E + M Classnotes October 3 ANNE SICKLES

FORCE TRANSFORMATION

$$\vec{F} = \frac{d\vec{p}}{dt}$$

consider a boost along z direction

$$F_z = \frac{dp_z}{dt} \quad (p_z = 0 - \text{particle's rest frame})$$

$$\frac{dp_z'}{dt'} = \frac{dp'}{dy}\frac{dy}{d\tau}\frac{d\tau}{dt'} \quad (\text{primes denote lab frame})$$

$$\sinh y = \gamma v$$

so $p_z' = m\gamma v$ and $\frac{dp'}{dy} = m\cosh y = m\gamma$

$= \underline{m\sinh y}$

$$\frac{dy}{d\tau} = \frac{dv}{dt}_{\text{rest frame}}$$

since $dy = dv$ for small shifts
and $d\tau$ is time as measured by the particle

and

$$d\tau = \frac{1}{\gamma}dt' \quad \text{so} \quad \frac{d\tau}{dt'} = \frac{1}{\gamma}$$

thus

$$\frac{dp_z'}{dt'} = m\gamma \frac{dv}{dt}_{\text{rest}} \frac{1}{\gamma} = m\frac{dv}{dt}_{\text{rest}} = F_z$$

so $F_z' = F_z$

all observers see same force
$\parallel$ to boost

Note this does not mean all observers see the same acceleration since in some frames v would then exceed c

$$p' = m\gamma v$$

$$\frac{dp'}{dt'} = m\frac{d(\gamma v)}{dt'}$$ γ and v both depend on t'
γm = inertial mass

Now: $F_\perp' = \frac{dp_\perp'}{dt'} = \frac{dp_\perp}{dt'}$ $p_\perp' = p_\perp$ because lengths $\perp$ to boost axis are not altered

force $\perp$ to boost

$$d\tau = \frac{1}{\gamma}dt'$$

$$F_\perp = \frac{dp_\perp}{d\tau} = \gamma\frac{dp_\perp}{dt'}$$

so

$$F_\perp' = \frac{1}{\gamma}F_\perp$$

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so $F_\parallel = F_\parallel'$ $F_\perp' = \frac{1}{\gamma}F_\perp$ where the primed forces are in the lab frame

in the parallel case
$p_\parallel' \neq p_\parallel$ and $dt \neq dt'$
but the changes cancel causing $F_\parallel = F'$

in the perpendicular case
$p_\perp = p_\perp'$ but again the times are different
so $F_\perp' = \frac{1}{\gamma}F_\perp$

if $\vec{E}' = (E_1', 0, 0)$ and $B' = (0,0,0)$ boost in x_3 direction in lab frame

$$F_1' = qE_1' \qquad F_1 = qE_1 + \beta q B_2 \quad \text{in rest frame}$$

$$F_\perp' = \frac{1}{\gamma} F_\perp$$

B_2 is only non-zero component in rest frame

$$F_\perp' = qE_1' = \gamma F_\perp$$

$$F_\perp = qE_1 + q\beta B_2 \Leftarrow \text{the Lorentz force}$$

so $E_1' = (E_1 + \beta B_2)\gamma$

if $E_1 = \gamma E_1'$ and $B_2 = -\beta\gamma E_1'$

then

$$E_1' = (\gamma E_1' + \beta(-\beta\gamma E_1'))\gamma$$

$$= \gamma^2 E_1' (1-\beta^2) \qquad \beta = \frac{v}{c} = v \; \binom{\text{when}}{c=1}$$

$$= \frac{(1-\beta^2)}{(1-\beta^2)} E_1'$$

$$\underline{E_1' = E_1'}$$

so a particle, which in the lab frame, is in electric field $E' = E_1' \hat{x}_1$ in it's rest frame is in electric field $E_1 = \gamma E_1'$ and magnetic field $\vec{B} = B_2 \hat{x}_2$ where $B_2 = -\beta\gamma E_1'$ and the force does transform according to $F_\perp' = \frac{1}{\gamma} F_\perp$ and $F_\parallel' = F_\parallel$