Limits on the Rest Mass of the Photon: from

to Vulcan-Solar Probe

Alfred S. Goldhaber and MMN
PRL 21, 567 (1968)
26, 1390 (1971)
RMP 43, 227 (1971)
PRD 9, 1119 (1974)
Sci. Am., May (1976)
with L. Davis, Jr. PRL 35, 1402 (1975)

New work with V. A. Kostelecky, PLB 317, 223(1993).

1) Maxwell Eqs. → Proca Eq.

($\mu \neq 0$)

2) Consequences of $\mu \neq 0$

3) Limits on μ from

   a) c = Const.

   b) Coulomb's Law

   c) Earth's magnetic field

   d) Jupiter's magnetic field

THIS CAN ALL BE DONE

RIGOROUSLY (See RMP)

Special relativity

Fields Linear in Currents } ⇒ **PROCA**

Energy Conservation

PROCA Eq. [Maxwell $\mu \neq 0$]

$$\partial^\lambda F_{\lambda\nu} + \mu^2 A_\nu = \left(\frac{4\pi}{c}\right) J_\nu$$

$$F_{\lambda\nu} = \partial_\lambda A_\nu - \partial_\nu A_\lambda$$

$$\left(\Box + \mu^2\right) A_\nu = \frac{4\pi}{c} J_\nu$$

[CHARGE CONSERVED]

$$\partial^\lambda A_\lambda = 0$$

[LORENTZ GAUGE
no gauge invariance]

New "Maxwell's Eq"

$$\left\{ \begin{array}{l} \nabla \cdot \underset{\sim}{E} = 4\pi\rho - \mu^2 V \\ \nabla \times \underset{\sim}{E} = -\frac{1}{c}\frac{\partial \underset{\sim}{H}}{\partial t} \\ \nabla \cdot \underset{\sim}{H} = 0 \\ \nabla \times \underset{\sim}{H} = \frac{1}{c}\frac{\partial \underset{\sim}{E}}{\partial t} + \frac{4\pi}{c}\underset{\sim}{J} - \mu^2 \underset{\sim}{A} \end{array} \right\}$$

STILL $\left\{ \begin{array}{l} \underset{\sim}{H} = \nabla \times \underset{\sim}{A} \\ \underset{\sim}{E} = -\nabla V - \frac{1}{c}\frac{\partial \underset{\sim}{A}}{\partial t} \end{array} \right\}$

b) $F \not\propto 1/r^2$

$$(\nabla^2 - \mu^2) A_\lambda = -4\pi J_\lambda / c$$

$$V = \frac{\exp(-\mu r)}{r}$$

YUKAWA POTENTIAL

$$F = -\nabla V$$

$$= \frac{e^{-\mu r}}{r^2}\left[1 + \mu r\right]$$

$$\approx \frac{1}{r^2}\left[1 + \frac{\mu^2 r^2}{2} + \cdots\right.$$

↖ THM

⇒ NEED HIGH PREC. OR BIG APPARATUS

c) 3rd Polarization ---

SIDELIGHT

μ_ν

① BIG APPARATUS
(ASG/MMM)

~ CLUSTERS
CDM ?

② SMALL APPARATUS
+ HIGH PRECISION

⟨merc⟩ — D. Groom

③ⓑ "Coulomb's Law"

I) PRIESTLY (1766-1767) "The History & Present State of Electricity" FROM FRANKLIN'S 1755 EXPT

XV. Experiments with an ELECTRIFIED CUP.

I shall close the account of my experiments with a small set, in which, as well as in the last, I have little to boast besides the honour of following the instructions of Dr. Franklin. He informed me, that he had found cork balls to be wholly unaffected by the electricity of a metal cup, within which they were held; and he desired me to repeat and ascertain the fact, giving me leave to make it public.

May we not infer from this experiment, that the attraction of electricity is subject to the same laws with that of gravitation, and is therefore according to the squares of the distances; since it is easily demonstrated, that were the earth in the form of a shell, a body in the inside of it would not be attracted to one side more than another.

II) Franklin. (single fluid ⇒ ±)

Franz Æpinus (1759 .. Later in St. Petersburg) speculates r^{-2}

JOHN ROBISON (classicist 1769) published in 1803 (Ency. Brit. supp.)

$$F \propto \frac{e \cdot e_2}{r^{2+q}} \qquad q = 0.06$$

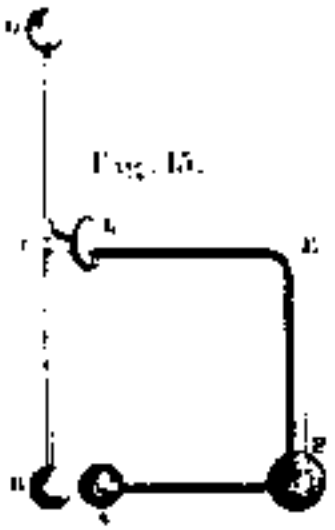


Fig. 15.

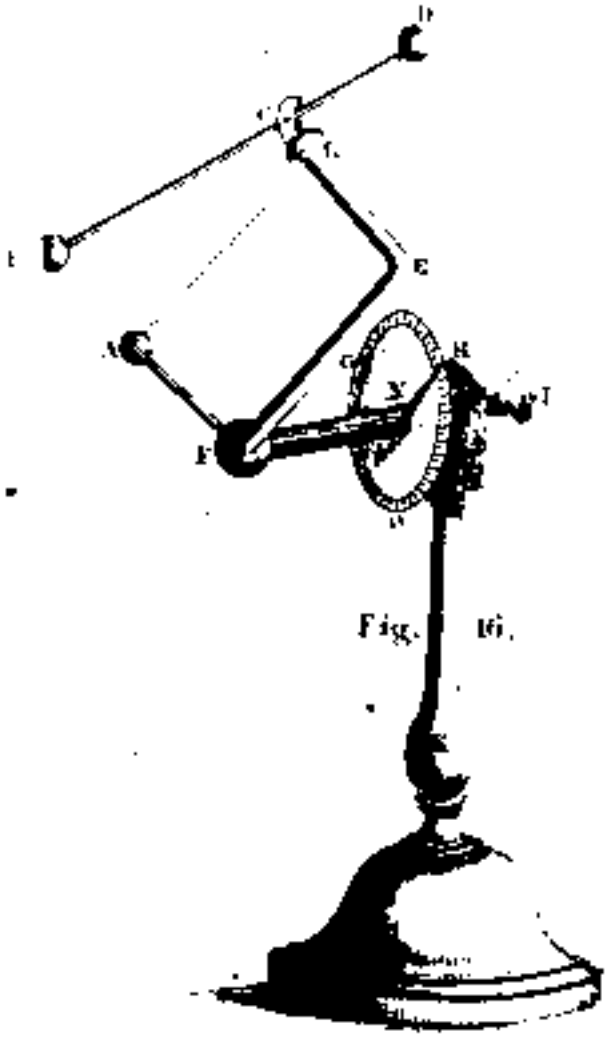


Fig. 16.

III) CAVENDISH (1773)

Published by Maxwell in 1879

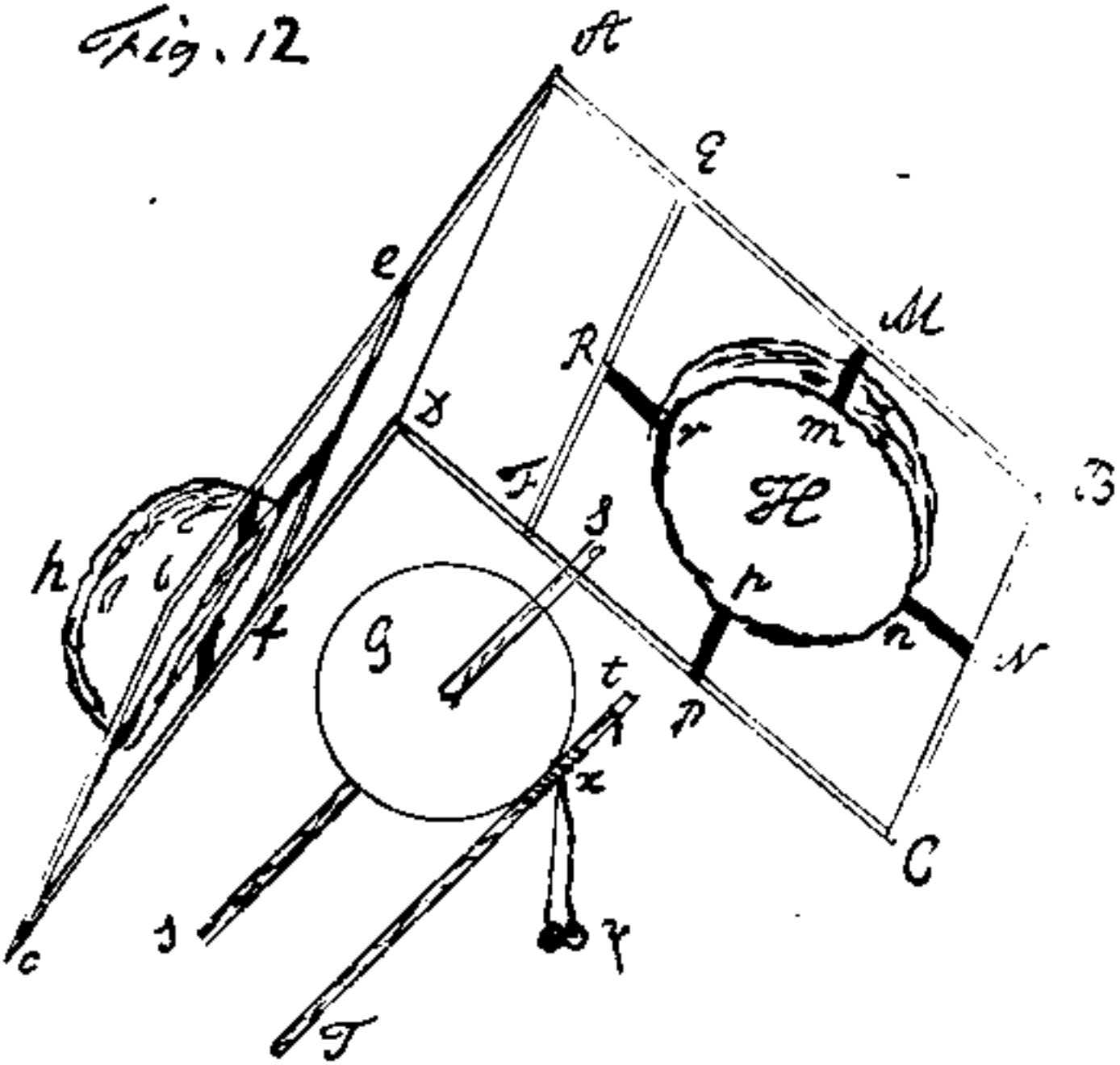
$|q| < \frac{1}{50} = 0.02$

① Shells closed with contact. Charge placed on outer sphere.

② Break connection

③ Open outer sphere + see if any charge on inner sphere.

Fig. 12



IV) COULOMB (1785- pub. 1789)

$q = 0.1$

Torsion balance at various distances --- Both repulsion + attraction

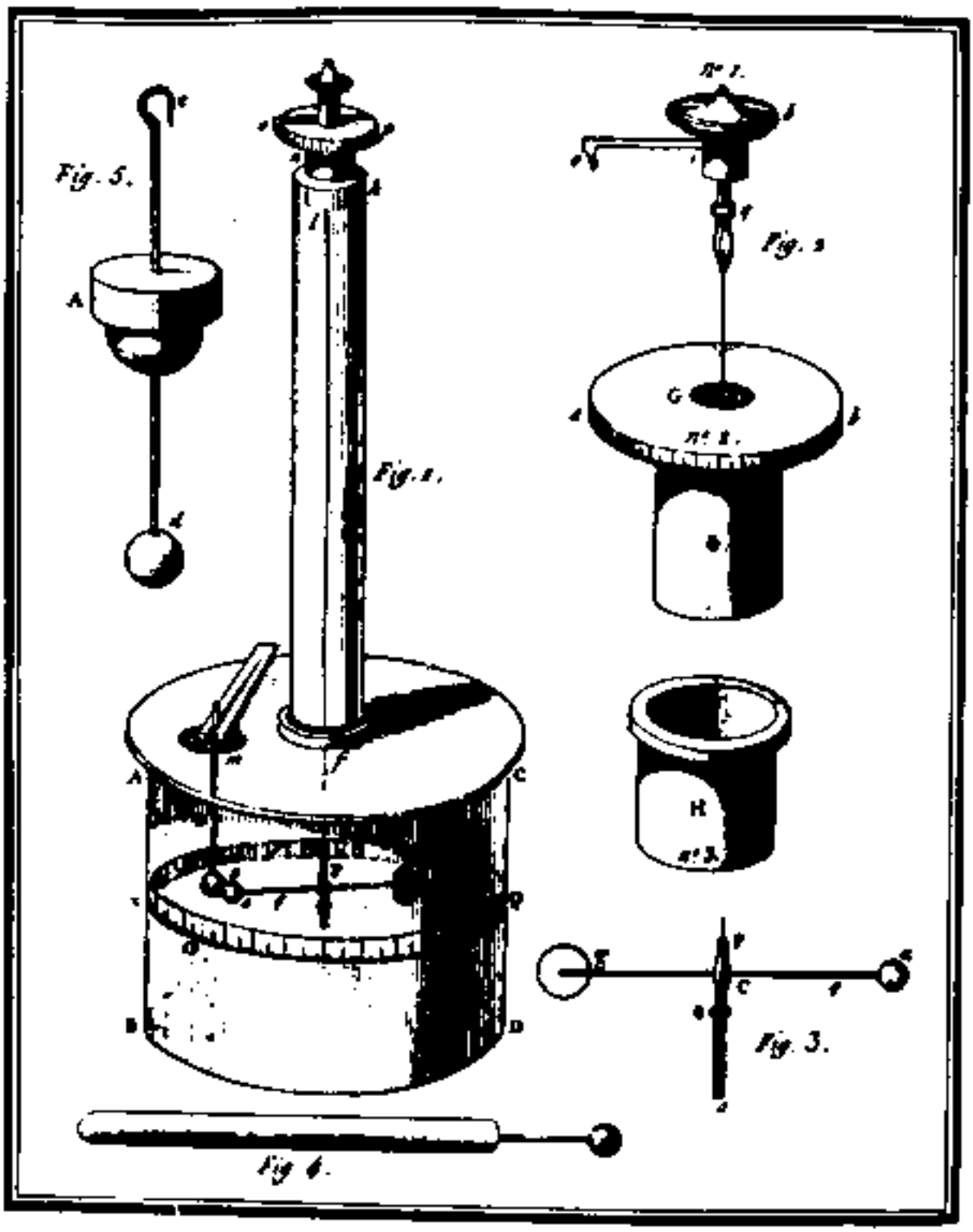


Fig. 5.
A
Fig. 1.
A
C
B
D
Fig 6.
N° 1.
Fig. 2
G
N° 2.
H
N° 3.
Fig. 3.

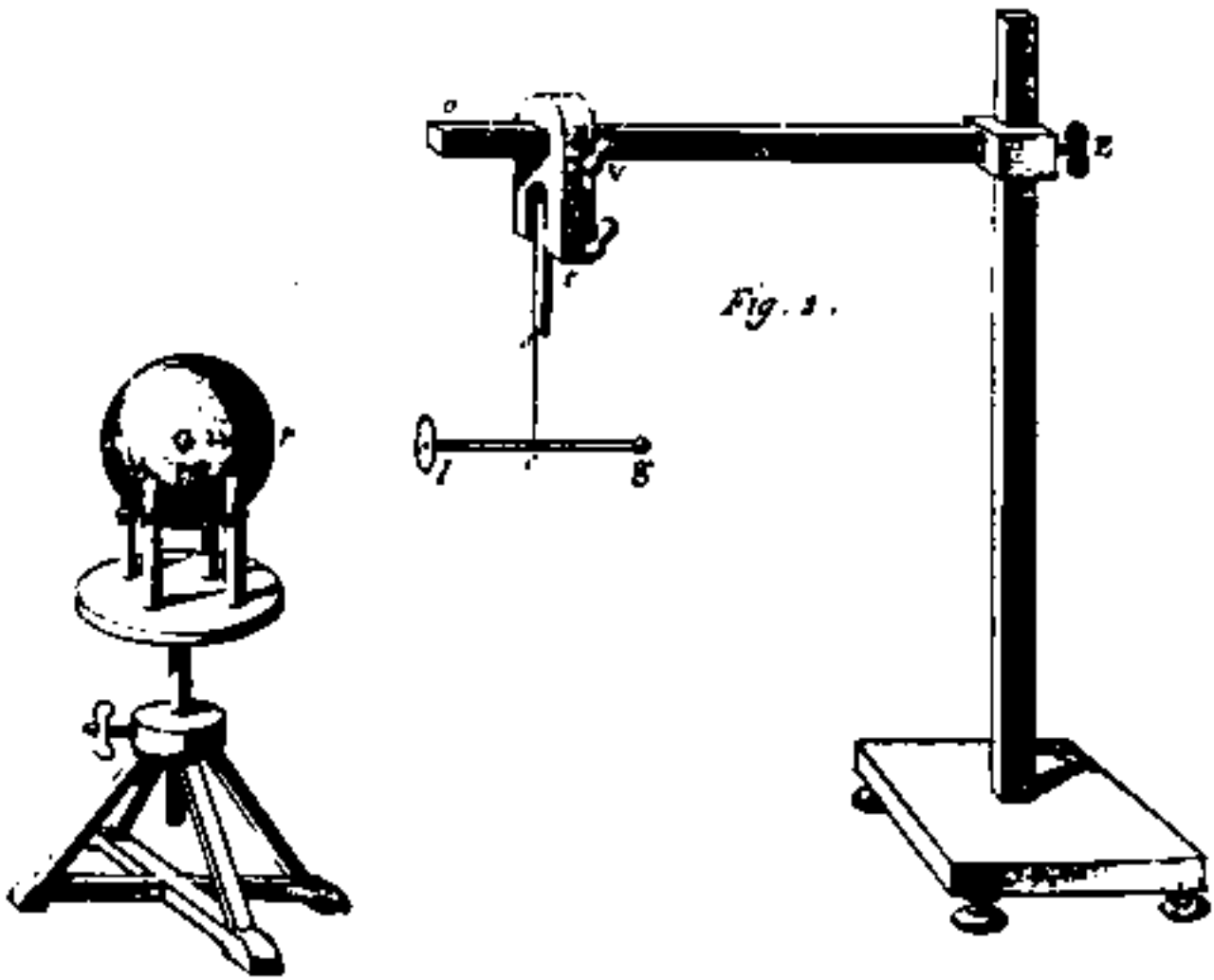
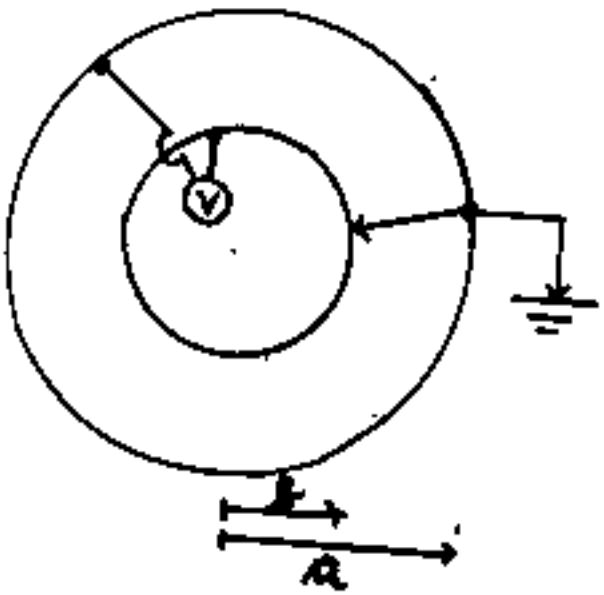
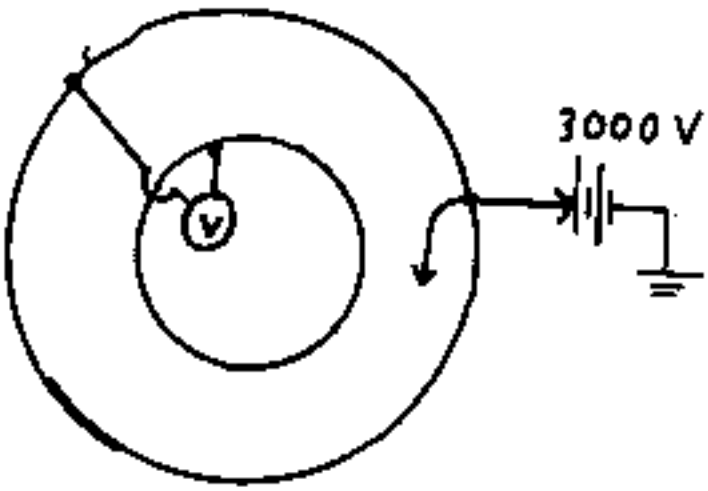


Fig. 2.

VI Plimpton & Lawton (1936)





$b = 2.5t$

$a = 2\frac{1}{2}.5t$

$\Delta V = 10^{-6}\ V$

$$q = F(a,b)\frac{\Delta V}{V}$$

$$\approx 2 \times 10^{-9}$$

NOW, in our Language

$$(\nabla^2 - \mu^2)\, V(r) = 0$$

$$\phi(r) = K\left[\frac{e^{\mu r} - e^{-\mu r}}{2\mu r}\right]$$

$a \le r \le b$

BECAUSE $\rightarrow V = \text{Const}\Big|_{a \le r \le b}$ $\mu = 0$

$V = \phi(a)$ $\qquad \Delta V = \phi(a) - \phi(b)$

$$\frac{\Delta V}{V} = \frac{1}{6}\mu^2 (a^2 - b^2) + \mathcal{O}\left[(\mu a)^4\right]$$

$\mu \le 10^{-6}\,\text{cm}^{-1} = 2 \times 10^{-11}\,\text{eV} = 4 \times 10^{-44}\,\text{g}$

VII Williams, Faller, Hill (1971)

In 1960's IMPROVE

① Lock in Detector
② $\omega \gg 0$, NOISE DOWN
③ 1, 2, 3 orders better

$\mu \leq 2 \times 10^{-47}$ g

$= 5 \times 10^{-10}$ cm^{-1}

$= 9 \times 10^{-15}$ eV

BEST LAB LIMIT

② CONSEQUENCES, $\mu \neq 0$

a) $v_g \neq c$

$$(\Box - \mu^2) A_\lambda = 0$$

$$\left(\frac{\omega}{c}\right)^2 - \underset{\sim}{k}^2 = \mu^2$$

$$v_g = \frac{d\omega}{dk} = \frac{c^2 k}{\omega} = c\,k \Big/ (k^2 + \mu^2)^{1/2}$$

DISPERSION OF LIGHT

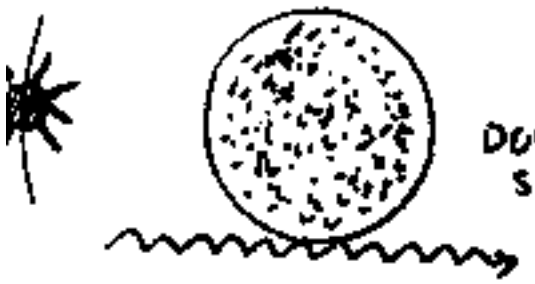
PHOTONS ARE REL. PARTICLES

③ a) dispersion of light signals

$$\delta t = \int dl \left[\frac{1}{v_R} - \frac{1}{v_B} \right] \sim \frac{L}{c^2}(v_B - v_R)$$

$$= \frac{\mu^2 L}{8\pi^2 c} \left(\lambda_B^2 - \lambda_R^2 \right) \left(\frac{c}{\lambda}\right)^2$$

De Broglie (1940)



$\lambda = 4000\,\text{Å}$
$8000\,\text{Å}$
$L = 10^3\,\text{light yr.}$
$\delta t < 10^{-3}\,\text{sec}$

$\mu < 0.78 \times 10^{-39}\,\text{gm}$
$\neq 10^{-44}\,\text{gm.}$

+ Pulsars Feinberg (1969) - Drake et al (1968)

ON OFF

λ_R λ_B

Problem is plasma dispersion

$$k^2 = \frac{\omega^2}{c^2}\left[1 - \frac{\omega_p^2}{\omega^2 \pm \omega\,\omega_B}\right]$$

$$\omega_p^2 = \frac{4\pi n e^2}{m} \qquad \omega_B = \frac{eB}{mc}\cos\theta$$

IGNORE

$$v(\omega) = \frac{d\omega}{dk} = c\sqrt{1 - \frac{\omega_p^2}{\omega^2}} \simeq c\left[1 - \frac{1}{2}\frac{\omega_p^2}{\omega^2}\right]$$

SAME AS Eq FOR v_g

COMPARE

$$\left(\frac{\mu c^2}{\hbar}\right) = 8.2\times10^{47}\,[\mu(\mathrm{gm})]\ \mathrm{sec}^{-1}$$

$$\left(\frac{4\pi n e^2}{m}\right)^{1/2} = 5.6\times10^{4}\,[n(\mathrm{cm}^{-1})]^{1/2}\ \mathrm{sec}^{-1}$$

$\Rightarrow$

$n = 0.028$ elect/cm^3

OR

$\mu \simeq 10^{-44}$ gm

$= 3\times10^{7}\ \mathrm{cm}^{-1}$

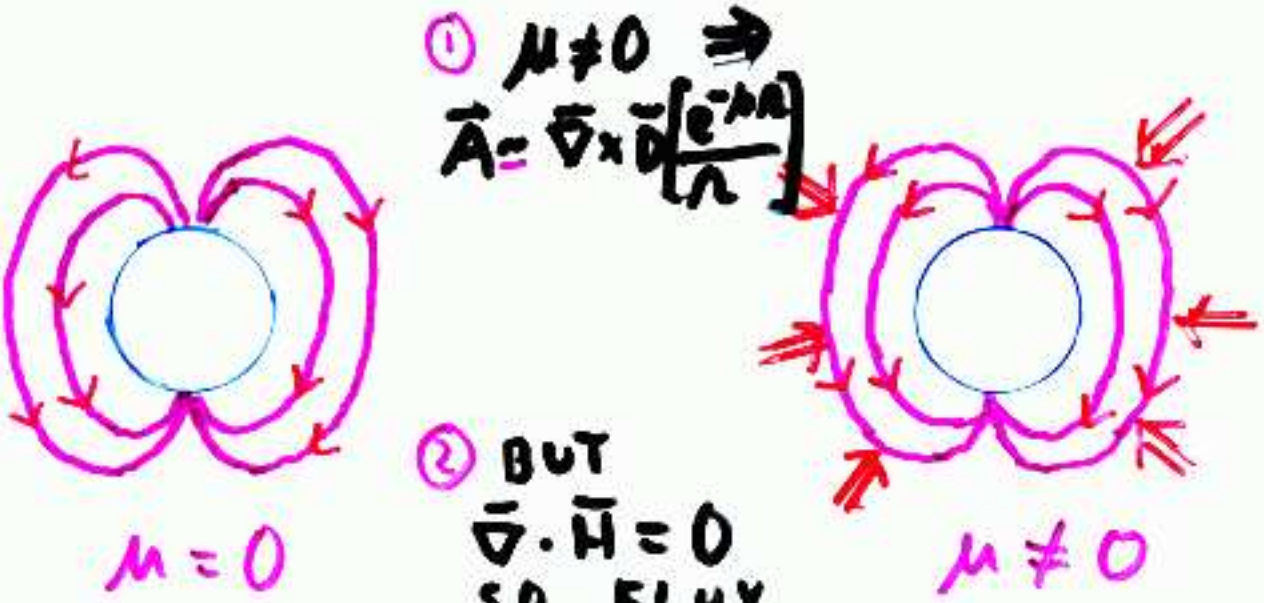
$= 6\times10^{-12}$ eV

CRAB PULSAR - NP0532

3-c) Earth's Magnetic Field

Schrödinger wanted a massive photon for his unified theory. (See Moore)

In 1943, inspired by McConnell's idea,



③ So, lines closer in mostly at Equator

$$D\hat{z} = \underset{\sim}{D} = -\frac{1}{2}\int d^3r\, \underset{\sim}{J} \times \underset{\sim}{r}\left(\frac{4\pi}{c}\right)$$

$$(-\nabla^2 + \mu^2)\,\underset{\sim}{A} = J\left(\frac{4\pi}{c}\right)$$

$$\Rightarrow \quad \underset{\sim}{A} = \nabla \times \underset{\sim}{D}\left[e^{-\mu r}/r\right]$$

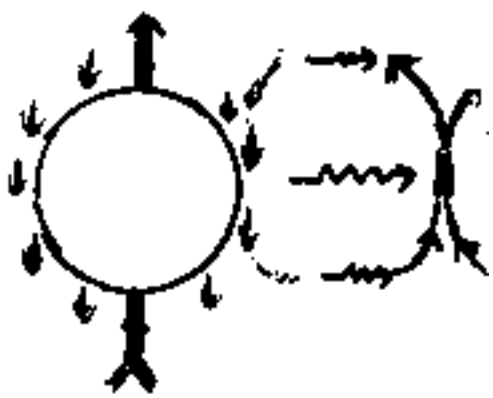
BUT $\quad \underset{\sim}{H} = \nabla \times \underset{\sim}{A}$

$$\underset{\sim}{H} = \left[D e^{-\mu r}/r^3\right]\left[(1+\mu r + \tfrac{1}{3}\mu^2 r^2)(3\hat{z}\cdot\hat{r}\,\hat{r} - \hat{z}) - \tfrac{2}{3}\mu^2 r^2 \hat{z}\right]$$

$$\underset{\sim}{H}_D = \left[D/r^3\right]\left[3\hat{z}\cdot\hat{r}\,\hat{r} - \hat{z}\right]$$

DIFFERENCES $\rightarrow$ $\underset{\sim}{H}_{\text{EXTERNAL}}$ (on sphere, anti-$\parallel$ to $\underset{\sim}{D}$ at equator best)

$$\frac{H_{ext}}{H_{D.E.}} = \frac{\frac{2}{3}\mu^2 R^2}{1 + \mu R + \frac{1}{3}(\mu R)^2}$$



I) Schrödinger had Ad Schmidt's {1885, 1922 Surveys}

$$\frac{H_{ext}}{H_{DE}} \sim \frac{539\,\gamma}{31089\,\gamma}$$

(Eq ABOVE) + (Factor of 2 later) $\Rightarrow$ $\mu \sim 10^{-47}$ gm

---

II) ASG + MMN had NASA's data (Cain et al)

From satellites + earthbd. observations,
FIT to ~ 100 spherical Harmonics.

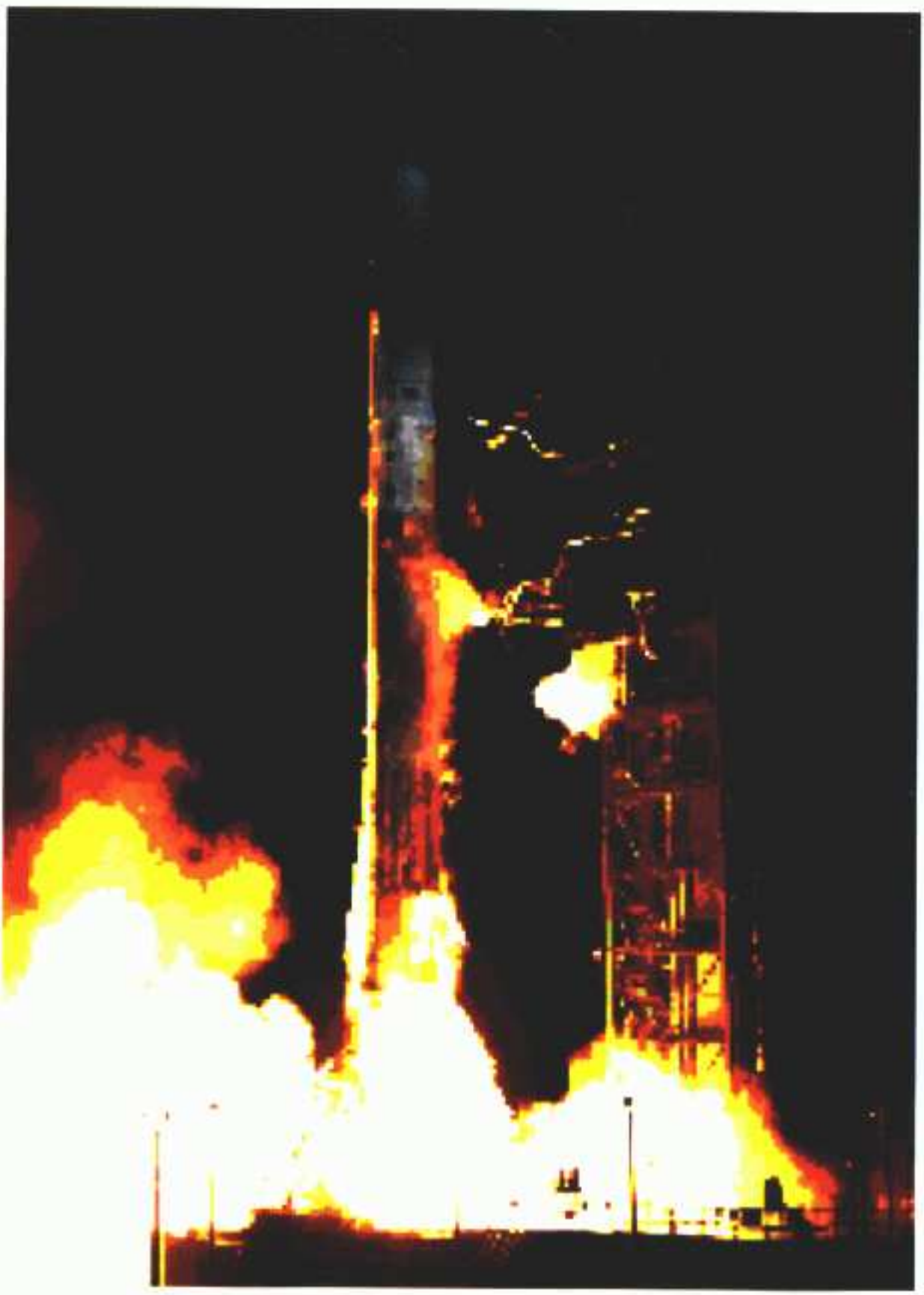
ERA (1960.0)

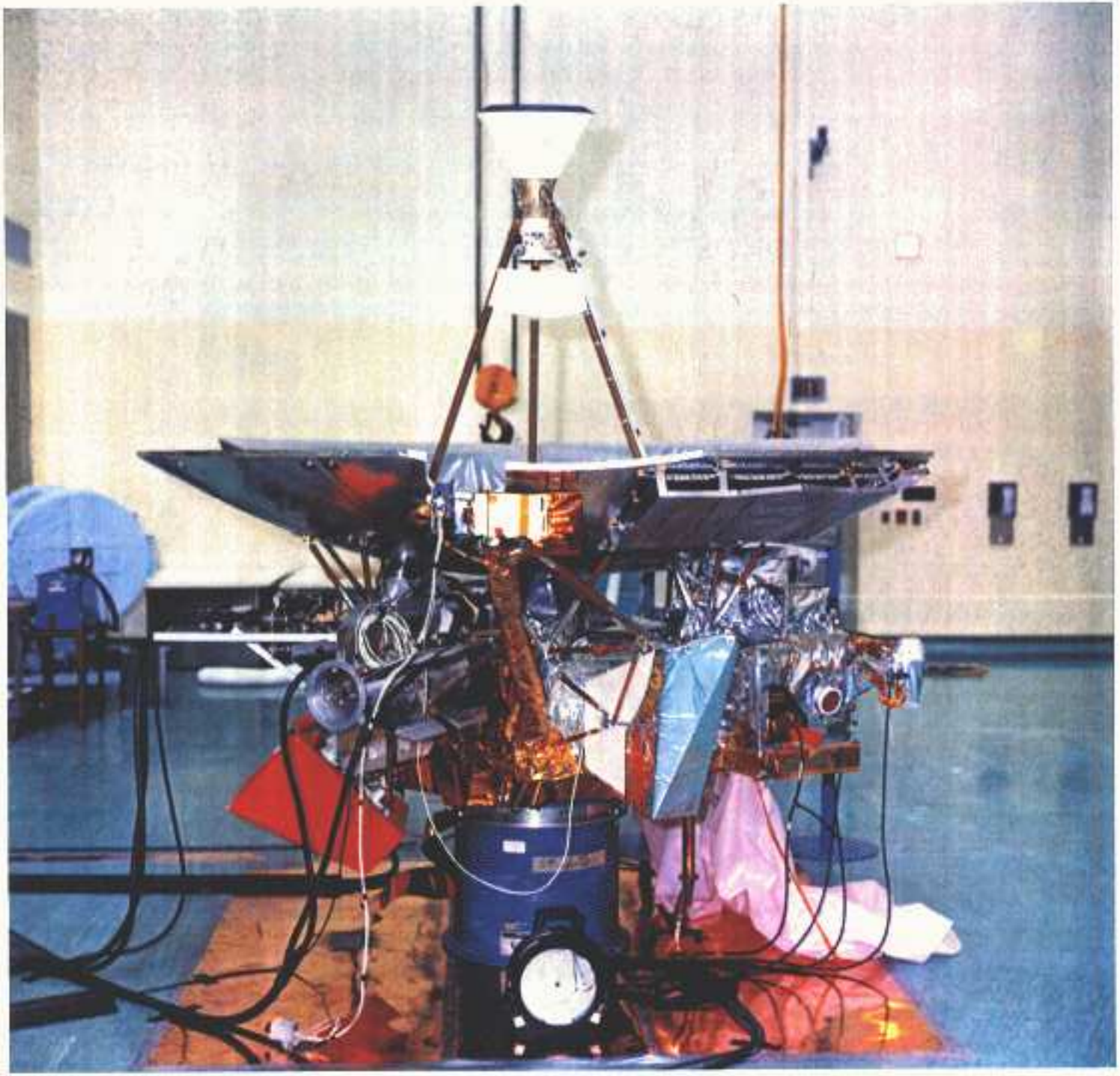
$$D = 31044\,\gamma R^3$$

$$\underline{H}_{ext} \cdot \underline{D} = (21 \pm 5)\,\gamma$$

$$\underline{H}_{ext} \cdot [\hat{S} \times \hat{D} / |\hat{S} \times \hat{D}|] \equiv \underline{H}_{ext} \cdot \hat{y} = (14 \pm 5)\,\gamma$$

$$\underline{H}_{ext} \cdot \hat{y} \times \hat{D} = (8 \pm 5)\,\gamma$$





SCIENCE

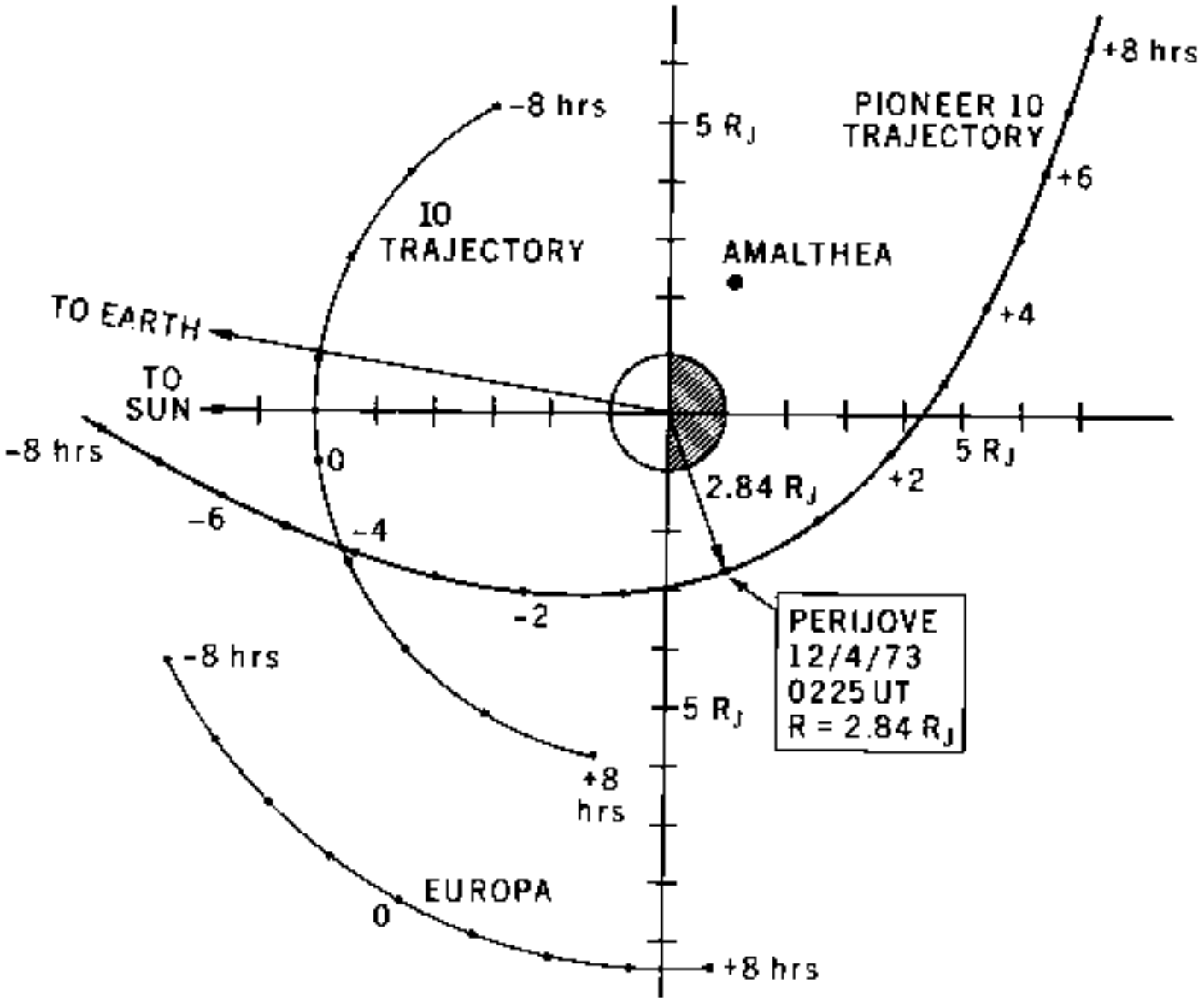
25 January 1974

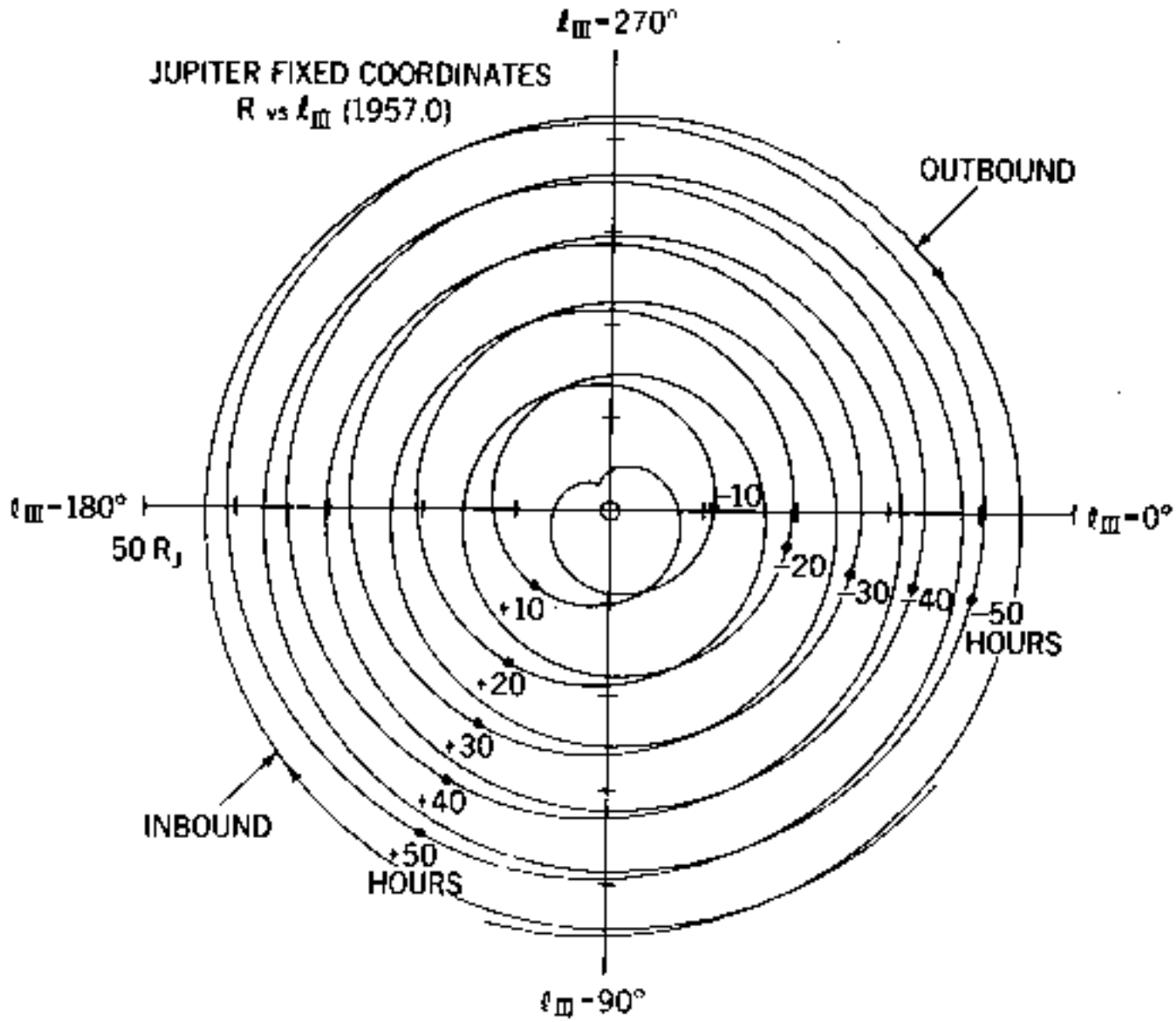
Vol. 183, No. 4122

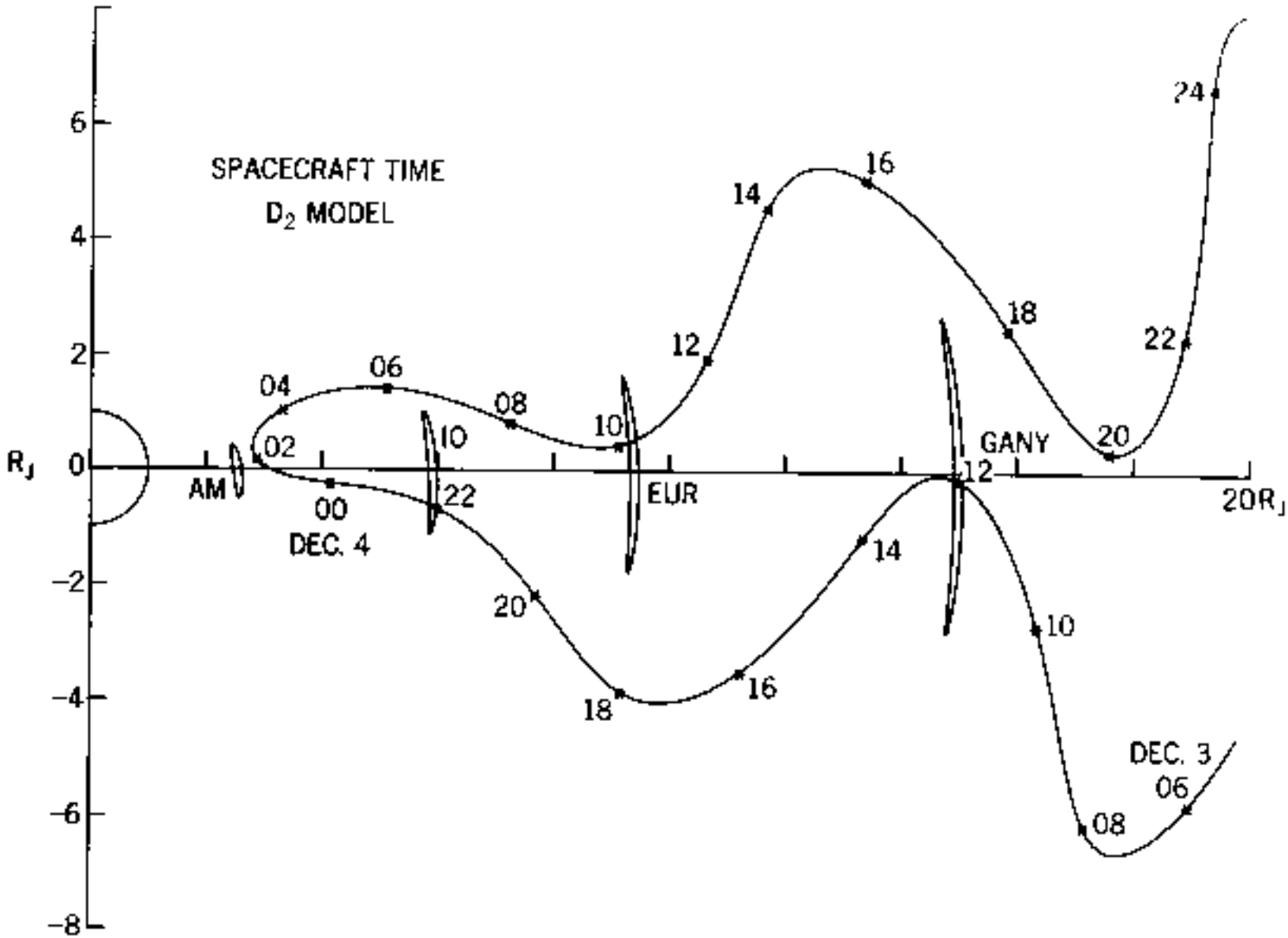
3-d) Jupiter's Magnetic Field

[L. Davis, Jr., ASG, MMN, Phys. Rev. Lett. 35, 1402, 1975.]

Fit to μ , internal & external dipoles and quadrapoles, taking out ring currents, etc.


+8 hrs
PIONEER 10
TRAJECTORY
+6
-8 hrs
5 R_J
IO
TRAJECTORY
AMALTHEA
+4
TO EARTH
TO
SUN
-8 hrs
0
5 R_J
2.84 R_J
+2
-6
-4
-2
PERIJOVE
12/4/73
0225 UT
R = 2.84 R_J
-8 hrs
5 R_J
+8
hrs
EUROPA
0
+8 hrs


JUPITER FIXED COORDINATES
R vs ℓ_{III} (1957.0)
ℓ_{III} - 270°
OUTBOUND
ℓ_{III} - 180°
50 R_J
ℓ_{III} - 0°
-10
-20
-30
-40
-50
HOURS
+10
+20
+30
+40
INBOUND
+50
HOURS
ℓ_{III} - 90°


SPACECRAFT TIME
D2 MODEL
RJ
6
4
2
0
−2
−4
−6
−8
AM
EUR
GANY
20RJ
00
DEC. 4
02
04
06
08
10
12
14
16
18
20
22
24
DEC. 3
06
08
10
12
14
16
18
20
22

VERY conservative fit obtained

$$\mu \leq 8 \times 10^{-49}\ g$$

$$= 6 \times 10^{-16}\ eV$$

$$= 2 \times 10^{-11}\ cm^{-1}$$

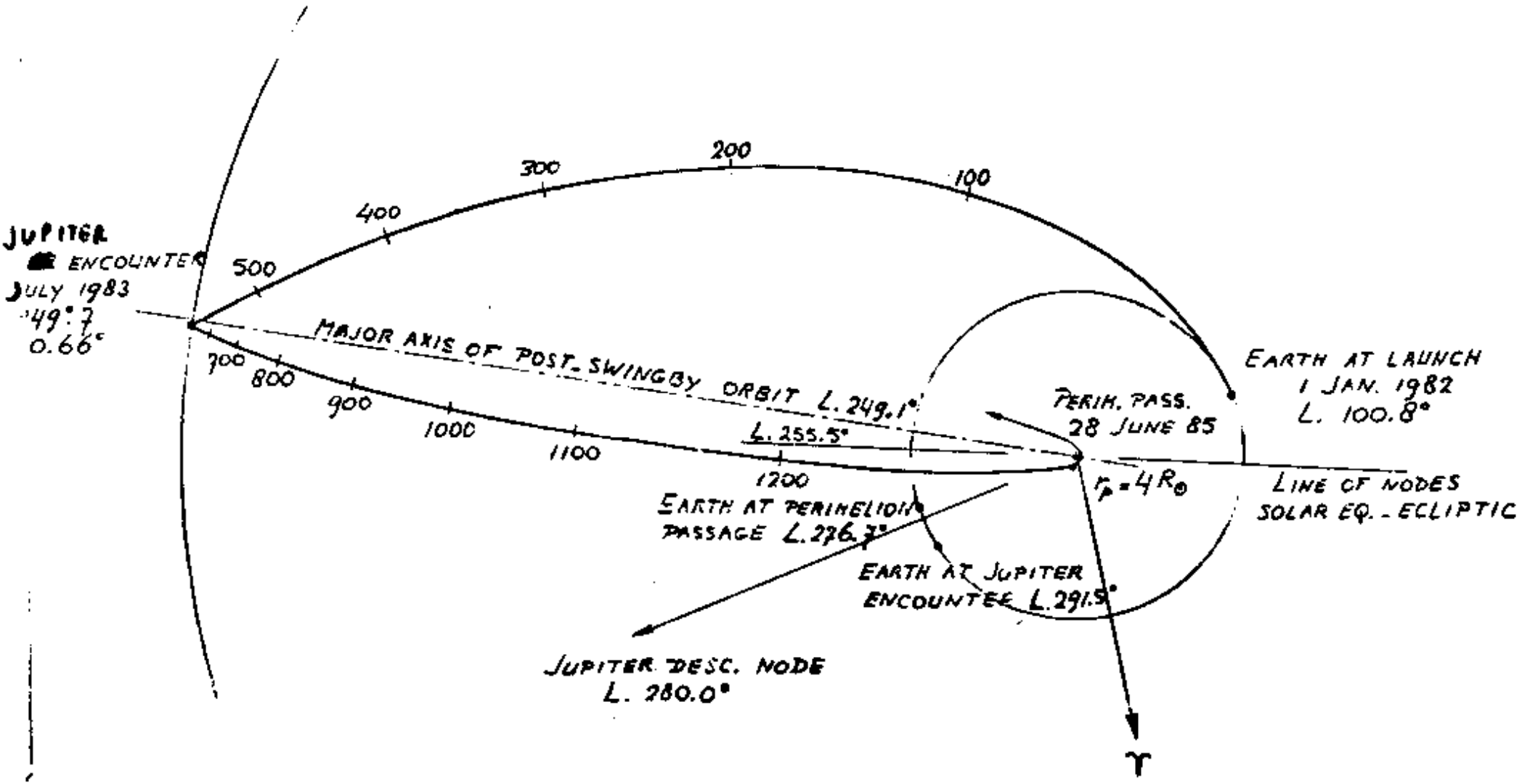
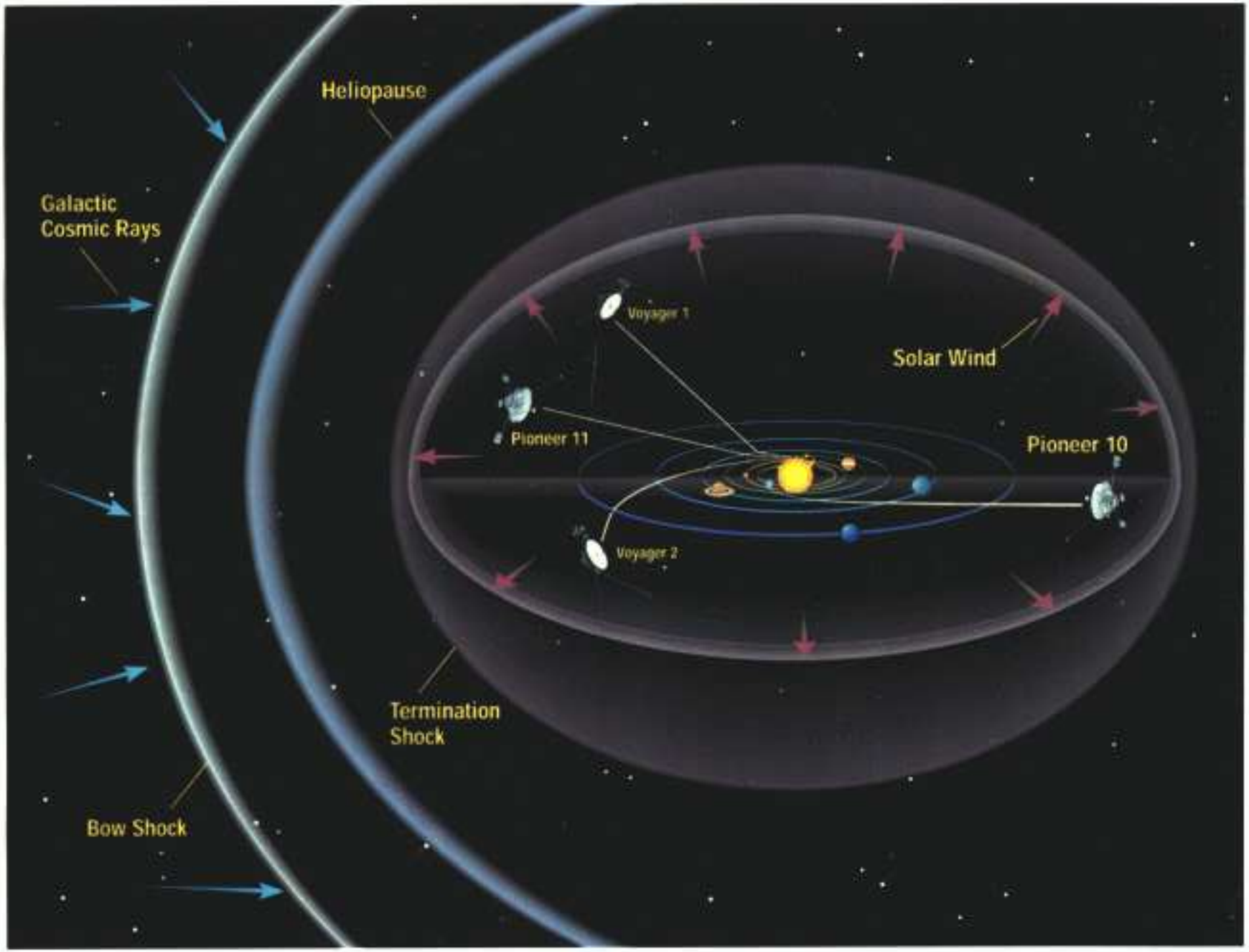


Fig 3.3 Trajectory geometry for in-ecliptic SPM with perihelion of 4 Solar radii

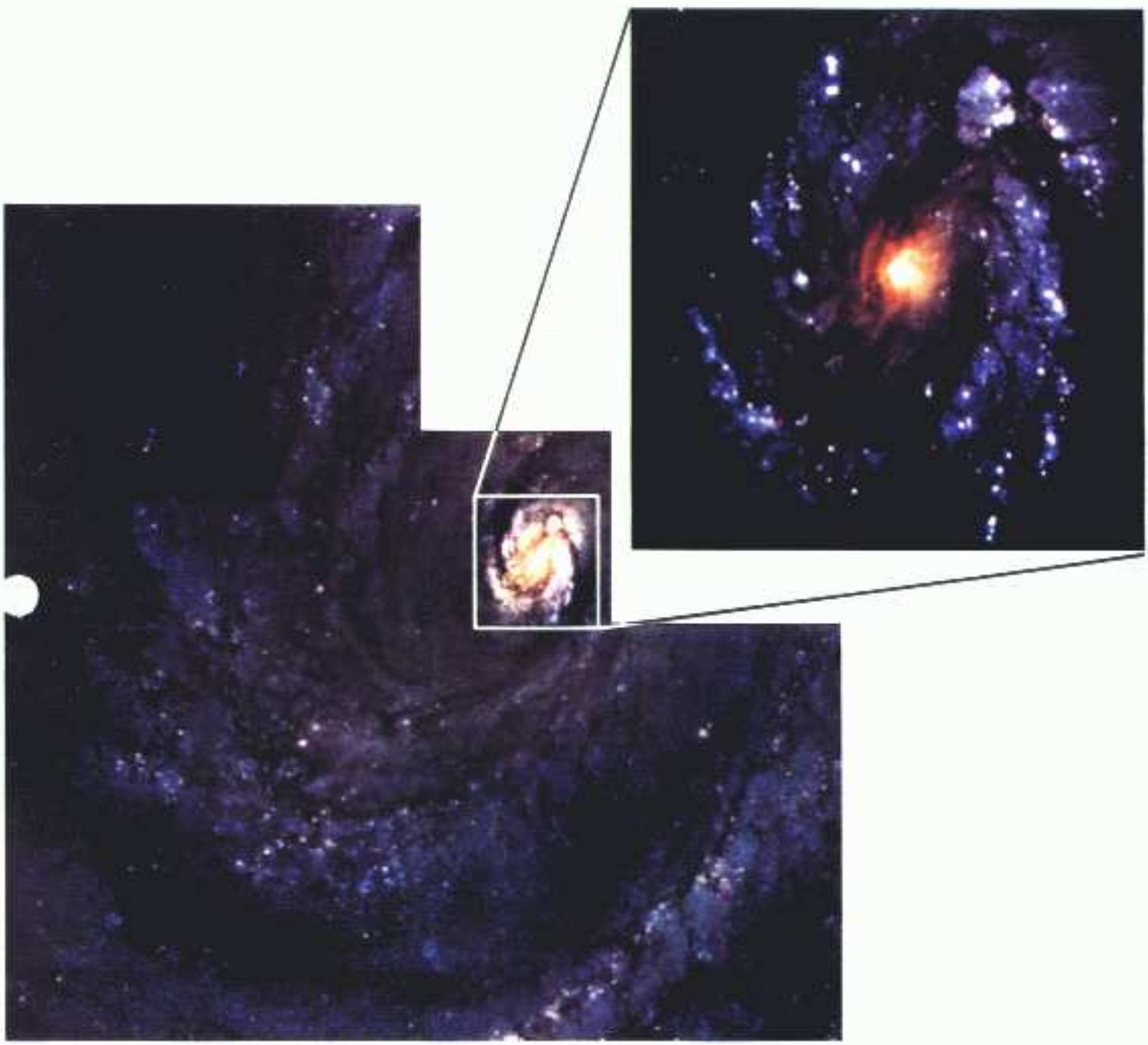

Heliopause
Galactic
Cosmic Rays
Voyager 1
Solar Wind
Pioneer 11
Pioneer 10
Voyager 2
Termination
Shock
Bow Shock

What don't we understand about gravity?

Actually, we don't understand gravity and matter (let alone antimatter) for almost all of the universe !!

DMP

SPIRAL GALAXY M100

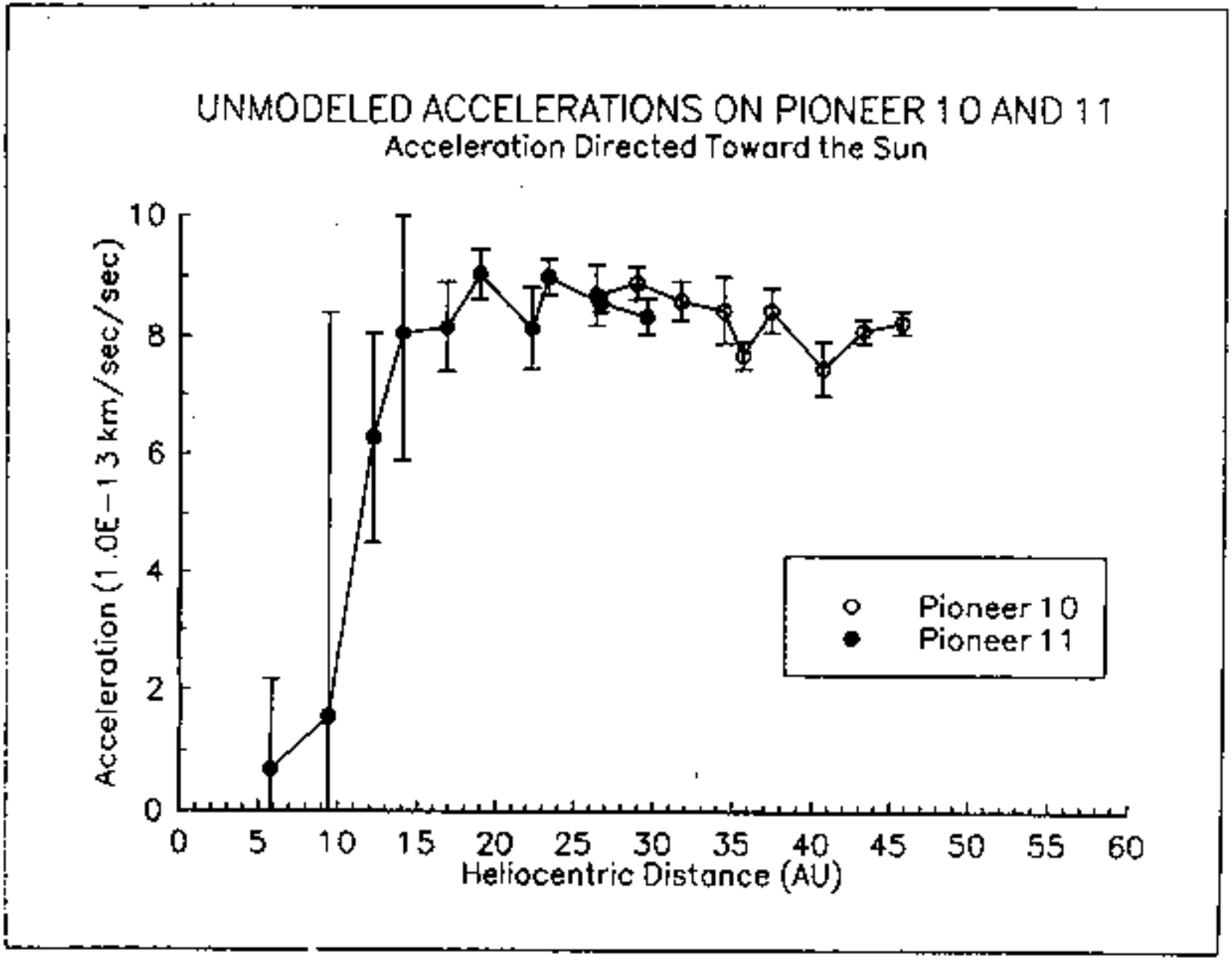


By the way, the biggest systematic in our acceleration residuals is a bias of 8 X 10^-13 km/s^2 directed toward the Sun.

1994
LEAP

$a_N = 5.93 \times 10^{-6} \frac{km}{s^2}$

Figure 1


UNMODELED ACCELERATIONS ON PIONEER 10 AND 11
Acceleration Directed Toward the Sun
Acceleration (1.0E−13 km/sec/sec)
0
2
4
6
8
10
0
5
10
15
20
25
30
35
40
45
50
55
60
Heliocentric Distance (AU)
Pioneer 10
Pioneer 11

JOHN D. ANDERSON (JPL)

PHILIP A. LAING (AEROSPACE)©

EUNICE L. LAU (JPL)

ANTHONY S. LIU (ASTRODY. SCI.)©

MMN (LANL)

SLAVA G. TURYSHEV (JPL)

AFTER 2 CODES ('98 PAL) ['87-'94]

PIONEER 10 (ODP)

$(8.09 \pm 0.20) \times 10^{-8}$ cm/sec^2

PIONEER 11 (ODP) ['87-'90]

$(8.56 \pm 0.15) \times 10^{-8}$ cm/sec^2

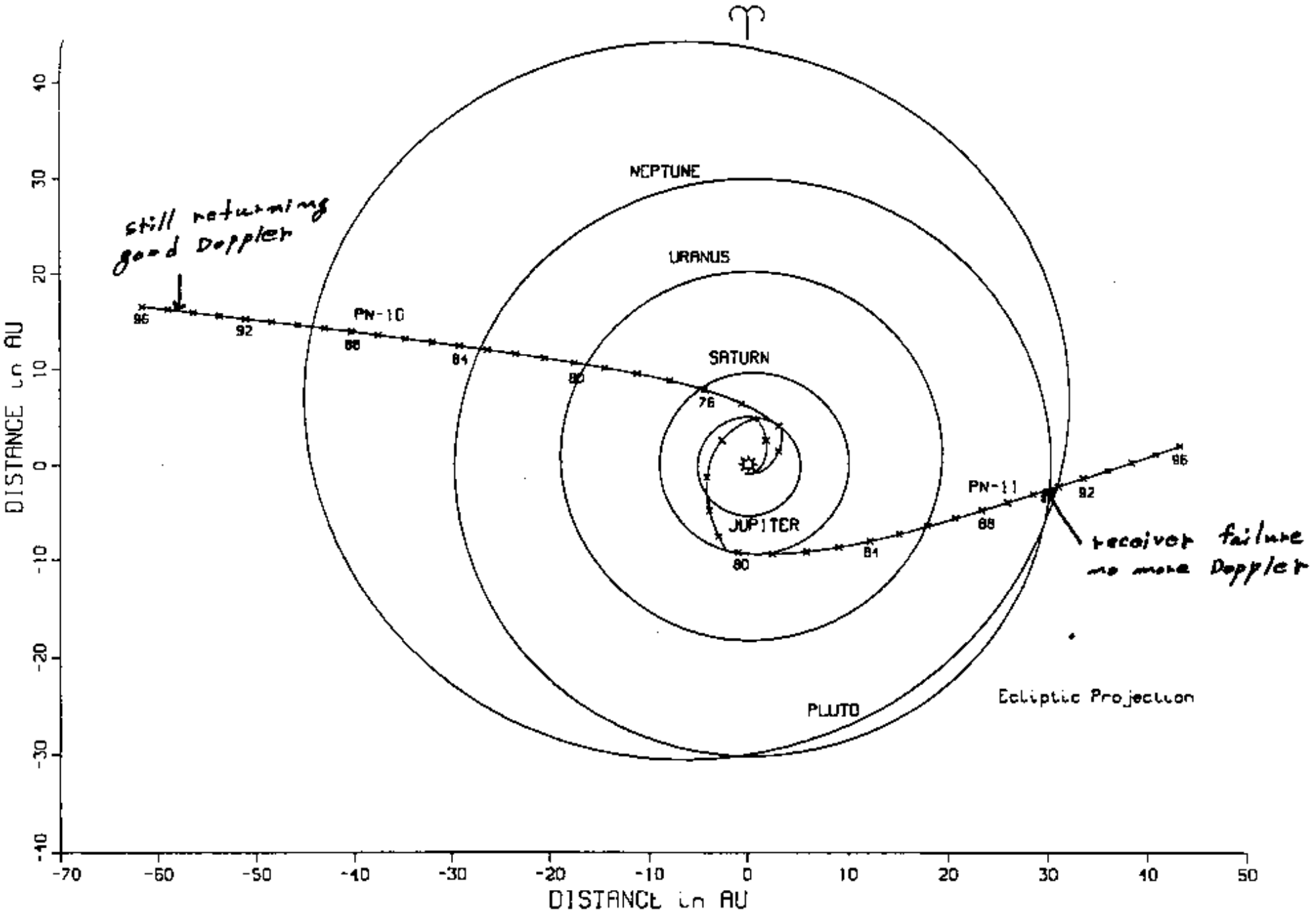
PIONEER 10 (CHASMP)

(8.65 ± 0.03)

NOW NEED SYSTEMATICS

+

P10 10 DATA → 1998


still returning good Doppler
NEPTUNE
URANUS
SATURN
JUPITER
PLUTO
PN-10
PN-11
receiver failure no more Doppler
Ecliptic Projection
DISTANCE in AU
DISTANCE in AU
NASA/AMES
PPO/88-1.1-2
D' 3/2/94

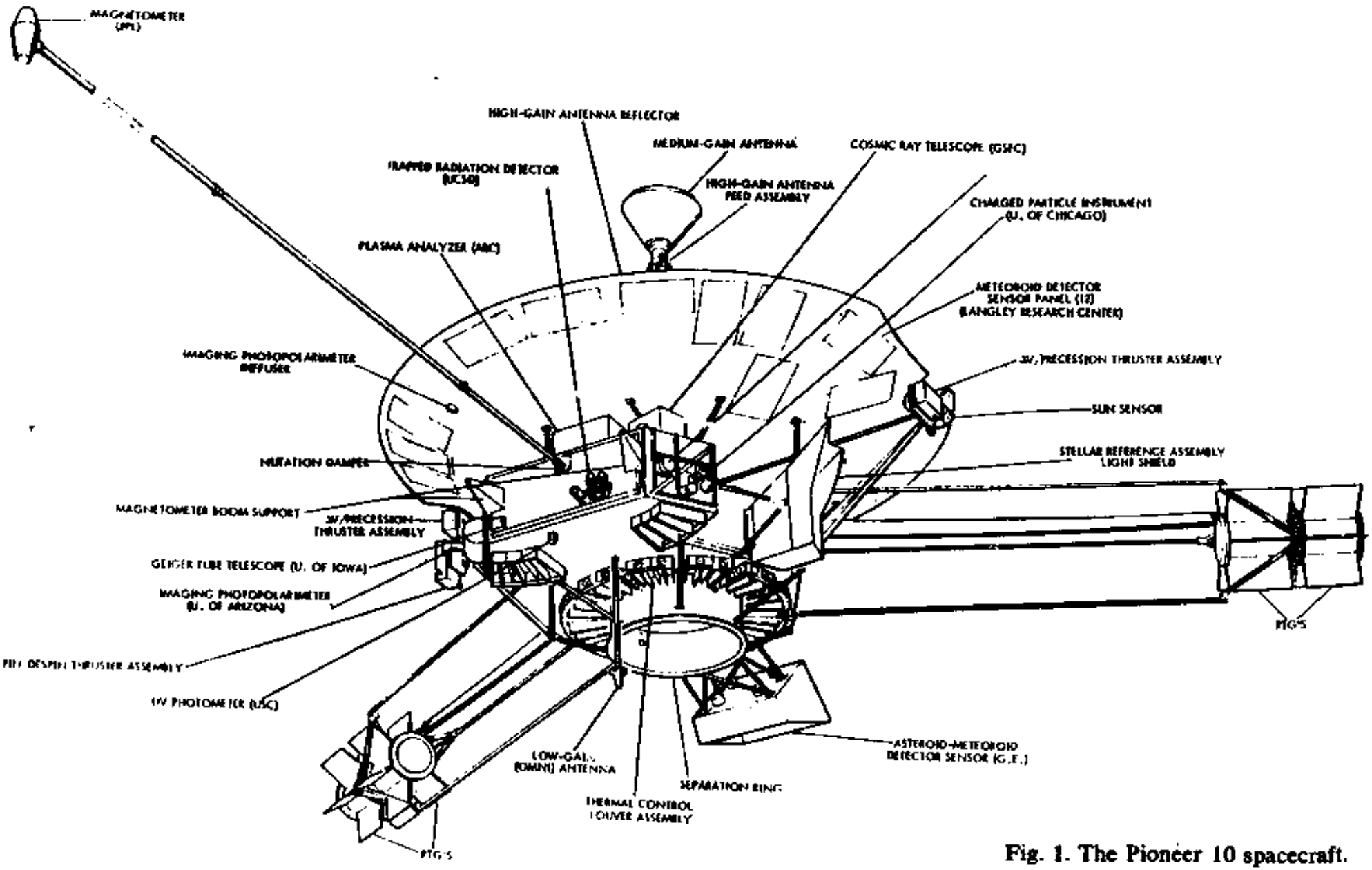
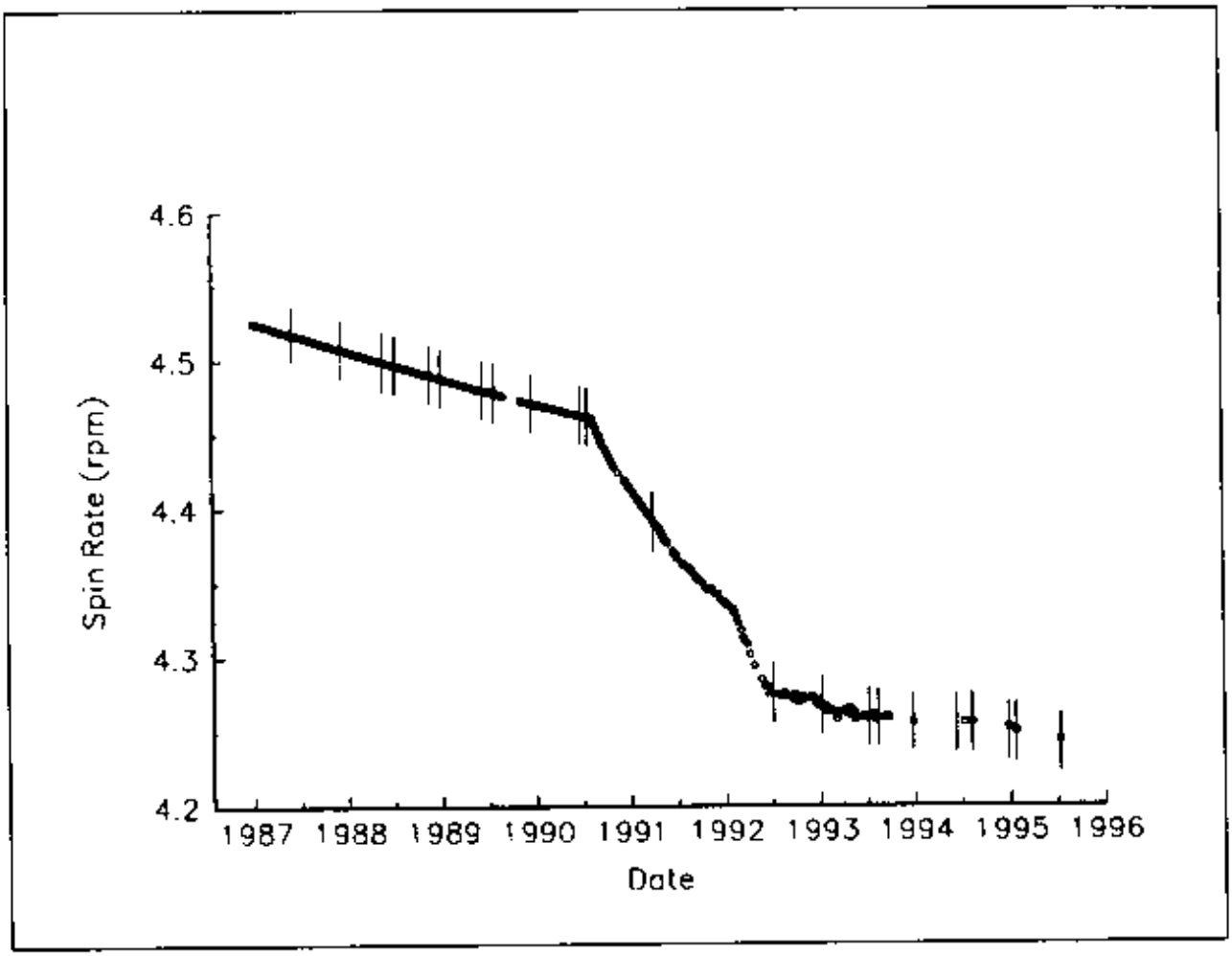


Fig. 1. The Pioneer 10 spacecraft.

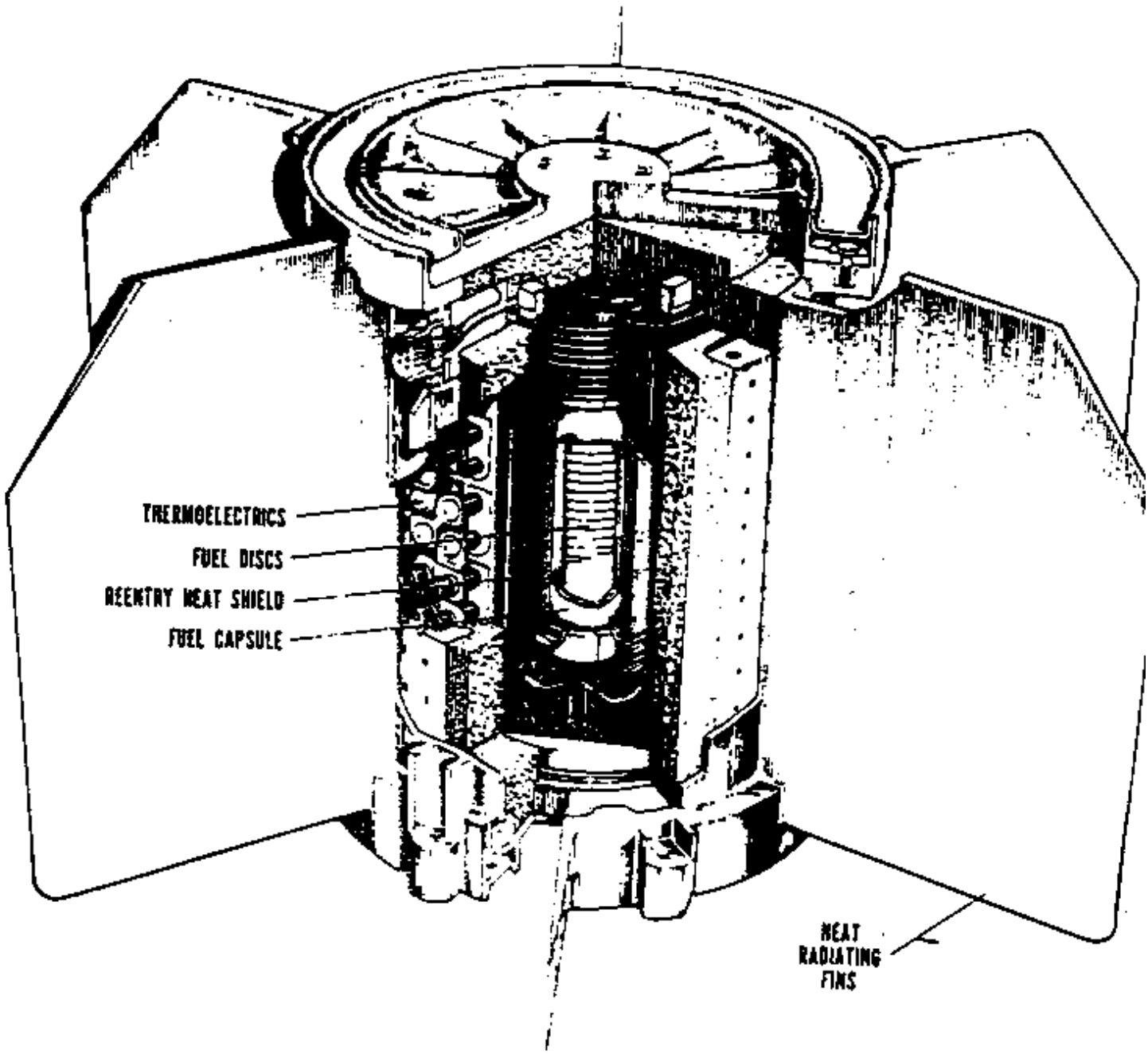


The spin history of Pioneer 10.

Program/Estimation method	Pio 10 (I)	Pio 10 (II)	Pio 10 (III)	Pio 11
Sigma, WLS, no solar corona model	8.02 ± 0.01	8.65 ± 0.01	7.83 ± 0.01	8.46 ± 0.04
Sigma, WLS, with solar corona model	8.00 ± 0.01	8.66 ± 0.01	7.84 ± 0.01	8.44 ± 0.04
Sigma, BSF, 1-day batch, with solar corona model	7.82 ± 0.29	8.16 ± 0.40	7.59 ± 0.22	8.49 ± 0.33
CHASMP, WLS, no solar corona model	8.25 ± 0.02	8.86 ± 0.02	7.85 ± 0.01	8.71 ± 0.03
CHASMP, WLS, with solar corona model	8.22 ± 0.02	8.89 ± 0.02	7.92 ± 0.01	8.69 ± 0.03
CHASMP, WLS, with corona, weighting, and F10.7	8.25 ± 0.03	8.90 ± 0.03	7.91 ± 0.01	8.91 ± 0.04

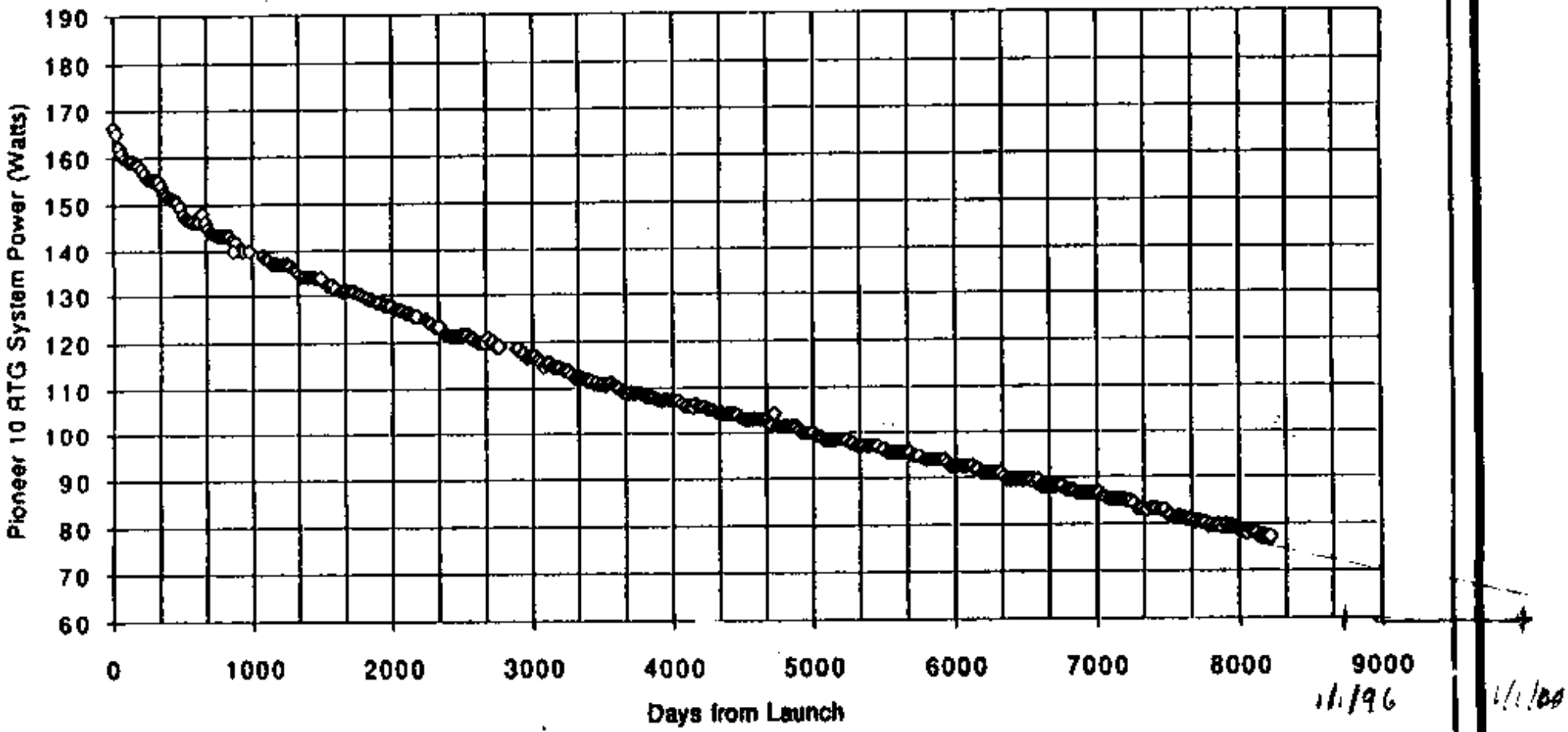
$$a_{P(EX)}^{IC} = (7.84 \pm 0.01) \times 10^{-8} \text{ cm/s}^2$$

$$a_{P(EX)}^{II} \rightarrow 5.55$$

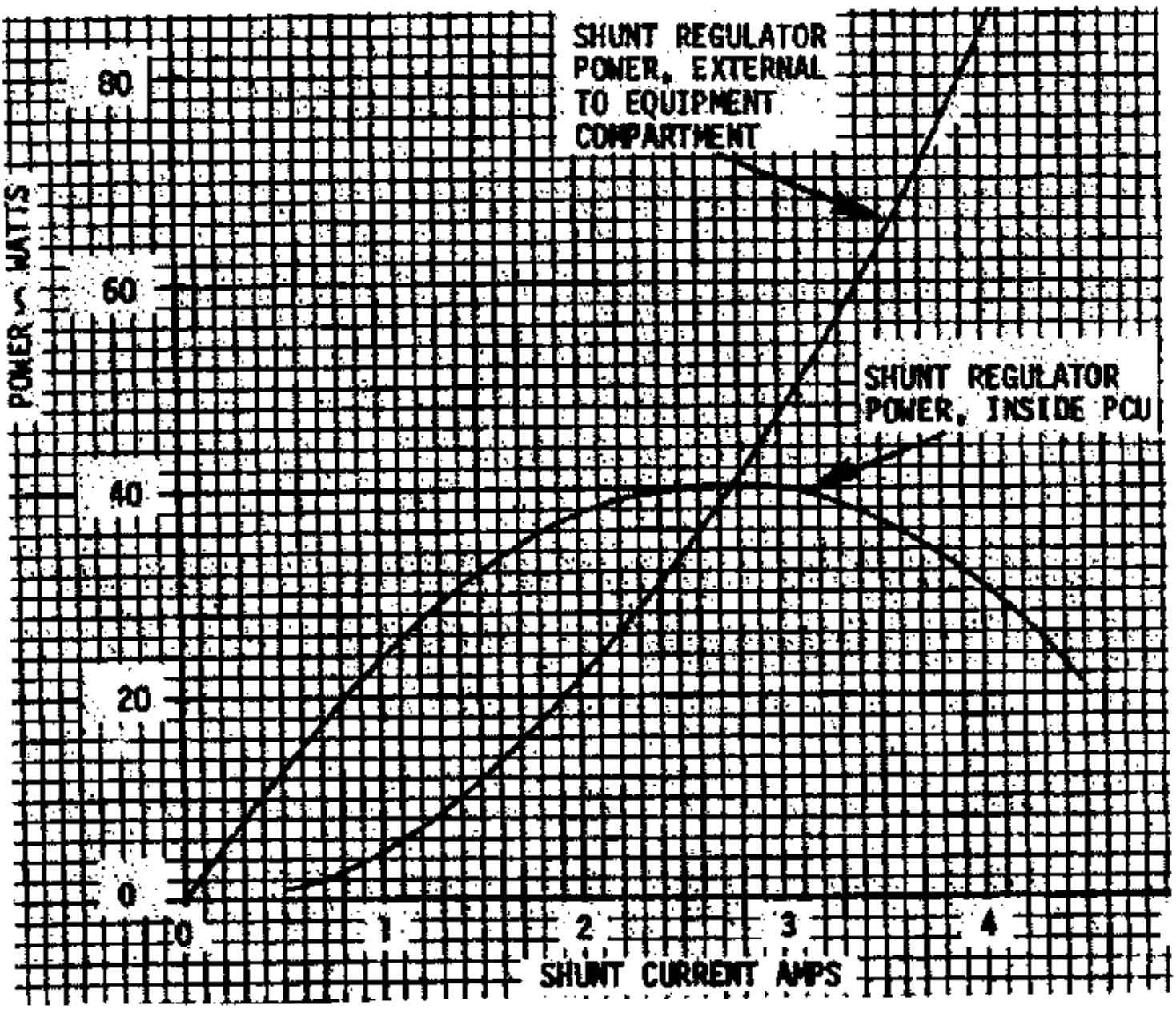


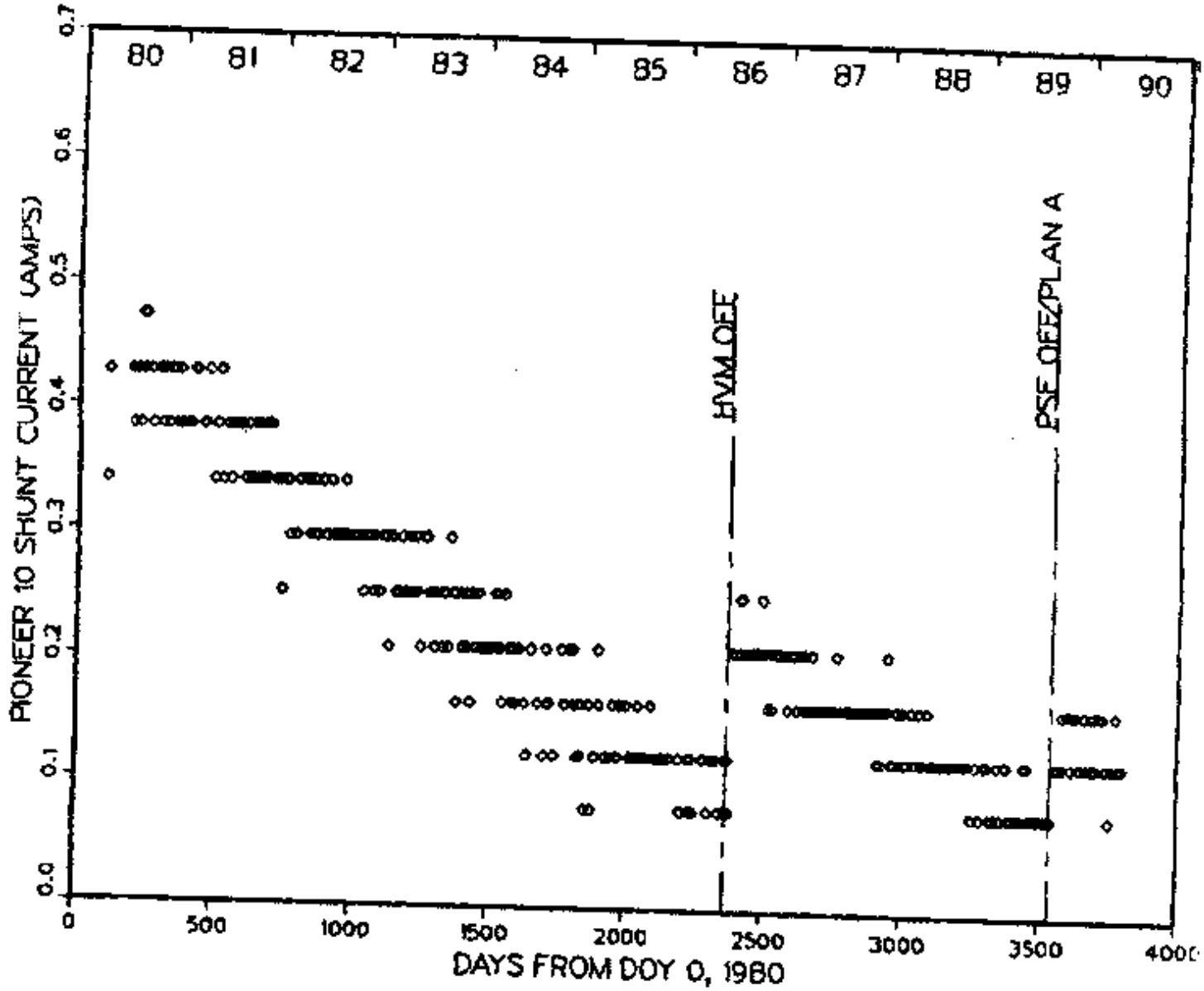
SNAP 19/PIONEER RADIOISOTOPE THERMOELECTRIC GENERATOR

Pioneer 10 RTG System Power History



NASA/AMES
PPO/691.2-1
BJK: 12/1/94


SHUNT REGULATOR POWER, EXTERNAL TO EQUIPMENT COMPARTMENT
SHUNT REGULATOR POWER, INSIDE PCU
POWER - WATTS
80
60
40
20
0
0
1
2
3
4
SHUNT CURRENT AMPS

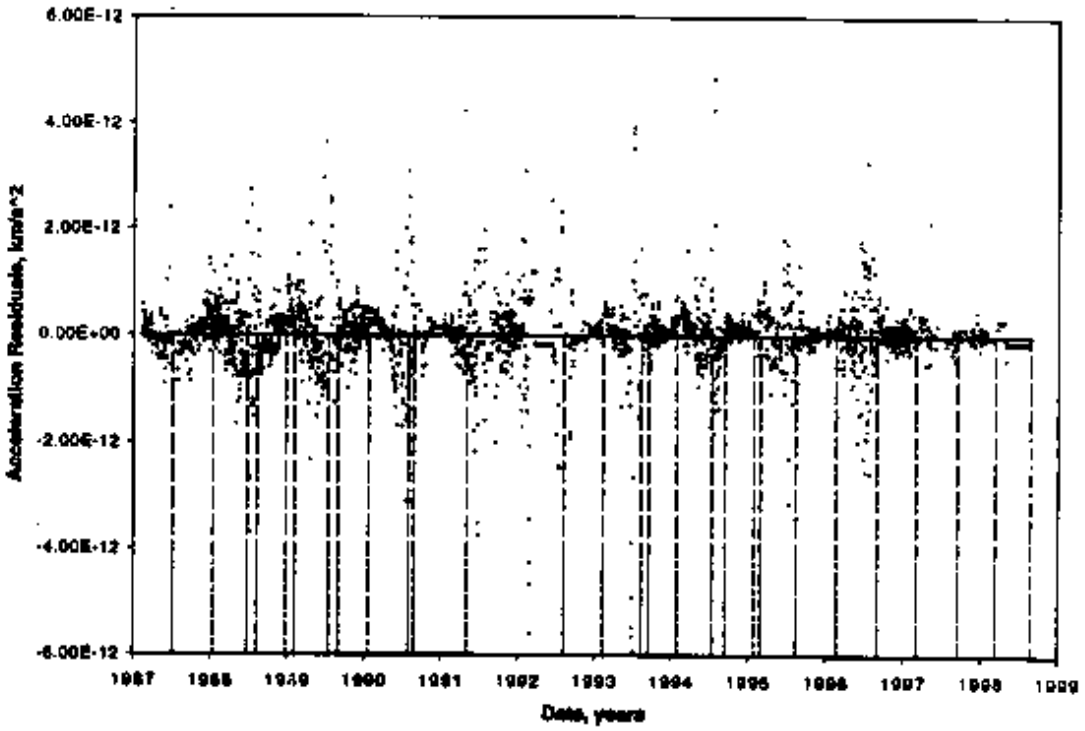

80
81
82
83
84
85
86
87
88
89
90
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
PIONEER 10 SHUNT CURRENT (AMPS)
HVM OFF
PSE OFF/PLAN A
0
500
1000
1500
2000
2500
3000
3500
4000
DAYS FROM DOY 0, 1980

2001 gr-qc/0104064

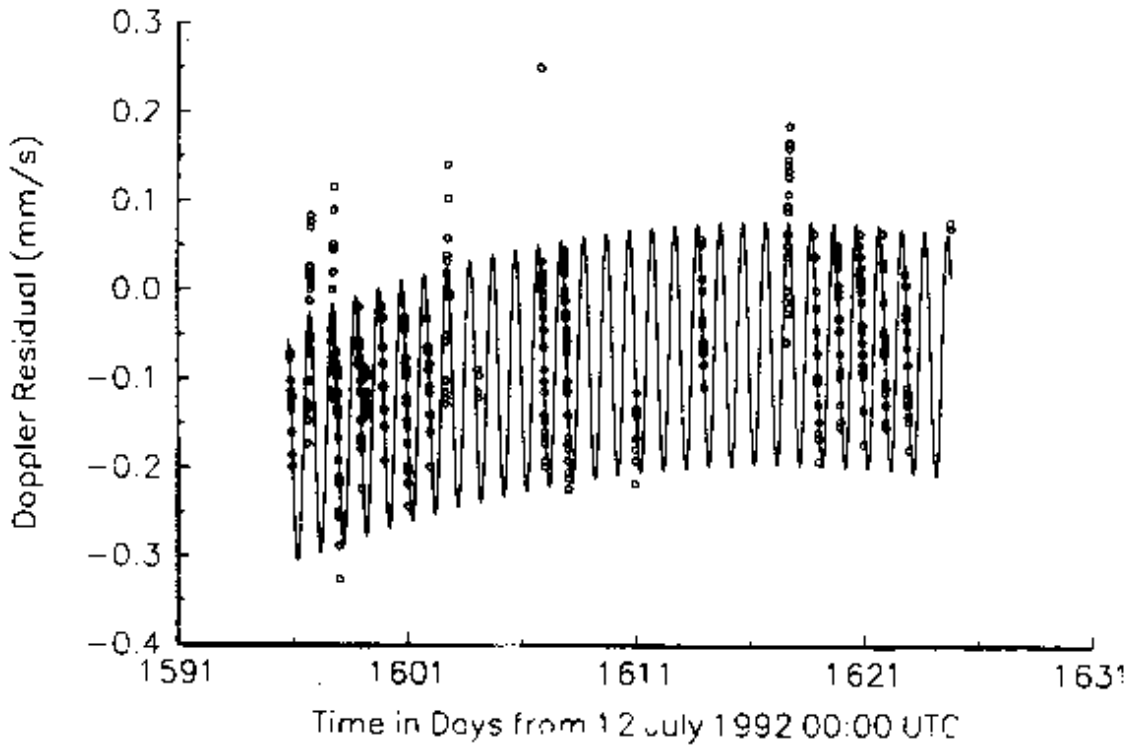
Table 2: Error Budget: A Summary of Biases and Uncertainties.

Item	Description of error budget constituents	Bias 10^{-8} cm/s^2	Uncertainty, 10^{-8} cm/s^2
1	Systematics generated external to the spacecraft:		
	a) Solar radiation pressure and mass	+0.03	±0.01
	b) Solar wind		$\pm < 10^{-5}$
	c) Solar corona		±0.02
	d) Electro-magnetic Lorentz forces		$\pm < 10^{-4}$
	e) Influence of the Kuiper belt's gravity		±0.03
	f) Influence of the Earth orientation		±0.001
	g) Mechanical and phase stability of DSN antennae		$\pm < 0.001$
	h) Phase stability and clocks		$\pm < 0.001$
	i) DSN station location		$\pm < 10^{-5}$
	j) Troposphere and ionosphere		$\pm < 0.001$
2	On-board generated systematics:		
	a) Radio beam reaction force	+1.10	±0.11
	b) RTG heat reflected off the craft	−0.55	±0.55
	c) Differential emissivity of the RTGs		±0.85
	d) Non-isotropic radiative cooling of the spacecraft		±0.16
	e) Expelled Helium produced within the RTGs	+0.15	±0.16
	f) Gas leakage		±0.56
	g) Variation between spacecraft determinations	+0.17	±0.17
3	Computational systematics:		
	a) Numerical stability of least-squares estimation		±0.02
	b) Accuracy of consistency/model tests		±0.13
	c) Mismodeling of maneuvers		±0.01
	d) Mismodeling of the solar corona		±0.02
	e) Annual/diurnal terms		±0.32
	Estimate of total bias/error	+0.90	±1.25

$$a_p = (8.74 \pm 1.25) \times 10^{-8} \text{ cm/s}^2$$

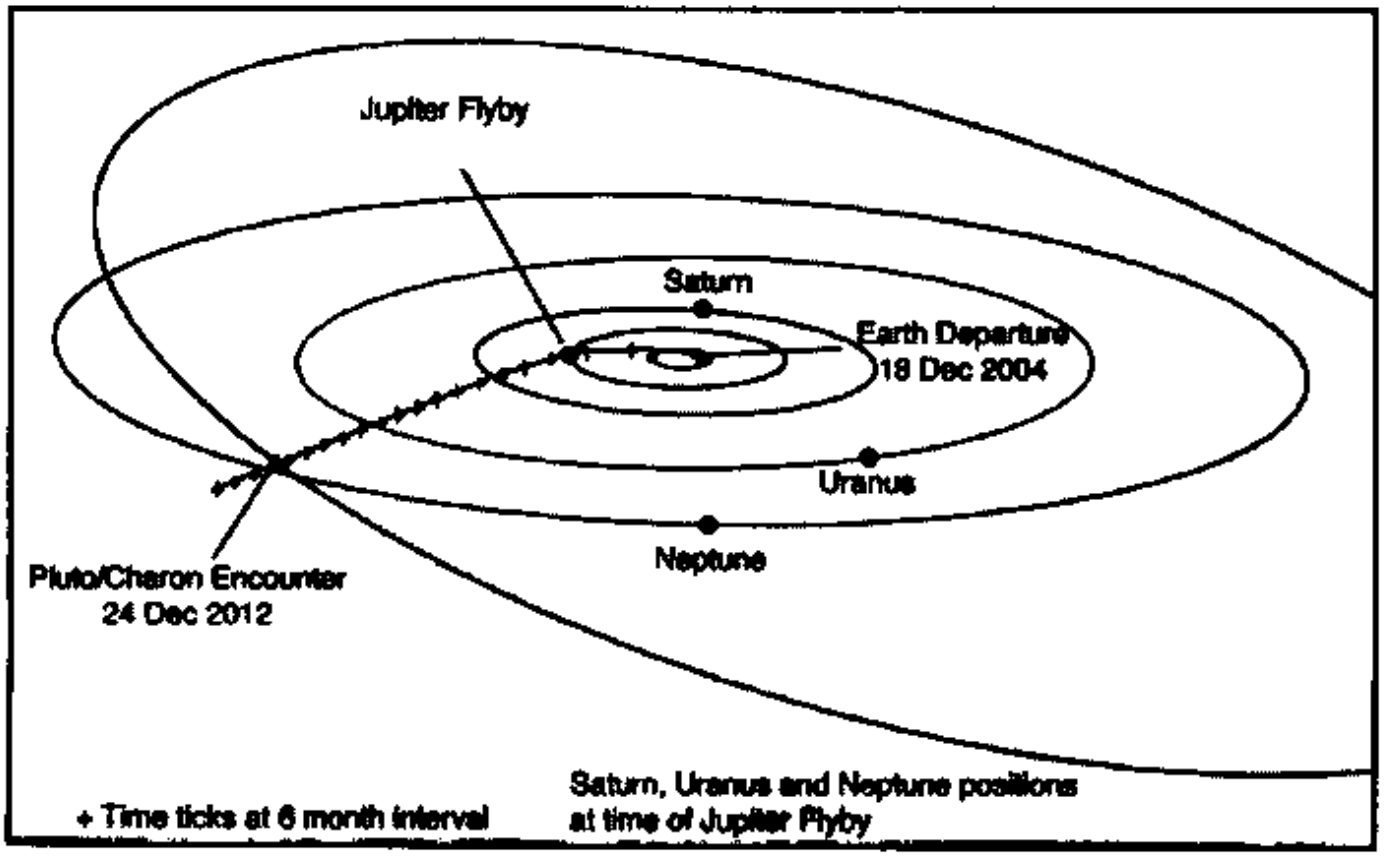


CHASMP Doppler Residuals For Interval III
30–Day Interval Surrounding 1996 Opposition



Example Pluto-Kuiper Express

8-year, Jupiter Gravity Assist* (JGA) trajectory to Pluto



A number of alternative trajectories have been studied for Pluto-Kuiper Express and the current favorable path for the 2004 launch date is a Jupiter Gravity Assist (JGA), arriving at Pluto/Charon 8 years later.