October 1

Effects of boosts on E, B

1) Let us consider infinitesimal boost $d\bar{v}$.

What is the change in $d\bar{E}$ and $d\bar{B}$

1) Effect must be linear in $d\bar{v}$.

(independent of frame)

2) Effect must be linear in $\bar{E}, \bar{B}$

3) Effect must preserve vector character under rotations

4) electro-magnetic duality.

Maxell equations give a linear relation of $\bar{E}$ & $\bar{B}$ to sources $\rho, \bar{J}$

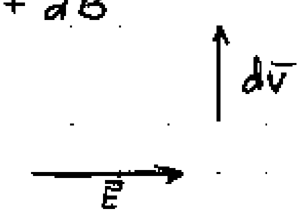
Non linearity is an important aspect of electrodynamics.

Any system, if pushed enough, has a non-linear response.

Let us start with some $\bar{E}, \bar{B}$

Transformation: $\bar{E}, \bar{B} \rightarrow \bar{E} + d\bar{E}, \bar{B} + d\bar{B}$

Then $dE = a\, d\bar{v} \times \bar{B}$, $dB = b\, d\bar{v} \times \bar{E}$



dE, dB - tensor quantities

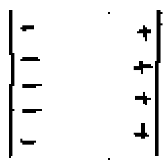
$P\,(x,y,z) \to (-x,-y,-z)$ – Parity

Magnetic field B cannot follow this.
(B is pseudovector)

Maxell equations are consistent with parity P
symmetry

Proportionality to $d\bar{v}$, $\bar{E}$, $\bar{B}$ not just linearity.

Even a "free" electro-magnetic field can be accurately approximated using suitable sources

$\bar{E}$ - const  $\to$ uniform $\bar{E}$ field

"Free" electro-magnetic field is ideal case

$d\bar{E} = -d\bar{v} \times \bar{B}$, $d\bar{B} = \xi\, d\bar{v} \times \bar{E}$. What is ξ

We make transformation ($c = 1$)

$$\bar{E} \to \cos\alpha\, \bar{E} + \sin\alpha\, \bar{B}$$

$$\bar{B} \to \cos\alpha\, \bar{B} - \sin\alpha\, \bar{E}$$

Symmetry of Maxell equations. $\xi = \pm 1$

$\xi = +1$ (from Looking at equations)

$$\left.\begin{aligned} \bar{E}' &= \bar{E} - d\bar{v} \times \bar{B} \\ \bar{B}' &= \bar{B} + d\bar{v} \times \bar{E} \end{aligned}\right\} \text{infinitesimal boosts}$$

Multiplying many *infinitesimal boosts*:

$\bar{E} = (E_1, 0, 0)$

$d\bar{v} = dv\, \bar{z}$

$\bar{B} = (0, B_2, 0)$

$\Rightarrow dE_1 = -dv\, (\bar{z} \times \bar{B}) \cdot \bar{x} \qquad dB_2 = dv\, E_1$

And this *gives us* the following *matrix* for infinitesimal *boosts*:

$$L \begin{pmatrix} E_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} 1 & dv \\ dv & 1 \end{pmatrix} \begin{pmatrix} E_1 \\ B_2 \end{pmatrix}$$

General expression

$$E_1' = (\cosh y) E_1 + (\sinh y) B_2 \qquad E_2' = E_3' = 0$$

$$B_2' = (\cosh y) B_2 + (\sinh y) E_1 \qquad B_1' = B_3' = 0$$