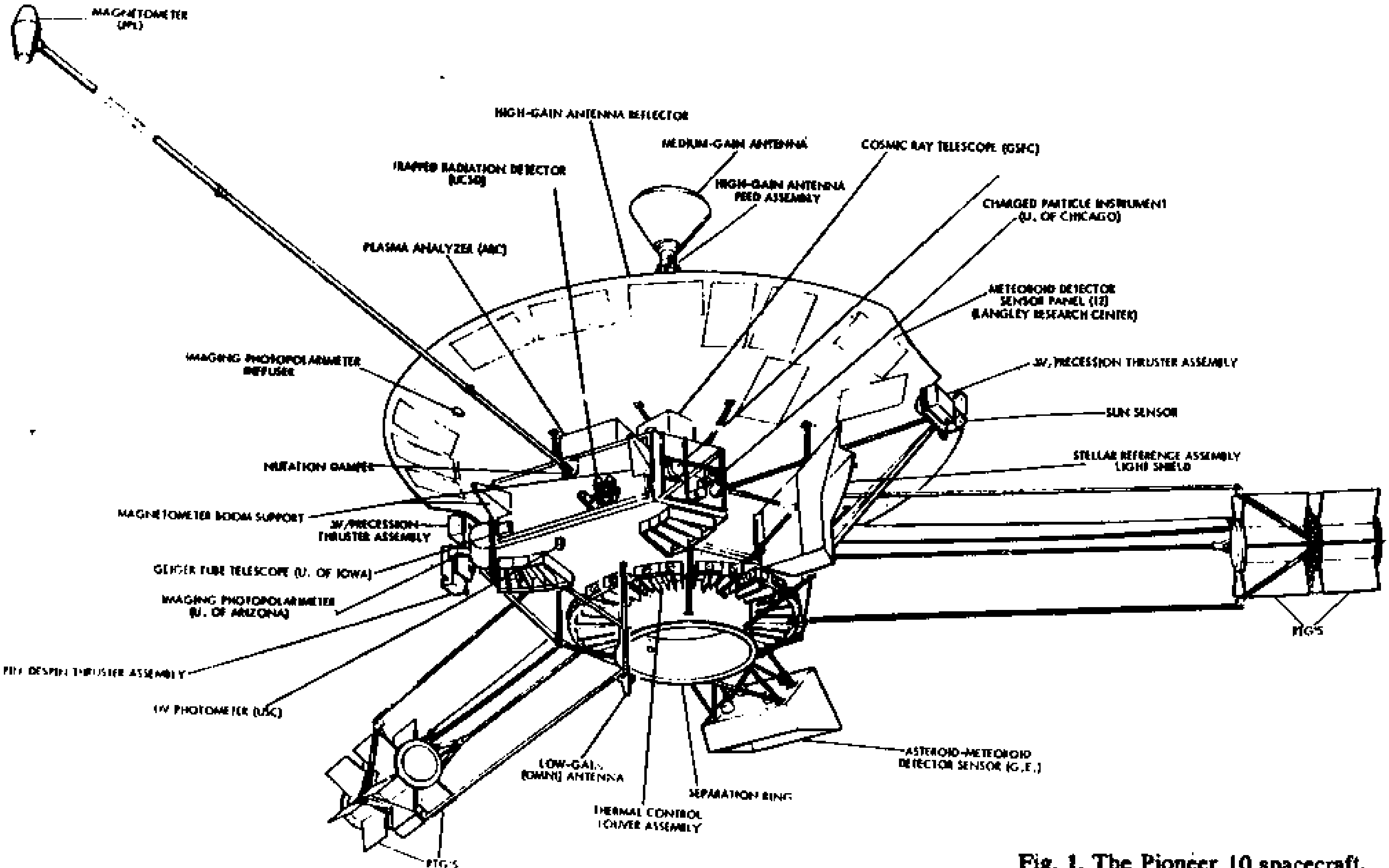
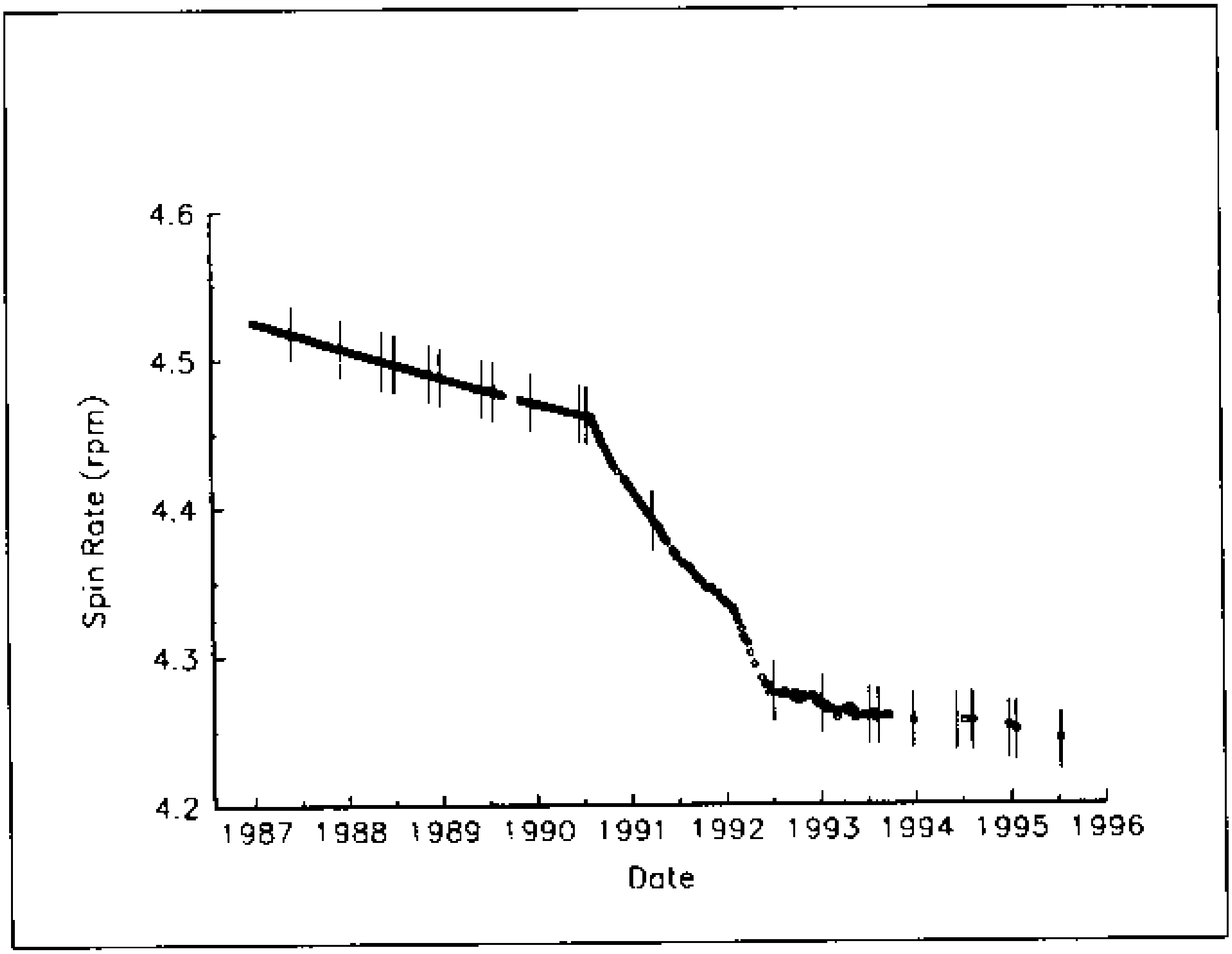


Fig. 1. The Pioneer 10 spacecraft.

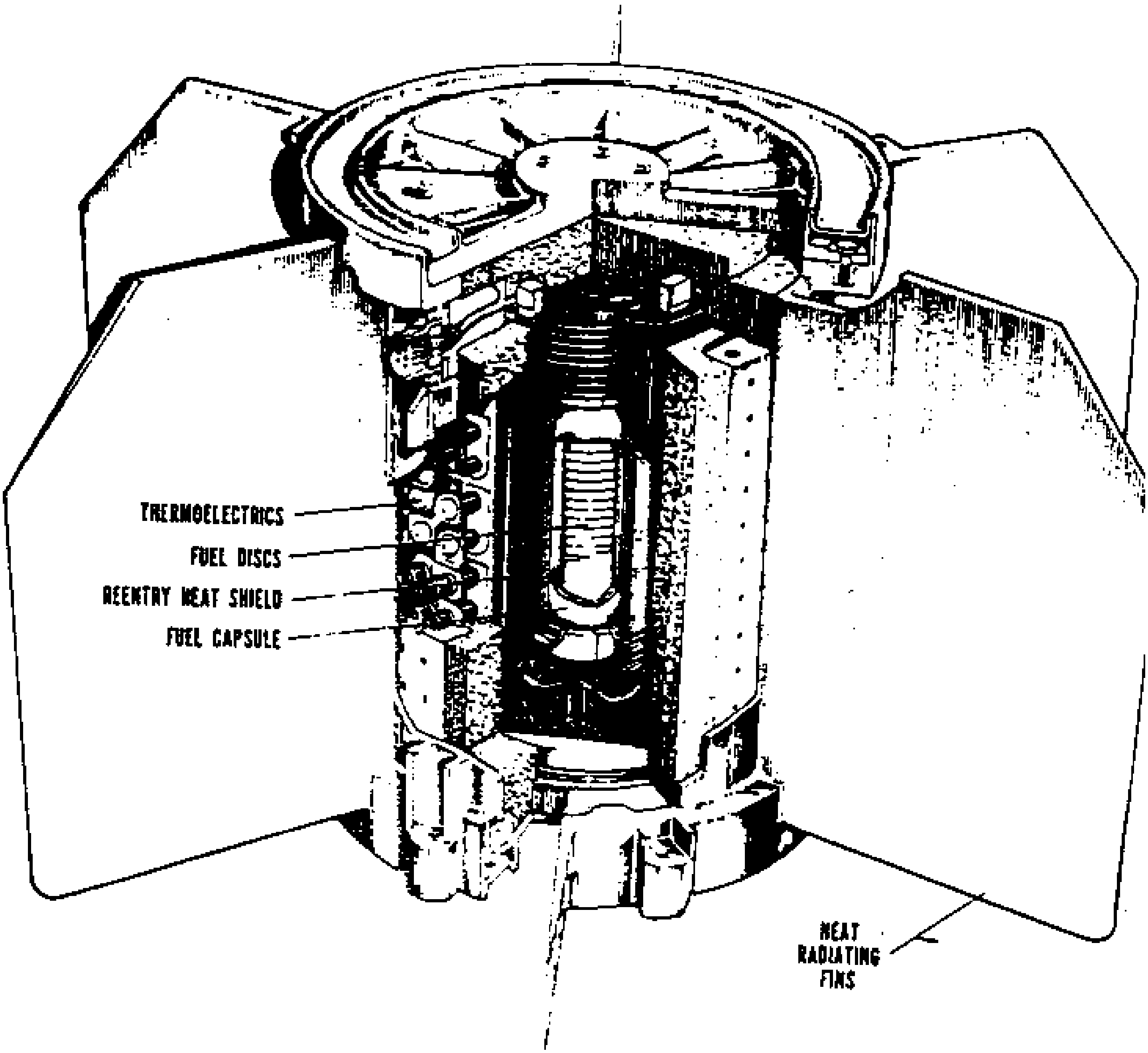


The spin history of Pioneer 10.

Program/Estimation method	Pio 10 (I)	Pio 10 (II)	Pio 10 (III)	Pio 11
Sigma, WLS, no solar corona model	8.02 ± 0.01	8.65 ± 0.01	7.83 ± 0.01	8.46 ± 0.04
Sigma, WLS, with solar corona model	8.00 ± 0.01	8.66 ± 0.01	7.84 ± 0.01	8.44 ± 0.04
Sigma, BSF, 1-day batch, with solar corona model	7.82 ± 0.29	8.16 ± 0.40	7.59 ± 0.22	8.49 ± 0.33
CHASMP, WLS, no solar corona model	8.25 ± 0.02	8.86 ± 0.02	7.85 ± 0.01	8.71 ± 0.03
CHASMP, WLS, with solar corona model	8.22 ± 0.02	8.89 ± 0.02	7.92 ± 0.01	8.69 ± 0.03
CHASMP, WLS, with corona, weighting, and F10.7	8.25 ± 0.03	8.90 ± 0.03	7.91 ± 0.01	8.91 ± 0.04

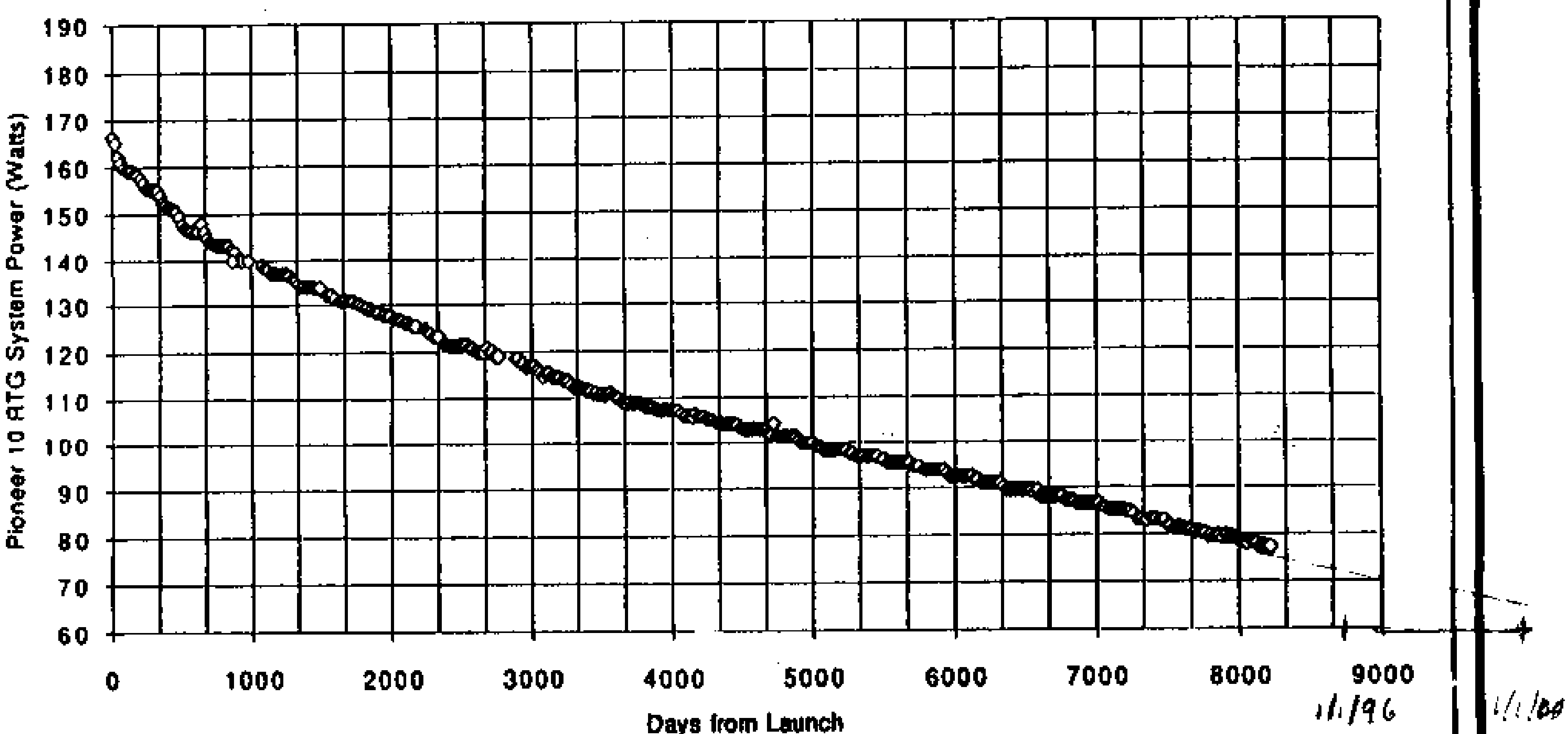
$a^{10}_{p(EX)} = (7.84 \pm 0.01) \times 10^{-8}$ cm/s^2

$a^{11}_{p(EX)} \rightarrow$ 8.55

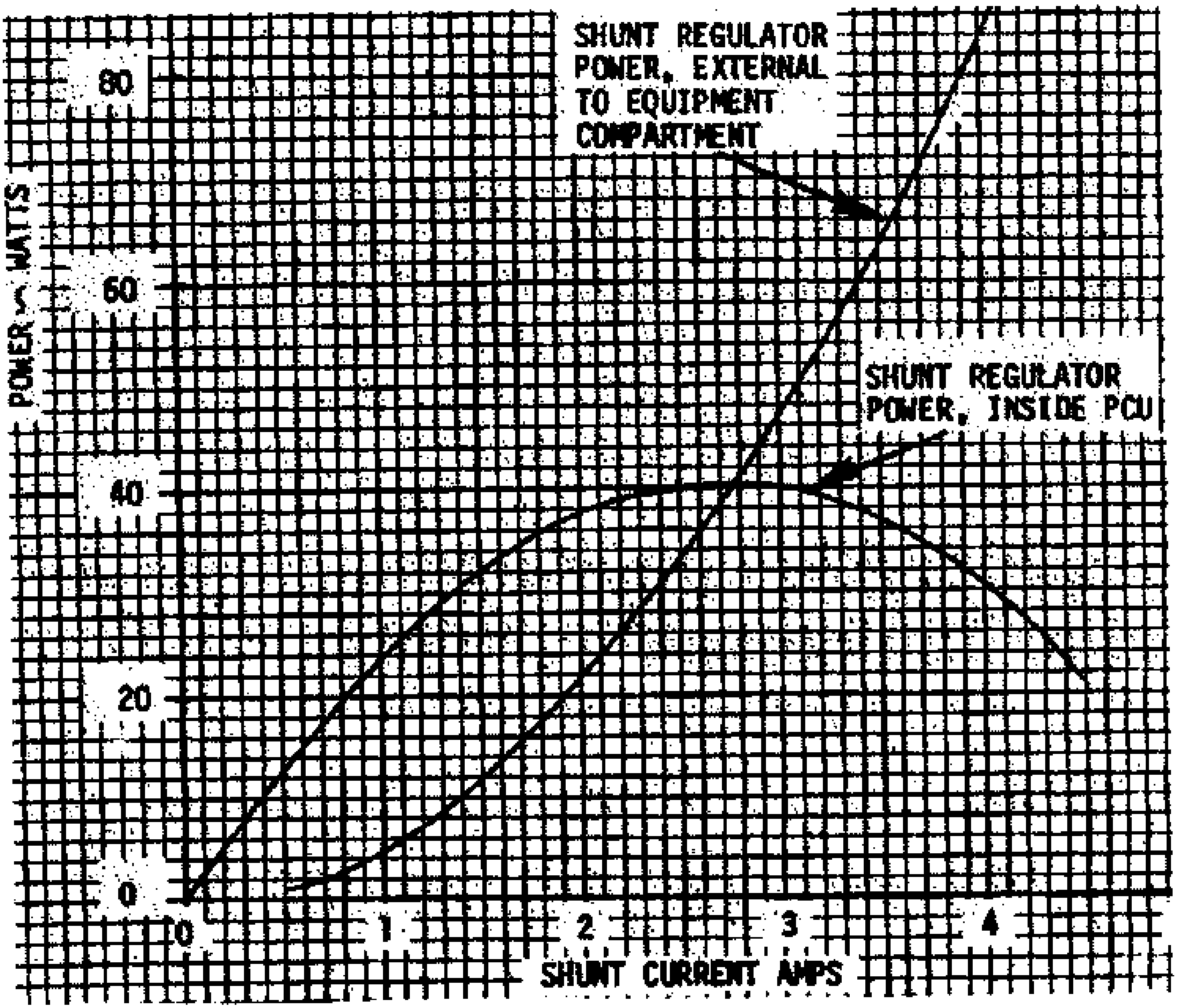


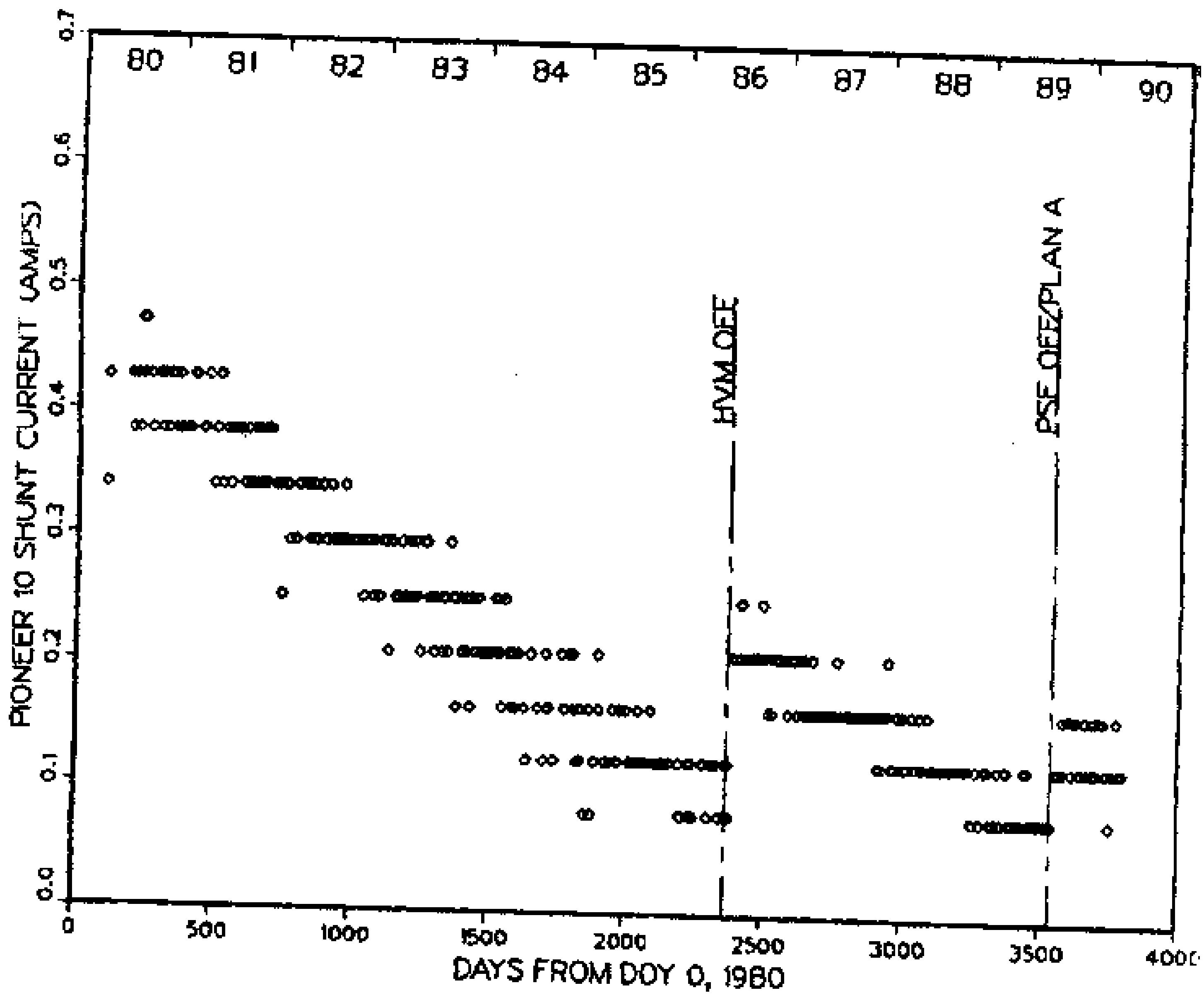
SNAP 19/PIONEER RADIOISOTOPE THERMOELECTRIC GENERATOR

Pioneer 10 RTG System Power History



NASA/AMES
PPO/891.2-1
BJK: 12/1/94


SHUNT REGULATOR POWER, EXTERNAL TO EQUIPMENT COMPARTMENT
SHUNT REGULATOR POWER, INSIDE PCU
POWER ~ WATTS
80
60
40
20
0
0
1
2
3
4
SHUNT CURRENT AMPS



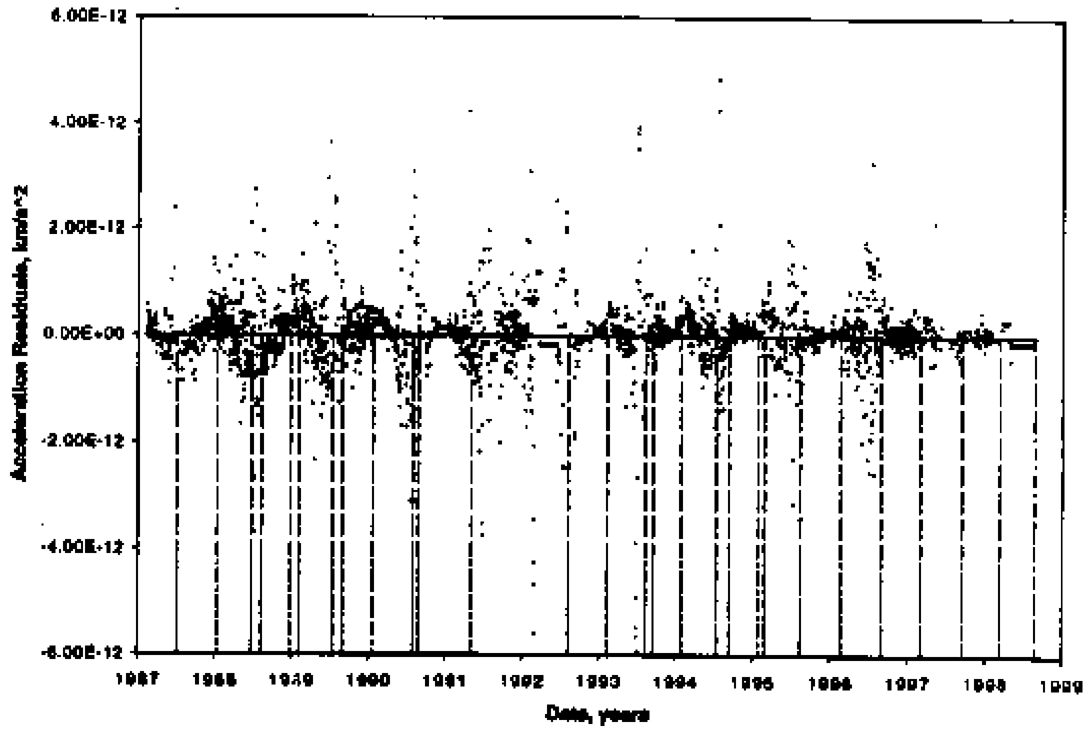
PIONEER 10 SHUNT CURRENT (AMPS)
DAYS FROM DOY 0, 1980
80
81
82
83
84
85
86
87
88
89
90
HVM OFF
PSE OFF/PLAN A
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
500
1000
1500
2000
2500
3000
3500
4000

2001 gr-qc/0104064

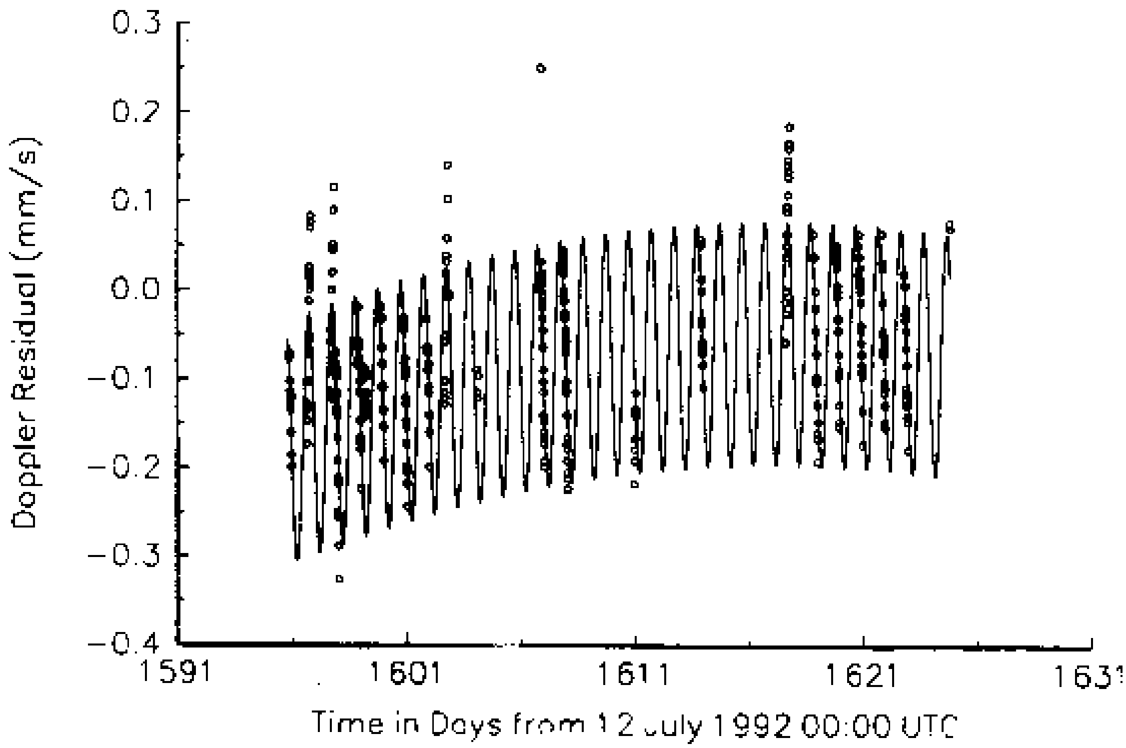
Table 2: Error Budget: A Summary of Biases and Uncertainties.

Item	Description of error budget constituents	Bias 10^{-8} cm/s^2	Uncertainty, 10^{-8} cm/s^2
1	Systematics generated external to the spacecraft:		
	a) Solar radiation pressure and mass	+0.03	±0.01
	b) Solar wind		$\pm < 10^{-5}$
	c) Solar corona		±0.02
	d) Electro-magnetic Lorentz forces		$\pm < 10^{-4}$
	e) Influence of the Kuiper belt's gravity		±0.03
	f) Influence of the Earth orientation		±0.001
	g) Mechanical and phase stability of DSN antennae		$\pm < 0.001$
	h) Phase stability and clocks		$\pm < 0.001$
	i) DSN station location		$\pm < 10^{-5}$
	j) Troposphere and ionosphere		$\pm < 0.001$
2	On-board generated systematics:		
	a) Radio beam reaction force	+1.10	±0.11
	b) RTG heat reflected off the craft	−0.55	±0.55
	c) Differential emissivity of the RTGs		±0.85
	d) Non-isotropic radiative cooling of the spacecraft		±0.16
	e) Expelled Helium produced within the RTGs	+0.15	±0.16
	f) Gas leakage		±0.56
	g) Variation between spacecraft determinations	+0.17	±0.17
3	Computational systematics:		
	a) Numerical stability of least-squares estimation		±0.02
	b) Accuracy of consistency/model tests		±0.13
	c) Mismodeling of maneuvers		±0.01
	d) Mismodeling of the solar corona		±0.02
	e) Annual/diurnal terms		±0.32
	Estimate of total bias/error	+0.90	±1.25

$$a_p = (8.74 \pm 1.25) \times 10^{-8} \text{ cm/s}^2$$

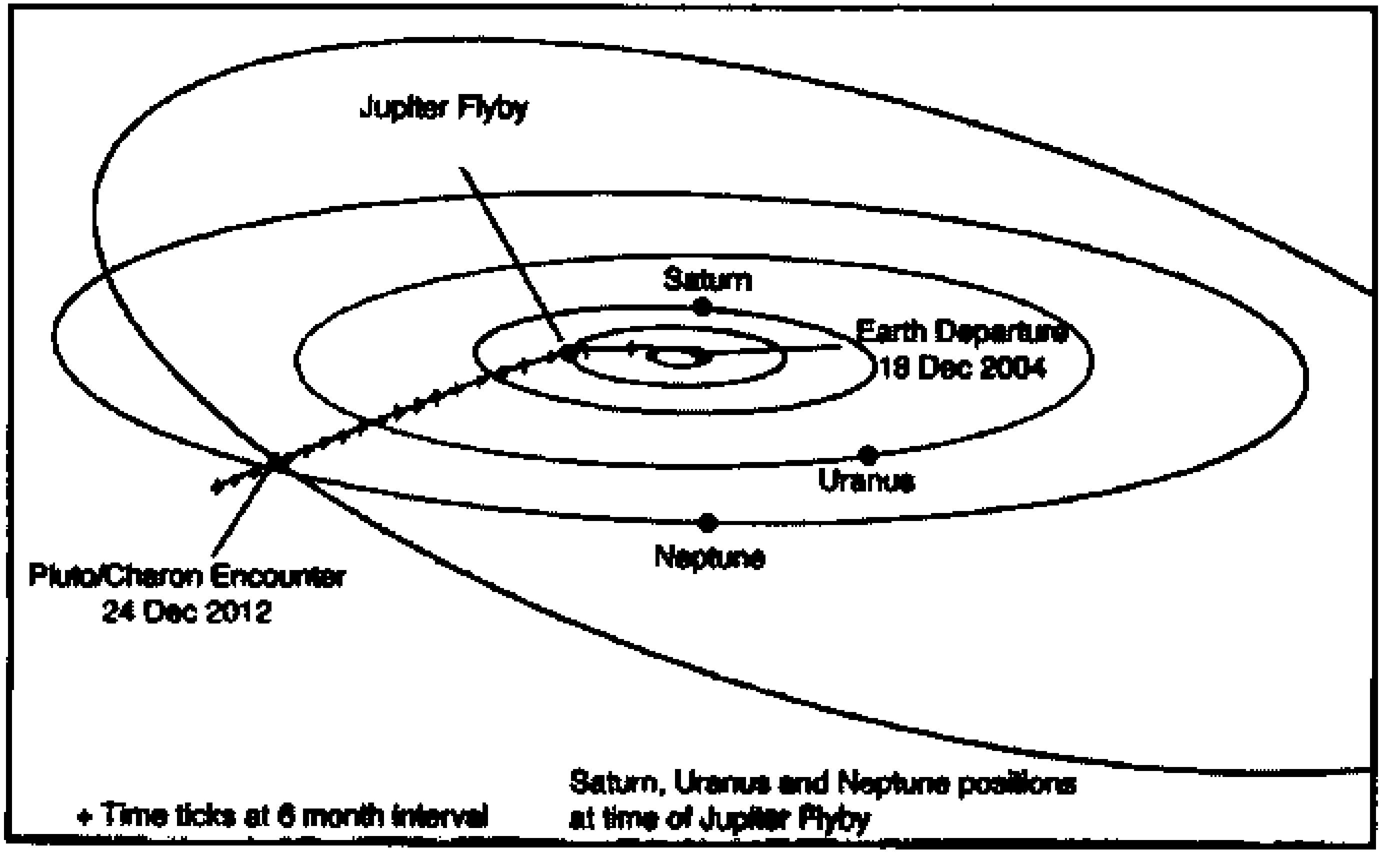


CHASMP Doppler Residuals For Interval III
30-Day Interval Surrounding 1996 Opposition



Example Pluto-Kuiper Express

8-year, Jupiter Gravity Assist* (JGA) trajectory to Pluto



A number of alternative trajectories have been studied for Pluto-Kuiper Express and the current favorable path for the 2004 launch date is a Jupiter Gravity Assist (JGA), arriving at Pluto/Charon 8 years later.