October 4

Effects of boosts on $E,B$

1) Let us consider infinitesimal boost $d\vec{\nu}$.
   - What is the change in $d\overline{E}$ and $d\overline{B}$?
     1) Effect must be linear in $d\vec{\nu}$.
        (independent of frame)
     2) Effect must be linear in $\overline{E}, \overline{B}$.

3) Effect must preserve vector character under rotations.

4) Electro-magnetic duality.

Maxwell equations give a linear relation of $\overline{E}\overline{B}$ to sources $p, \overline{J}$.
Non-linearity is an important aspect of electrodynamics. Any system, if pushed enough, has a non-linear response.

Let us start with some $\overline{E}, \overline{B}$.

Transformation: $\overline{E}, \overline{B} \rightarrow \overline{E} + d\overline{E}, \overline{B} + d\overline{B}$

Then $d\overline{E} = a d\vec{\nu} \times \overline{B}, d\overline{B} = b d\vec{\nu} \times \overline{E}$

$d\overline{E}, d\overline{B}$ - tensor quantities

$\overrightarrow{E}$
\[ P(x, y, z) \rightarrow (-x, -y, -z) \] - Pavity

Magnetic field \( \mathbf{B} \) cannot follow this.

(\( \mathbf{B} \) is pseudovector)

Maxell equations are consistent with parity \( P \) symmetry.

Proportionality to \( d\mathbf{V} \), \( \mathbf{E}, \mathbf{B} \) not just linearly.

Even a "free" electromagnetic field can be accurately approximated using suitable sources.

\[ \mathbf{E} \text{ - const} \rightarrow \text{uniform } \mathbf{E} \text{ field} \]

"Free" electromagnetic field is ideal case

\[ d\mathbf{E} = -d\mathbf{V} \times \mathbf{B} \quad d\mathbf{B} = \gamma d\mathbf{V} \times \mathbf{E} \]

What is \( \gamma \)?

We make transformation \( (c=1) \)

\[ \mathbf{E} \rightarrow \cos \chi \mathbf{E} + \sin \chi \mathbf{B} \]

\[ \mathbf{B} \rightarrow \cos \chi \mathbf{B} - \sin \chi \mathbf{E} \]

Symmetry of Maxwell equations: \( \gamma = \pm 1 \)

\( \gamma = +1 \) (from looking at equations)

\[ \mathbf{E}' = \mathbf{E} - d\mathbf{V} \times \mathbf{B} \] \quad \text{infinitesimal}

\[ \mathbf{B}' = \mathbf{B} + d\mathbf{V} \times \mathbf{E} \] \quad \text{boosts}
Multiplying many infinitesimal boosts,

\[ E = (E_1, 0, 0) \]
\[ B = (0, B_2, 0) \]

\[ dE_1 = -dv \ (\bar{z} \times \bar{B}) \cdot \bar{x} \]
\[ dB_2 = dv E_1 \]

And this gives us the following matrix for infinitesimal boosts:

\[
L \begin{pmatrix} E_1' \\ B_2' \end{pmatrix} = \begin{pmatrix} 1 & dv \\ 0 & 1 \end{pmatrix} \begin{pmatrix} E_1 \\ B_2 \end{pmatrix}
\]

General expression:

\[ E_1' = (\cosh \gamma) E_1 + (\sinh \gamma) B_2 \]
\[ B_2' = (\cosh \gamma) B_2 + (\sinh \gamma) E_1 \]

\[ E_2' = E_3' = 0 \]
\[ B_1' = B_3' = 0 \]