GENERAL RELATIVITY,

EXPERIMENT

AND

GRAVITATIONAL WAVES

Thibault Damour

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EINSTEIN'S VISION

BERN 1907

\[ x'^1 = x - \frac{1}{2} g^{ij} t^2 \]

RIEMANN ~ 1856

\[(V_n, g)\]

LOCAL EFFACEMENT OF \( g \)

\[ \exists x'^\mu, g'_{\mu\nu}(x'^{\mu}) = g_{\mu\nu} + O(x'^2) \]

LOCAL EFFACEMENT OF \( g_{\mu\nu} \)

UNIVERSALITY OF FREE FALL \( \longrightarrow \) UNIVERSAL COUPLING OF MATTER

(HYPOTHESIS OF EQUIVALENCE) \( \leftarrow \) TO ONE \( g_{\mu\nu}(x^1) \)

('EQUIVALENCE PRINCIPLE')

\[
S_{\text{tot}} = \frac{c^4}{16\pi G} \int \frac{\text{d}^4x}{c} R(g) + S_{\text{matter}} [\xi, A, H; g_{\mu\nu}] 
\]

TWO SORTS OF EXPERIMENTAL TESTS

- MATTER-COUPILING TESTS (RHS)
- TESTS OF THE DYNAMICS OF \( g_{\mu\nu} \) (LHS)
TESTS OF THE COUPLING MATTER GRAVITY 

"EQUIVALENCE PRINCIPLE" \( S_{\text{MATTER}}[\phi, A, H; g_{\mu
u}] \)

- UNIVERSALITY OF FREE FALL \( \mathbf{a}_A \downarrow \mathbf{a}_B \)
  Adelberger's group \( \frac{\Delta a}{a} \) \(_{Fe-Si} = (3.6 \pm 5.0^{\text{stat}} \pm 0.7^{\text{sys}}) \times 10^{-13} \)
  Lunar Laser Ranging group \( \frac{\Delta a}{a} \) \(_{\oplus} = (-1.0 \pm 1.4) \times 10^{-13} \)

- CONSTANCY OF "CONSTANTS" \( \alpha_{\text{EH}} = \frac{e^2}{\hbar c} \)
  Atomic Clock Tests
  Masion '03, Bize '03, Fischer '05
  Oklo's natural fusion reactor
  Shlyakhter '76, Zmievsky '96, Fuji '60
  Quasar absorption lines
  Quast '104, Srianand '104

- GRAVITATIONAL REDSHIFT
  Vessot, Levine '79

\[ \frac{\Delta \nu}{\nu} = (1 \pm 10^{-4}) \frac{\Delta U}{c^2} \]
Fontaines Atomiques

8 fountains in operation at SYRTE, PTB, NIST, USNO, Penn St, IEN. 5 with accuracy at $1 \times 10^{-15}$. More than 10 under construction.

BNM-SYRTE, FR

PTB, D

NIST, USA
Franges de Ramsey dans une fontaine atomique

S/N = 2000

T = 0.5 s
H = 30 cm

G. Santarelli et al., PRL 82, 4619 (1999)
UNIVERSALITY OF FREE FALL

"GALILEO" ~ 1630

NEWTON ~ 1686

1 \times 10^{-3}

\sim 10^{-7}

LAPLACE ~ 1805

\sim 10^{-8}

[POINCARE' 1906]

\sim 10^{-8}

DICKE ET AL. 1962

BRAGINSKY AND PANOV 1972

\sim 3 \times 10^{-11}

\sim 10^{-12}

\sim 10^{-13}

LUNAR LASER RANGING

1969 - NOW

DICKY ET AL. '94

WILLIAMS ET AL. '96

SPACE TESTS OF THE EQUIVALENCE PRINCIPLE

MICROSCOPE 2009

STEP 2007

\sim 10^{-15}?

\sim 10^{-18}?
Dynamics of the gravitational field: Weak field regime

One-graviton exchange

Linearized Einstein's equations

\[ \Box h_{\mu \nu} = -\frac{16\pi G}{c^4} \left( T_{\mu \nu} - \frac{1}{2} \Box T_{\mu \nu} \right) \]

\[
S_{\text{int}} = 2G \int ds_A ds_B \sum_{n=1}^\infty \left( \frac{m_A}{r_{AB}} \right)^n \left( \frac{m_B}{r_{AC}} \right)^n \mathcal{P}_{\rho \sigma}^{\mu \nu} \left( \chi_A - \chi_B + \chi_C \right) \int m_B u_B^\rho u_B^\sigma
\]

\[
L^{2\text{-body}} = \frac{1}{2} \sum_{A \neq B} \frac{G m_A m_B}{r_{AB}} \left[ 1 + \frac{3}{2c^2} \left( \frac{v_A^2 + v_B^2}{2c^2} \right) - \frac{7}{2c^2} \frac{v_A \cdot v_B}{2c^2} - \frac{1}{2c^2} \frac{(\mathbf{v}_A \cdot \mathbf{v}_B)}{(\mathbf{v}_A \cdot \mathbf{v}_B)} \right] + O(\frac{1}{c^4})
\]

\[
L^{3\text{-body}} = -\frac{1}{2} \sum_{3 \neq A \neq C} \frac{c^2 m_A m_B m_C}{r_{AB} r_{AC} c^2} + O(\frac{1}{c^4})
\]
Tests of the Dynamics of the Grav. Field

Solar-System Tests:

Weak ($h_{\mu\nu} < 10^{-6}$) and Quasi-Static ($\frac{\partial h_{\mu\nu}}{\partial x} \approx 10^{-4}$) Fields

- Perihelion Advance of Mercury
  I. Shapiro '90; assuming $J_2 \sim 2 \times 10^{-7}$
  \[ \Delta \dot{\varpi} = 42.98 \left( 1.000 \pm 0.001 \right) \]

- Light Deflection (VLBI)
  S. S. Shapiro... '04

- Orbital Motion of the Moon
  (Nordtvedt '68) Lunar Laser Ranging
  Williams... '04

- Varying Frequency Shift of Radio Links: Cassini Spacecraft
  (Bertotti, Ieds, Tortora '03)
  \[ \Delta v / v = 1 + \left( 1.0 \pm 1.1 \right) \times 10^{-5} \]

Quasi-Static, Weak-Field Einstein Gravity OK at

$10^{-5}$ Level
VARYING FREQUENCY SHIFT OF RADIO LINKS WITH THE CASSINI SPACECRAFT
(Bertotti, Iess, Tortora 103)

Two-Way Relativistic Frequency Shift

\[ \gamma = \left( \frac{\gamma_{\text{grav}}}{\gamma_{\text{grav}}} \right)^{\text{two-way}} = -4(1+\delta) \frac{GM_{\text{sun}}}{c^3 b} \frac{\text{d}b}{\text{d}t} = -0.2 \mu \text{arc} (1+\delta) \frac{1}{b} \frac{\text{d}b}{\text{d}t} \approx 10^{-9} \]

As Earth moves

\[ \gamma - 1 = (2.1 \pm 2.3) \times 10^{-5} \]
Binary Pulsars: First Possibility of Probing the Full Structure of Relativistic Gravity

- Radiative Effects [Field Propagation]
- Highly Non-Linear Effects [Strong Fields]

Propagation of Gravit. Interaction
+ Cubic Nonlinearities of Weak-Field Exp

Strong Gravit. Field
\[ \left( \frac{GM}{c^2r} \right)_{\text{PbR}} \approx 0.2 \]

Theoretical Aspects of Binary Pulsars:

1. Motion of Two Strongly Self-Gravitating Bodies
   (T.D. & Deruelle '81, T.D. '82, '83)

2. Relativistic Timing of a Binary Pulsar
   (Blandford & Teukolsky '76, T.D. & Deruelle '85, '86)

3. Use of Binary Pulsars as Probes of Relativistic Gravity
   (Eardley '75, Will & Chandra '77, T.D. '88, T.D. & Taylor '92)
TESTING RELATIVISTIC GRAVITY WITH BINARY PULSAR DATA

TWO APPROACHES

- "THEORY-INDEPENDENT" OR "PHENOMENOLOGICAL"
  - PARAMETRIZED POST-KEPLERIAN

- "THEORY-DEPENDENT"
  - BEYOND USUAL POST-NEWTONIAN PARAMETERS
  - CLASSES OF TENSOR-SCALAR THEORIES
USING BINARY PULSAR MEASUREMENTS TO PROBE RELATIVISTIC GRAVITY

TWO COMPLEMENTARY APPROACHES

1. PHENOMENOLOGICAL ANALYSIS OF BINARY PULSAR DATA
   "PARAMETERIZED POST-KEPLERIAN FORMALISM" (PPK)

RAW PULSAR DATA \{ \text{t}_{\text{obs}} \}

FIT

\text{t}_{\text{N}} = F[ N, p_i ] \quad i = 1, \ldots, 19

\text{THEOR}

\text{GENERAL RELATIVISTIC TIMING FORMULA}

\text{DATA (D+Deruelle 1986)}

\text{OBS} \quad ( = \text{FIT}) = \text{...} \pm \text{...}

\text{EACH RELATIVISTIC THEORY OF GRAVITY PREDICTS}

\text{Pi}^{\text{THEOR}} = \int_{D_i} \text{Pi}^{\text{THEORY}} (m_1, m_2 (\alpha, \eta))

\text{REDUNDANCY} : 19 - 2 = 17 \text{ TESTS OF RELATIVISTIC GRAVITY}

\text{MOST PROBE STRONG-FIELD ASPECTS OF GRAVITY}

\text{NB. EACH SUCH TEST IS A POTENTIAL KILLER OF G.R.}
Damour and Deruelle [36, 47] proved that it is possible to describe all of the independent $O(u^2/c^2)$ timing effects in a simple mathematical way common to a wide class of alternative theories. This made it possible to revert to a theory-independent analysis of timing data, and led to the possibility of working within a strong-field analog of the PPN formalism, the so-called [37] "parametrized post-Keplerian" approach. The part of the Damour–Deruelle phenomenological timing model describing orbital effects reads

\[ t_s - t_0 = F[T; \{p^K\}; \{p^{PK}\}; \{q^{PK}\}] , \]  

where \( t_s \) denotes the solar-system barycentric (infinite frequency) arrival time, \( T \) the pulsar proper time (corrected for aberration, see below),

\[ \{p^K\} = \{P_0, T_0, e_0, \omega_0, z_0\} \]  

is the set of Keplerian parameters,

\[ \{p^{PK}\} = \{k, \gamma, \dot{P}_0, \tau, e, \delta_0, \delta, \dot{\delta}\} \]  

the set of separately measurable post-Keplerian parameters, and

\[ \{q^{PK}\} = \{\delta_r, A, B, D\} \]  

the set of not separately measurable post-Keplerian parameters. The right-hand side of Eq. (2.1a) is given by

\[ F(T) = D^{-1} [T + \Delta_R(T) + \Delta_E(T) + \Delta_s(T) + \Delta_A(T)] , \]

\[ \Delta_R = x \sin \omega [\cos u - e (1 + \delta)] + z [1 - e^2 (1 + \delta)^2]^{1/2} \cos \omega \sin u , \]

\[ \Delta_E = \gamma \sin u , \]

\[ \Delta_s = -2 \tau \ln (1 - e \cos u - z [\sin \omega (\cos u - e) + (1 - e^2)^{1/2} \cos \omega \sin u]) , \]

\[ \Delta_A = A \{\sin [\omega + A_s(u)] + e \sin \omega \} + B \{\cos [\omega + A_s(u)] + e \cos \omega \} , \]

where

\[ x = x_0 + \dot{x}(T - T_0) , \]

\[ e = e_0 + \dot{e}(T - T_0) , \]

and where \( A_s(u) \) and \( \omega \) are the following functions of \( u \),

\[ A_s(u) = 2 \arctan \left[ \left( \frac{1 + e}{1 - e} \right)^{1/2} \tan \frac{u}{2} \right] , \]

\[ \omega = \omega_0 + k A_s(u) , \]

and \( u \) is the function of \( T \) defined by solving the Kepler equation

\[ u - e \sin u = 2 \pi \left[ \left( \frac{T - T_0}{P} \right) - \frac{1}{2} \dot{P}_0 \left( \frac{T - T_0}{P} \right)^2 \right] . \]
$3 - 2 = 1$
$4 - 2 = 2$
$6 - 2 = 4$

1. Radiative + Strong Field

$10^{-3}$, $10^{-2}$

2. Pure Strong Field Tests

PSR B1913+16

PSR J1141-6545

PSR J0737-3039
SOME HIGH-PRECISION BINARY-PULSAR TESTS

1913+16:
Weisberg, Taylor '04

\[ \frac{P_{\text{obs}}^* - P_{\text{galactic}}^*}{P_{\text{obs}}^* \left[ R_{\text{obs}}^*, \varphi_{\text{obs}}^* \right]} = 1.0013 \pm 0.0021 \]

1534+12:
Taylor, Weisberg, Damon, Weisberg '92
Steins et al. '92

\[ \frac{S_{\text{obs}}^*}{S_{\text{GR}} \left[ R_{\text{obs}}^*, \varphi_{\text{obs}}^* \right]} = 1.000 \pm 0.007 \]

0737-3039
Lyne et al. '04, Kramer et al. '04

\[ \frac{S_{\text{obs}}^*}{S_{\text{GR}} \left[ R_{\text{obs}}^*, R^* \right]} = 0.9998 \pm 0.0006 - 0.0011 \]

RADIATIVE AND STRONG-FIELD EINSTEIN GRAVITY OK AT 10^{-3} LEVEL
FIRST APPROACH TO Theory-Dependent Analysis

Idea: Generalize Parametrized Post-Newtonian Framework (Eddington '24, Schiff '60, Baierlein '67, Nordtvedt '68, Will '72)

Solar system ⇒ Weak field \( \frac{GM}{c^2 r} \ll 10^{-6} \ll 1 \)

Main first-order corrections parametrized by:
\[
\begin{align*}
\bar{\gamma} &= \gamma_{\text{PN}} - 1 \\
\bar{\beta} &= \beta_{\text{PN}} - 1 
\end{align*}
\]

: Light deflection

: Periastron precession

? Generalization of \( \bar{\beta} \) and \( \bar{\gamma} \) to second-order corrections \( \propto \left( \frac{GM}{c^2 r} \right)^2 \)?

Seek inspiration from simplest class of theories: Tensor-scalar \( g_{\mu \nu}, \varphi_A, \varphi_B, \ldots \)

Second-order (2PN) corrections parametrized by only two parameters

(Damour, Esposito-Farèse '96)

E.g.
Effective gravitational coupling between A and B

\[
\frac{G_{AB}}{G} = 1 + (4\bar{\beta} - \bar{\gamma}) \left( \frac{E_A^\text{grav}}{m_A c^2} + \frac{E_B^\text{grav}}{m_B c^2} \right) + 4 \zeta \left( \frac{E_A^\text{grav}}{m_A c^2} \right) \left( \frac{E_B^\text{grav}}{m_B c^2} \right) + \left( \frac{\varepsilon}{2} + 3 \right) \frac{\left( U_A^2 \right)}{c^4} + \left( U_B^2 \right) + \ldots
\]

2PN (Damour, Esposito-Farèse '96)

• 2PN terms, \( \alpha, \varepsilon, \zeta \), are too small to be measurable in solar system (they do not enter light deflection!)

• Binary pulsars: \( \frac{E_A^\text{grav}}{m_A c^2} \sim 0.15 \) ⇒ analyze data as constraints on \( \varepsilon \) >
Binary pulsar constraints on the 2PN parameters
\[ \epsilon (\infty) \quad \text{and} \quad \zeta (\infty) \]

[Damour & Esposito-Farèse, PRD 53 (1996) 5541]

\[ \Rightarrow 2\times \text{tighter constraints on } \epsilon; \quad 15\times \text{tighter constraints on } \zeta \]

\[ -4 \times 10^{-2} < \epsilon < 3 \times 10^{2} \quad -2 \times 10^{-4} < \zeta < -4 \times 10^{-4} \]
SECOND APPROACH TO PROBING GRAVITY WITH BINARY PULSAR DATA

THEORY-DEPENDENT ANALYSIS OF PSR DATA

Choose a class of simple alternatives to GR containing a small number of parameters, but sufficiently many to exhibit interesting effects.

Tensor-scalar gravity:

\[ S = \frac{1}{16\pi G} \int d^4x \sqrt{g} \left[ R(g) - 2(\partial \phi)^2 \right] + \int d^4x \sqrt{g} \left[ \tilde{g}^{\mu\nu} \frac{\partial}{\partial x^\mu} \phi \frac{\partial}{\partial x^\nu} \phi \right] \]

Scalar field \( \phi \)  
Coupling function \( \alpha(\phi) \)  
Potential: fixes \( \phi_0 \)

Two-parameter coupling function:

\[ \alpha(\phi) = \alpha_0 (\phi - \phi_0) + \frac{1}{2} \beta_0 (\phi - \phi_0)^2 \]

Simple generalization of Jordan-Brans-Dicke

\[ \alpha^{\text{JBD}}(\phi) = \alpha_0 (\phi - \phi_0) \quad ; \quad \alpha_0^2 \equiv \frac{1}{2\omega + 3} \]

Minimal theory leading to PPN parameters

\[ \gamma_{\text{PPN}} = \gamma - 1 = -2 \frac{\alpha_0^2}{1 + \alpha_0^2} \]

\[ \beta_{\text{PPN}} = \beta - 1 = \frac{1}{2} \frac{\beta_0 \alpha_0^2}{(1 + \alpha_0^2)^2} \]

Features interesting non-perturbative strong-field effects

(Edington, Nordtvedt, Will...
Tensor–scalar theories

\[ S = \frac{1}{16 \pi G} \int \sqrt{-g} \left\{ R - 2 \left( \partial_{\mu} \phi \right)^2 \right\} + \mathcal{S}_{\text{matter}} \left[ \text{matter} ; \tilde{g}_{\mu \nu} = e^{2a(\phi)} g_{\mu \nu} \right] \]

\[ a(\phi) = \alpha_0 (\phi - \phi_0) + \frac{1}{2} \beta_0 (\phi - \phi_0)^2 + \ldots \]

\[ a(\phi) \]

\[ \gamma_{\text{PPN}} - 1 \propto \alpha_0^2 \]

\[ \beta_{\text{PPN}} - 1 \propto \alpha_0^2 \beta_0 \]

SOLAR SYSTEM (WEAK-FIELD) TESTS

Vertical axis \((\beta_0 = 0)\) : Jordan–Fierz–Brans–Dicke theory \( \alpha_0^2 = \frac{1}{2\omega_{\text{ad}} + 3} \)
No deviation from General Relativity in weak-field conditions

large slope \( \alpha_A \)
\( \Rightarrow \) large deviations from General Relativity for neutron stars

\[ E = \int \left[ \frac{1}{2} (\nabla \phi)^2 + \rho e^{\phi R^2} \frac{1}{2} \right] \]
\[ \Rightarrow \frac{1}{2} R \phi^2 + m e^{\phi R^2} \frac{1}{2} \]
parabola Gaussian if \( \beta_0 < 0 \)

"spontaneous scalarization" [T. Damour & G.E.J. 1993]

```
neutron star
--
scalar charge \( |\alpha| \)
```

```
baryonic mass \( \frac{m_\Lambda}{m_e} \)
critical mass
maximum mass in GR
maximum mass
```

```
0 0.2 0.4 0.6
```

```
0.5 1 1.5 2 2.5 3
```

```
critical mass
maximum mass
```
Strong-field effects

\[ G_{AB}^{\text{eff}} = G \left( 1 + \alpha_A \alpha_B \right) \]

depends on internal structure of bodies A & B

Similarly for \((\gamma_{\text{PPN}} - 1)\) and \((\beta_{\text{PPN}} - 1)\) \(\Rightarrow\) all post-Newtonian effects

Energy flux = \[\frac{\text{Quadrupole}}{c^5} + O\left(\frac{1}{c^7}\right) \text{ spin 2}\]
\[+ \frac{\text{Monopole}}{c} \left(0 + \frac{1}{c^2}\right)^2 + \frac{\text{Dipole}}{c^3} + \frac{\text{Quadrupole}}{c^5} + O\left(\frac{1}{c^7}\right) \text{ spin 0}\]
\[\propto (\alpha_A - \alpha_B)^2\]
Solar-system and best binary pulsar constraints on tensor-scalar theories (updated April 2005)

The graph shows the relationship between $|\alpha_0|$ and $\beta_0$ for various pulsar systems. The curves represent different pulsars with labels such as J1411-6545, B1534+12, and others. The plot includes a comparison with general relativity and matter contributions.
Was Einstein 100% right?

Should we stop testing special and general relativity?
String-inspired phenomenology

- No clear understanding of how to fit our world within string theory
  ⇒ Discuss phenomenological possibilities; open new experimental opportunities

"String" Theory

- Extended massless spectrum
- Light spectrum
- Branes, large extra-dimensions
- Short distance effects $\ell_s$
- String cosmology

\[ g_{\mu \nu}(z) \]
\[ \Phi(z) \]
\[ B_{\mu \nu}(z) \]

Moduli($z$)

Kaluza-Klein gravity

Brane-modified gravity

Long-range modification of gravity

Short-range modification of gravity

\(?\) effect in cosmology and relativistic astrophysics

E.g., GW signals from early cosm or cosmic superstrings

Incomplete decoupling of $M_s$ effects

New or modified primordial cosmology scenarios
BRANES AND GRAVITY

"LARGE" BUT COMPACT EXTRA-DIMENSIONS
Antoniadis, Arkani-Hamed, Dimopoulos, Dvali

INFINITE EXTRA-DIMENSIONS
BUT "MISMATCHED" GRAVITY
Randall, Sundrum

MULTI-BRANES
Kogan, Neveu, Papazoglou, Ross, Santiago, Gregory Rubakov, Sibiryakov

Dvali, Guevara, Gubser, Henningson, Vachaspati

INDUCED GRAVITY
$G_5$/$G_4$ $L$

TUNNELING (LINEAR GRAVITON WAVES)
BETWEEN EXTRA DIMENSIONS

$U = \frac{GM}{r} \left[ 1 - \frac{1}{L} \left( \frac{r^2}{c^2} \right) \right]$

Effect in solar system, lunar ranging...

$\text{\textsuperscript{1}}$Pauli-Fierz

$\text{\textsuperscript{2}}$Type massive gravity

HIGHER-DIMENSIONAL

GRAVITY WHEN

$\gamma < L_c$

AND (if $ls \sim \text{TeV}$)
INTERESTING OBSERVABLE EFFECTS IN LHC
Figure 5: 95% confidence-level constraints on ISL violating Yukawa interactions with 1 mm < \lambda < 1 cm. The lower curves give experimental upper limits (the Lamoroux constraint was compiled in Reference (151)). Theoretical expectations for extra dimensions (56), model (101), dilaton (102), and radon (83) are shown as well.
**Intuitive Meaning of** $g_{\mu\nu}(x) + \Phi(x) + ...$

<table>
<thead>
<tr>
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<th><strong>Geometry</strong></th>
<th><strong>Coupling Constants</strong></th>
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<tbody>
<tr>
<td><strong>Newton</strong></td>
<td>Rigid</td>
<td>Rigid</td>
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<tr>
<td><strong>Einstein</strong></td>
<td>Soft</td>
<td>Rigid</td>
</tr>
<tr>
<td><strong>String Theory</strong></td>
<td>Soft</td>
<td>Soft</td>
</tr>
</tbody>
</table>

\[ g_{\mu\nu}(x) \sim g^2(x) \sim G(x) \]

**But then one would expect:**

- **Non-universality of Free Fall** $\frac{\Delta a}{a} \sim 10^{-5}$

- **Cosmological Variation of Coupling Constants**
  \[ \frac{\dot{a}}{a} \sim \frac{\dot{H}}{H} \sim 10^{-10} \text{yr}^{-1} \]

- **Modification of Post-Newtonian Gravity**
  \[ \gamma - 1 \sim \mathcal{O}(1) \]
1. \( m_\Phi \neq 0; V(\Phi) \neq 0 \) in low-energy world \( \Rightarrow \) short-range effects \( \propto e^{-m_\Phi r} \)

Recent experiments:
- Hoyle 2001
- Chiusanelli 2003
- Long 2003

\( \Rightarrow \) \( \lambda_\Phi = \frac{1}{m_\Phi} < 0.1 \text{ mm} \) \( \Rightarrow m_\Phi > 10^{-3} \text{ eV} \)

The value of \( m_\Phi \) is model-dependent. Some models need to fix \( \Phi \) early on (before inflation) \( \Rightarrow m_\Phi \sim M_s \gg H_{\text{inf}} \)

In some models, \( m_\Phi \) is linked to SUSY breaking: \( V(\Phi) \sim M_{\text{susy}}^4 \left( \frac{\Phi}{M_p} \right) \)

\( \Rightarrow m_\Phi \sim \frac{M_{\text{susy}}^2}{M_p} \approx \frac{(1 \text{ TeV})^2}{2.4 \times 10^{18} \text{ GeV}} \sim 10^{-3} \text{ eV} \)

\( \Rightarrow \) possible modification of Cavendish experiments just below 0.1 mm current data

2. \( m_\Phi = 0; V(\Phi) = 0 \), but \( \exists \) non-trivial coupling function \( B(\Phi) \)

\( L_{\text{eff}} \downarrow \Rightarrow B_R(\Phi) R(g) + B_\Phi(\Phi)(\Phi)^2 + B_F(\Phi) F_{\nu}^2 + \ldots \)

\( \Rightarrow \) presence of matter

\( \Phi \) mechanism of natural cosmological attraction: \( \Phi \rightarrow \Phi_m \)

\( \Rightarrow \) nearly decouples from matter when \( \Phi = \Phi_m \)

\( \Rightarrow \) naturally suppressed modifications of long-range gravity

3. Both a quintessence-like \( V(\Phi) \neq 0 \) and coupling to matter \( B(\Phi) \)

\( \Rightarrow m_\Phi \) depends on surrounding matter density, so that \( \Phi \) is short-ranged in earth-bound expts. Khoury, Weltman, Byr...
\[ \mathcal{L}_{\text{eff}} = B_R(\Phi) R + B_\Phi(\Phi)(\nabla \Phi)^2 + B_F(\Phi) F^2 + B_X(\Phi) (\nabla X)^2 + B_V(\Phi) X^n \]

Dilaton coupling functions

\[ B_0(\Phi) = C_i + \Delta(\omega^{-2}) \]

As \( \Phi \to \infty \),

\[ g_s = e^\Phi \to 0 \]

Inflation

Coupling to matter

\( \alpha \propto B_i(\Phi) / \Phi \) is small but \( \neq 0 \)

Vacuum

Observational consequences today

Residual coupling

\[ \alpha_{\text{had}}^2 (\Phi_{\text{end}}) \sim 10 \left( \frac{b_F}{b_s c} \right)^2 \left( \frac{8}{n+2} \right) \sim 2.5 \times 10^{-8} \quad V(\chi) \sim \chi^n \]

If \( n = 2 \)

\[ \gamma_{\text{PPN}} - 1 \sim -2 \alpha_{\text{had}}^2 \sim -5 \times 10^{-8} \]

\[ \frac{\Delta a}{a} \sim 5 \times 10^{-5} \alpha_{\text{had}}^2 \sim 10^{-12} \]

\[ \frac{a_{\text{EM}}}{a_{\text{EM}}} \sim \pm \sqrt{1 + q_0 - \frac{3\Delta a}{2}} \sqrt{10^{12} \frac{\Delta a}{a}} 10^{-16} \text{ yr}^{-1} \]
FUTURE EXPERIMENTS ON GRAVITY

- Gravity Probe B
- Comparison of atomic clocks
- Exploring sub-micron deviations from Newton's law
- Old and new binary pulsars
- Improved solar-system tests
- More
  - Gaia
  - LATOR
  - $Y \approx 10^{-9}$
  - $\beta \approx 5 \times 10^{-6}$
- Gravitational waves
  - LIGO/Virgo/GEO
  - LISA
  - Coalescence of binary black holes
  - Coalescence of binary neutron stars
  - GW bursts from cusps on massive strings
- Improved (satellite) tests of the equivalence principle
  - Microscope (2017)
    - Onera/CNES
    - $\frac{\Delta \alpha}{\alpha} \approx 10^{-15}$
  - STEP
    - Stanford/NASA/ESA/LISA
    - $\frac{\Delta \alpha}{\alpha} \approx 10^{-18}$
- Improved CMB measurements
  - Planck
Fig. 3.2: Equivalence principle violation: The figure shows the relative motion of free masses where the ratio of inertial mass to gravitational mass depends on the composition of the masses. These test masses are constrained by linear magnetic bearings and sensing circuits. Here, the Equivalence Principle violation signal appears at the orbital frequency. In the normal mode of operation the spacecraft would be spun about an axis perpendicular to the orbital plane at a non-integral multiple of the orbital frequency, shifting the Eötvös signal frequency to the spin-frequency = the orbital frequency (depending on the spin sense).
CONSTRAINTS ON $\alpha_0^2 = \alpha_{\text{had}}^2$ FROM PRESENT AND FUTURE EXPERIMENTS (LONG-RANGE DEVIATIONS)

Composition-independent

LIGO/VIRGO

Present solar system expts

Binary pulsar expts

Present VLBI tests $d\theta_0$

$\alpha_{\text{had}}^2$

Compositional-dependent

OKLO

$\times 10^{-3}$

$\times 10^{-4}$

Cassini

GPB

More

Potsdam astrometry

Points

Gaia

Time delay from laser links to heliocentric clocks

Sort

LATOR

$\times 10^{-9}$

$\times 10^{-10}$

$\times 10^{-11}$

$\times 10^{-12}$

$\times 10^{-13}$

USCOPE

$\times 10^{-14}$

STEP

Using

$(t_0 - t)^2 \sim 10^{-17}$

Ground clock (differential)*

Geocentric clock (differential)*

Present EP expts

Webb et al. $(\Delta \alpha/\alpha)_2 \sim 10^{-5}$

Present EP expts

Geocentric clock (differential)*

Space time

Ultimate heliocentric clocks (differential)
Gravitational Radiation

Excitation of Spacetime Geometry

\[ \delta_{\rho\nu} = \gamma_{\rho\nu} + h_{\rho\nu} \]

\[
\begin{pmatrix}
1 & x & y & z \\
0 & 0 & 0 & 0 \\
0 & h_{+} & h_{\times} & 0 \\
0 & -h_{\times} & h_{+} & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

Two Transverse Polarizations

Strain of Space Distances:

\[ \frac{\delta L}{L} \sim h \]

Interferometric Detector

Several Independent Observational Proofs of Reality of Grav. Radiation

PSR 1913+16 0.3% proof that \( c_{\gamma} = c + \frac{d}{c^2} \)

+ PSR 1534+12, PSR J1141-6545, PSR J0737-3039

+ PSR 1913+16 \( ^{\text{No}} \quad \frac{2GM}{c^2R} = 0.4 \Rightarrow 1\% \) test of strong-field regime of General Relativity
The problem of motion in Newtonian and Einsteinian gravity

The equation of motion of a body in a gravitational field is given by

\[ \ddot{r} = -A \frac{\ddot{r} - \dot{r} \cdot \dot{r}}{c^2}, \]

where \( A \) is the gravitational potential. The term \( \ddot{r} - \dot{r} \cdot \dot{r} \) represents the acceleration due to the gravitational field. The equation can be written as

\[ \ddot{r} = -A \left( \frac{\ddot{r} - \dot{r} \cdot \dot{r}}{c^2} \right). \]

The parameter \( A \) is given by

\[ A = \frac{Gm}{r^2} \left( 1 - \frac{2m}{r} \right), \]

where \( G \) is the gravitational constant, \( m \) is the mass of the body, and \( r \) is the distance from the center of mass. The acceleration due to gravity is

\[ \ddot{r} = -Gm \left( 1 - \frac{2m}{r} \right) \frac{\ddot{r} - \dot{r} \cdot \dot{r}}{c^2}. \]

The two parameters \( m \) and \( m' \) appearing in eqs. (154) to (161) are the 'Schwarzschild masses' of the condensed bodies. They are two constants which appear in the external gravitational field, in which are hidden many internal structure effects (see the discussion of the 'effacement of internal structure' in Section 6.14). On the other hand, the spin tensors undergo a slow evolution (on the post-Newtonian time scale, i.e., \( t^2 \) times the orbital period) which is also obtained in the Finsler–Infeld–Hoffmann Kerr-type approach (Damour, 1982, and references therein). Introducing \( \ddot{S} \), a suitable spin-vector, \( \dot{S} \), associated with \( S_{\text{in}} \), the law of evolution (spin precession) reads for the first body (see also references in Section 6.13.2)

\[ \frac{dS}{dt} = \left[ \frac{Gm}{c^2}, \dot{S} \right] \times \dot{S}. \]
GROUND-BASED NETWORK OF INTERFEROMETRIC DETECTORS

LIGO / VIRGO / GEO / TAMA / ...

2 sites (Hanford, Livingston)
3 interferometers
4 km + 2 km, 4 km

1 site
1 interferometer
3 km
600 m
300 m

Need 3 sites for localizing the source

SPACE-BASED INTERFEROMETRIC DETECTOR

LISA (ESA/NASA)

2 interferometers
2 arms

\( \frac{8L}{c \Delta x} \leq 10^{-21} \)

\( 8L \leq 4 \times 10^{-16} \text{ cm} \)

\( \sim 5 \times 10^6 \text{ km arms} \)
FIG. 3. The effective noise $h_n = \sqrt{\int S_n(f)}$ in various ground-based interferometers.
Coalescing Binary

Inspiral

Merge + Ringdown

Plunge
COALESCING BINARY BLACK HOLE

\[ m_1 + m_2 \sim 20 \text{ to } 30 \, M_\odot \]

- EVENT RATE: \( \approx 1 \text{ / yr} @ 200 \text{ Mpc} \)
  - (Lipunov, Postnov, Prokhorov '97)
  - Mc Neill, Portegies Zwart '00

- THEORETICAL CHALLENGES

  USEFUL SIGNAL COMES FROM LAST FEW ORBITS + PLUNGE
  (Flanagan, Hughes '97, Damour, Iyer, Sathyaprakash '01)

\[ \mathcal{L}_{\text{GW}} = 2 \int L_{\text{LHO}} \sim 5000 \frac{M_\odot}{(m_1 + m_2)} \, \text{Hz} \approx 167 \left( \frac{30 \, M_\odot}{m_1 + m_2} \right) \, \text{Hz} \]

- NEW ANALYTICAL METHODS:
  - VERY HIGH-ORDER PERTURBATION CALCULATIONS OF DYNAMICS AND RADIATION: “3-LOOP” 
    - WITH DIMENSIONAL REGULARIZATION (Damour, Jaranowski, Shapiro '01, Blanchet, Damour, Esposito-Ferjani, Iyer, ‘05)
  - RESUMMATION OF PERTURBATIVE EXPANSIONS: PADE APPROXIMANTS
    (Damour, Iyer, Sathyaprakash)
  - “EFFECTIVE ONE BODY” APPROACH (Buonanno, Damour)
    [Similar to: QED O(\alpha) (Brozić, Itzykson, Zinn-Justin, Todorov ...)]
"3-LOOP" CALCULATION OF 2 POINT-MASS GRAVIT. INTERACTION

\[
\text{MATTER (POINT-MASS)} \quad \text{GRAVITY}
\]

\[
\begin{align*}
\text{\textit{ZERO-LOOP}} & = \text{BNG-GRAVITON EXCHANGE} \\
& = - \frac{G m_1 m_2}{r_{12}} \left[ 1 + \frac{c^2}{2} \frac{c^4}{4} + \ldots \right]
\end{align*}
\]

\[
\begin{align*}
\text{\textit{1-LOOP}} & = \ldots \\
\left\{ \begin{array}{l}
\text{\textit{2-LOOP}} \\
\text{\textit{3-LOOP}} \\
\end{array} \right. \\
& = \ldots \\
\end{align*}
\]

Finite (unambiguous) result in Dimensional Regularization

(Damour, Javanainen, Schäfer'01)
Effective One Body Dynamics

Resummed Radiation Reaction (Quasi-Circular Orbit)

Transition from plunge with arbitrary mass ratio

Initial Dynamical Data (q, q, p, p)

At beginning of plunge: 0.6 orbit light

First Estimate of Full Waveform;

"6M" → "2M" zeroth order
\( h(t) = v_0^2 \sin 2\phi(t) \)
\( v_0 = (\phi)^{1/3} \)

MATCHING TO LEAST-DAMPED QUASI-NORMAL MODE OF A KERR BH
\[ \begin{align*}
\mathcal{M} &= \frac{E_{\text{light-ring}}}{m^2} \\
J &= \frac{J_{\text{light-ring}}}{m} \\
\end{align*} \]
AT THE (\( \nu \)-DEFORMED) LIGHT-RING

\( \nu = 1/4 \)

- **inspiral + plunge**
- **merger + ring-down**
- naive LSO
  - \( r \)-LSO
  - \( j \)-LSO
  - \( E \)-LSO
  - \( \omega \)-LSO

CLOSE LIMIT APPROXIMATION
Smarr, Bace, Pullin...
readly: Baker et al.
 Initial data: Cook, Bensinger, Gundlach, et al.
FIG. 1. Signal-to-noise ratios in GEO, LIGO-I and VIRGO when using as Fourier-domain templates the post-Newtonian model Eq. (3.6) ($T^3$), truncated at the test-mass $f_{\text{esc}} = 4000\, m_\odot/m$ Hz (in thin lines), compared to the optimal one obtained when the template coincides with the fiducial "exact" (effective one-body) signal (thick lines). As usual, we averaged over all the angles. The overlaps are maximized over the time lags, the phases, and the two individual masses $m_1$ and $m_2$. The plots are jagged because we have computed the SNR numerically by first generating the fiducial "exact" waveform in the time-domain and then using its discrete Fourier transform in Eq.(3.3). The greater SNR achieved by effective one-body waveforms for higher masses, as compared to Fig. 1 of DIS2, is due to the plunge phase present in these waveforms. Observe that the presence of the plunge phase in the latter significantly (up to a factor of 1.5) increases the SNR for masses $m > 35\, m_\odot$. Using the effective one-body templates will, therefore, enhance the search volume of the interferometric network by a factor of 3 or 4.
COSMIC SUPERSTRINGS!

Witten '85; Dvali, Tye, Tye, ...; KKLMMT; Copeland, Myers, Polchinski; Dvali, Vilenkin

10 dim spacetime:
\[ X^M = (x^\mu, y^a) \]
\[ 4 \text{ flat}, 6 \text{ compact} \]

\[ \Gamma \sim |x_1 - y_2| \]

\[ V(y) = A - \frac{B}{y^4} \]

SLOW ROLL INFLATION
\[ \eta(x) = f(x) \]

OUR WORLD

COSMOLOGICAL NETWORK OF MASSIVE (F OR D) STRINGS
WITH STRING TENSION

\[ 10^{-11} \leq G \rho \leq 10^{-6} \]

\[ G \rho \approx 10^{-8} \]

KKLMMT
Copeland IP

10^{-11} \leq G \rho \leq 10^{-6} \]

G \rho \approx 10^{-8} \]

KKLMMT
Copeland IP

GRAVITATIONAL WAVE BURSTS

\[ \dddot{h}(t) \]

\[ \ddot{h}(t) \sim \frac{1}{(t-t_c)^{5/3}} \]

RECURRENT CUSPS

POTENTIALLY DETECTABLE IN LIGO/VIRGO/...; LISA; PULSARTIMING

Dvali, Vilenkin
FIG 1. Gravitational wave amplitude of bursts emitted by cosmic string cusps (upper curves) and kinks (lower curve) in the LIGO/VIRGO frequency band, as a function of the parameter $\alpha = 50G\mu$ (in a base-10 log-log plot). The upper curve assumes that the average number of cusps per loop oscillation is $n = 1$. The middle curve assumes $n = 0.1$. The lower curve gives the kink signal (assuming only one kink per loop). The horizontal dashed lines indicate the one sigma noise level (after optimal filtering) of LIGO 1 (initial detector) and LIGO 2 (advanced configuration). The short-dashed line indicates the "confusion" amplitude noise of the stochastic GW background.

FIG 2. Gravitational wave amplitude of bursts emitted by cosmic string cusps (upper curves) and kinks (lower curve) in the LISA frequency band, as a function of the parameter $\alpha = 50G\mu$ (in a base-10 log-log plot). The meaning of the three solid curves is as in Fig 1. The short-dashed short-dashed curve indicates the confusion noise. The lower long-dashed line indicates the one sigma noise level (after optimal filtering) of LISA.
GRAVITATIONAL WAVE BURSTS FROM COSMIC STRING STRINGS

(STRONGLY) NON GAUSSIAN
STOCHASTIC BACKGROUND OF GRAV. WAVES

UNKNOWN PARAMETERS: \( \mu, p, \varepsilon \)

- STRING TENSION \( \mu \): RECENT SCENARIOS \( \sim 10^{-11} \leq \mu \leq 10^{-6} \)
- RECONNECTION PROBABILITY \( 0 \leq p \leq 1 \): RECENT SCENARIOS \( 10^{-3} \leq p \leq 1 \)
- TYPICAL LENGTH OF NEWLY FORGED LOOPS \( l = \varepsilon \times 50 \, G_{\mu} t \): RECENT SCENARIOS \( \varepsilon \ll 1 \)

POSSIBILITY OF DETECTING SUCH GW BURSTS
IN LIGO1, LIGO2, LISA AND PTA

\[ \mathcal{O}_{g} (\Omega_p) h^2 \sim 10^{-9.13} c \left( \frac{G \mu}{10^{-10}} \right)^{2/3} p^{-1} e^{-\frac{1}{3}} \left( \frac{f}{(10 \text{yr})^{-1}} \right)^{-1/2} \]

Damour, Vilenkin '04

# OF GUSPS \( c \leq 1 \) TEND TO INCREASE THE SIGNAL!
Suggested (allowed) range
CONCLUSION: Gravity: A New Frontier

- For a long time, gravity and General Relativity were
  - Experimentally badly tested
  - Astrophysically nearly useless \( \frac{v^2}{c^2} \frac{GM}{c^2R} \ll 1 \)
  - Cosmologically useful, but poor data
  - Theoretically isolated from rest of physics

- Today, the situation is quite different
  - Experimentally:
    - High-precision confirmations \( 10^{-5} \) weak-field
    - Grav. waves: A new window on the universe
  - Astrophysically: Crucially useful: NS, BH, GW, ...
  - Cosmological data: Now high-precision (few%) \( \rightarrow \) dark energy...
  - Theoretically: GR is central in string theory:
    - \( G_{\mu\nu} \) as massless excitation of closed strings
    - But “gravity sector” of string theory is potentially much richer than GR

\[ \Rightarrow \text{? Beyond Einstein’s GR?} \]

New experimental opportunities: e.g. short-range deviation, \( E_P \)

At the same time: High-precision tests + theoret. interest \( \Rightarrow \text{Can trust GR predictions?} \)