Science dealing with mat
energy in terms of motion an

Better :

a) Method of prediction -
To predict, must first descri
Choice of variables makes it

b) Experimental

c) Mathematical

d) Restricted (not, e.g., b
where things are still too comp
for precise physics methods)

Themes of Physics :

Scale extension - Desc
phenomena over a range o
distance or energy scales

Conservation...
E+M: Flux lines ~ electric
on charges; magnetic never en

Unification
E+M: Magnetism = electri
Changing $\vec{B} \to \vec{E}$
Changing $\vec{E} \to \vec{B}$
Light = E+M wave
Relativity: c = universal
(Unifies mechanics and

Symmetry
E&M: Gauge invariance

$$\vec{E} = -\nabla\varphi - \frac{\partial \vec{A}}{\partial t} \qquad \vec{B} = \nabla \times \vec{A}$$

$$\vec{A}' = \vec{A} + \nabla\Lambda, \quad \varphi' = \varphi - \frac{\partial \Lambda}{\partial t}$$

$$\vec{E}' = \vec{E}, \quad \vec{B}' = \vec{B}$$

We'll explore all thes
themes

$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$

Force density: $\vec{f} = \rho\vec{E} + \vec{J}\times$

Equation of continuity
(Local conservation of

$\frac{\partial\rho}{\partial t} + \nabla\cdot\vec{J} = 0$; (L+L) $\frac{\partial\rho}{\partial t} + \mathrm{div}$

Integral form:

$$\frac{dQ}{dt} + I_{out} = 0$$

$Q = \int dV\rho$, $I_{out} = \int d\vec{S}\cdot$

Integral forms:

$$\oint \vec{E} \cdot d\vec{l} = \mathcal{E}mf = 0$$

$$\oint \vec{E} \cdot d\vec{S} = 4\pi k Q$$

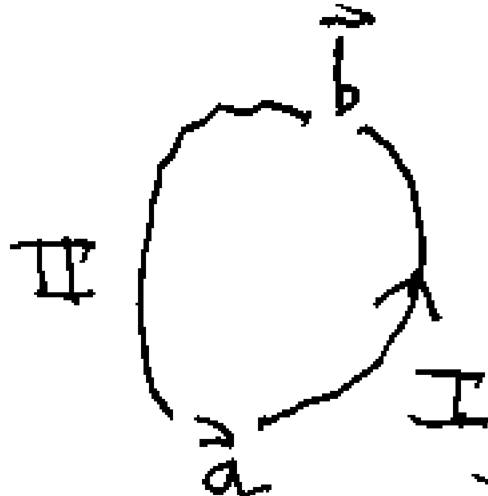
If (a) $[\,\vec{E}$ everywhere c

then $\vec{E} = -\nabla \varphi = -\text{grad}\, \varphi$

Proof:

$\int_{\vec{a}}^{\vec{b}} d\vec{l} \cdot \vec{E}$ independe

path



$$\int_{I} - \int_{II} = \oint d\vec{l} \cdot \vec{E} = 0$$

$$\therefore \int_{\vec{a}}^{\vec{b}} = \varphi(\vec{r} = \vec{a}) - \varphi$$

$$\nabla\cdot\vec{E} = \underbrace{-\nabla^2\varphi = 4\pi k\rho}_{\text{Poisson Equation}}$$

$\rho = 0$: $\nabla^2\varphi = 0$ (Lapla

$\nabla^2\varphi = $ (L+L) $\Delta\varphi = $ laplacia

Point charge $\vec{E} = \frac{kq\hat{r}}{r^2}$

$$= \frac{q\hat{r}}{4\pi\epsilon_0 r^2}$$

$$\varphi = \frac{kq}{r} = \frac{q}{4\pi\epsilon_0 r}$$

$$\nabla^2\varphi = -\frac{q}{\epsilon_0}\,\delta^{(3)}(\vec{r})$$

Dirac de

$$\delta^{(3)}(\vec{r}) = 0 \,,\ \vec{r} \neq 0$$

$$\text{undefined } \vec{r} = 0$$

NOT A FUNCTION!!

from the function $f[\vec{r}]$ to
number $f(\vec{r}=0)$. Obviously
assumes f is reasonably
e.g., continuous.

Thought question:
Is this the precise crite

Look at this another way u
the divergence theorem

$$\int_V dV\, \nabla\cdot\vec{E} = \int_{S\text{ of }V} d\vec{S}\cdot\vec{E}$$

Let V = ball centered on $\vec{r}=$
w/ radius R

$$\int_S d\vec{S}\cdot\vec{E} = \int d\Omega\, R^2\, \hat{r}\cdot\frac{q}{4\pi}$$

$= \frac{q}{\epsilon_0}$. However, outside
$\nabla\cdot\vec{E} = 0$ (check this), so

Equivalent to the previo
discussion, where now
function $f(\vec{r}) = 1$ for $\vec{r}$
V, $f = 0$ otherwise.

Potential energy i
electrostatics:

$$U(\vec{r}) = q\,\varphi(\vec{r}) \text{ for } p$$

charge q at $\vec{r}$.

Collection of charge

$$U = \sum_{i<j} q_i \varphi_j(\vec{r}_i)$$

Continuous density —

$$U = \frac{1}{2} \int dV \rho(\vec{r})\,\varphi($$

Reciprocity theorem

$$q_i \varphi_j(\vec{r}_i) = q_j \varphi_i(\vec{r}_j)$$

So
$$q_i \varphi_i(\vec{r}_j) = \frac{q_i q_j}{4\pi\epsilon_0 |\vec{r}_i}$$
$$= q_j \varphi_j(\vec{r}_i)$$

Reciprocity is more gene
e.g., in presence of cond

Earnshaw's thm:

No static arrangeme
point charges can be in s
equilibrium for finite separa
of the charges

Proof: First, a lemma
For charge q_i the pot
at r_i from other charges a

(Mean value theorem)

Proof:

$$\int_{|\vec{r}-\vec{r}_i|\le R} dV \left(\nabla^2 \varphi \left[\frac{1}{|\vec{r}-\vec{r}_i|} - \frac{1}{R}\right] - \varphi \nabla \right.$$

$$\nabla^2\varphi \overset{?}{=} 0$$

$$= 4\pi \, \varphi(\vec{r}_i) \qquad - \ldots$$

$$= \int_{|\vec{r}-\vec{r}_i|=R} d\vec{S} \cdot \left(\nabla \varphi \left(\frac{1}{R} - \frac{1}{R}\right) - \ldots\right.$$

$$\left(\frac{1}{R} - \frac{1}{R}\right) = 0$$

$$= \int d\Omega \, R^2 \hat{r} \cdot \varphi \left(-\nabla \frac{1}{|\vec{r}-\vec{r}_i|} \ldots\right.$$

$$= \int d\Omega \, R^2 \hat{r} \cdot \varphi(\vec{r}) \frac{\hat{r}}{|\vec{r}-\vec{r}_i|^{\ldots}}$$

$$[\hat{r} = (\vec{r}-\vec{r}_i)/|\vec{r}-\vec{r}_i|]$$

$$= \int d\Omega \, \varphi(\vec{r}) = 4\pi \langle \varphi(\vec{r}\ldots$$

Done!

must be a lower value
$U(q_i)$ somewhere else.
only exception would be
were constant, giving neu
equilibrium, but that is no
sible if any other charges
present, because φ actual
blows up at the location o
source charge.

Thus it is always poss
to find a place for q_i wit
lower potential energy, un
q_i is right on top of an o
sign charge.

This proves that any
atom is unstable, but it
too much, because the same
for the planets in the solar s

What is missing?

big orbit the radiation h
little effect for a very
time. Not only are the
orbits very big, but also th
radiation is less efficient
gravity is weak and the
radiation is quadrupole,
dipole (discussed later
course).

$\boxed{\text{Magnetostatics}}$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = 4\pi k' \vec{J} = \mu_0 \vec{J}$$

Stokes' Theorem

$$\oint_{\text{Border of } S} \vec{B} \cdot d\vec{l} = \int_S d\vec{S} \cdot \nabla \times \vec{B}$$

$$= 4\pi k' I = \mu$$

By symmetry, $\vec{B} = B(r)\,\hat{\varphi}$

$$\int \vec{B}\cdot d\vec{l} = 2\pi B(r) = 4\pi$$

$$B(r) = \frac{2k'}{r} I = \frac{\mu_0 I}{2\pi r}$$

<u>Homework</u>

1. Find $\nabla\cdot\vec{f}$ in spherical coordinates.
2. Find $\nabla^2 g$ in spherica coordinates.

Background:

$$dg = dr\,\partial_r g + d\theta\,\partial_\theta g + d\varphi$$

$$= d\vec{R}\cdot\nabla g$$

$dl_r = dr$, $dl_\theta = r\,d\theta$, d
$r\sin\theta\, d\varphi$

So $\nabla g_r = \partial_r g$, $\nabla g_\theta = \frac{1}{r}\partial_\theta$
$\nabla g_\varphi = \frac{1}{r\sin\theta}\partial_\varphi g$