

direction according to this viewer? }
the direction in terms of the tangent
the x-axis.

given:

$$\omega = 2\pi \cdot 2000 \text{ Hz}, \quad \vec{k} = \begin{pmatrix} k \\ 0 \\ 0 \end{pmatrix}$$

boost: $\Lambda = \begin{pmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{pmatrix}$ where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

We want to discuss two ways of solving the

- Knowing the kinematic energy-momentum
can derive a four-vector k^μ which
and wave vector \vec{k} :

$$p^\mu = \begin{pmatrix} \frac{E}{c} \\ \vec{p} \end{pmatrix} \quad \text{with } E: \text{ energy, } \vec{p}:$$

$$\Rightarrow k^\mu = \frac{p^\mu}{\hbar} = \begin{pmatrix} \frac{\omega}{c} \\ \vec{k} \end{pmatrix}$$

$$\vec{k}' = \begin{pmatrix} 0 \\ \gamma\beta \frac{\omega}{c} \end{pmatrix} \implies \underline{\underline{\tan \theta}}$$

where θ is the angle to the x axis
that photons have a linear dispersion relation

- Another way of solving the problem is to pick two points in four-space that are separated by a wavelength λ :

$$x_1^M = (0, 0, 0, 0), \quad x_2^M = (\lambda, \lambda, 0, 0)$$

$$\implies \Delta x^M = x_2^M - x_1^M = (\lambda, \lambda, 0, 0)$$

The four-vector Δx^M undergoes the boost

$$\Delta x^{M'} = \Lambda \Delta x^M = \begin{pmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} \lambda \\ \lambda \\ 0 \\ 0 \end{pmatrix}$$

The space part yields the angle θ to the x axis

$$\underline{\underline{\tan \theta = \frac{\beta\gamma\lambda}{\lambda} = \gamma\beta}}$$

are invariant under boosts we can use the following equation:

$$k_{\mu} x^{\mu} = 0 + k_x \cdot 2 + 0 + 0 = k'_x$$

$$\Rightarrow k_x = k'_x = |\vec{k}'| \cos \theta = \frac{\omega'}{c} \cos \theta$$

$$= \frac{\omega'}{c} \cdot \sqrt{\frac{1}{1 + \tan^2 \theta}} = \frac{\omega'}{c} \sqrt{\frac{1}{1 + \beta^2}}$$

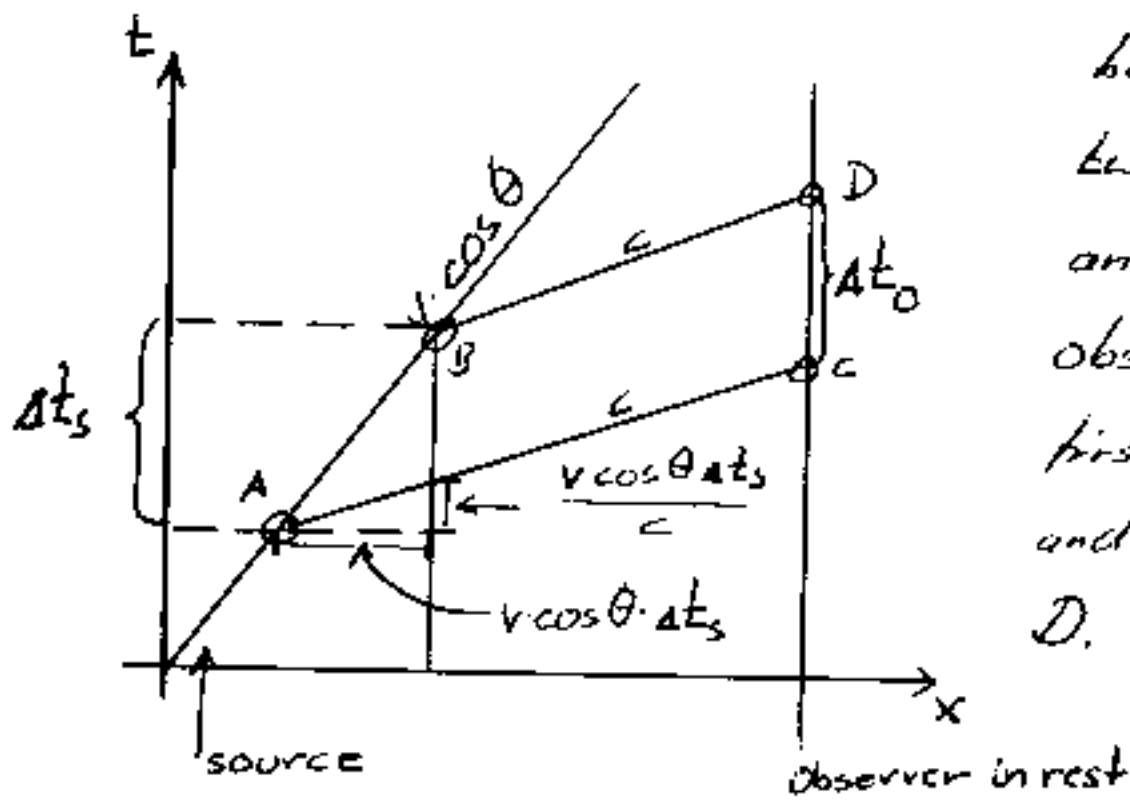
$$= \frac{\omega'}{c} \sqrt{\frac{1}{1 + \frac{\beta^2}{1 - \beta^2}}} = \frac{\omega'}{c} \sqrt{\frac{1 - \beta^2}{1 - \beta^2 + \beta^2}}$$

$$= \frac{\omega'}{c} \sqrt{1 - \beta^2} = \frac{\omega'}{c} \cdot \frac{1}{\gamma} = k_x$$

with $k_x = k = \frac{\omega}{c}$ as $\vec{k} = (k, 0, 0)$

$$\underline{\underline{\omega' = \gamma \cdot \omega}}$$

component of \vec{v} which is parallel to the line of sight between source and observer is $v \cdot \cos \theta$. The x axis be the line of sight. The



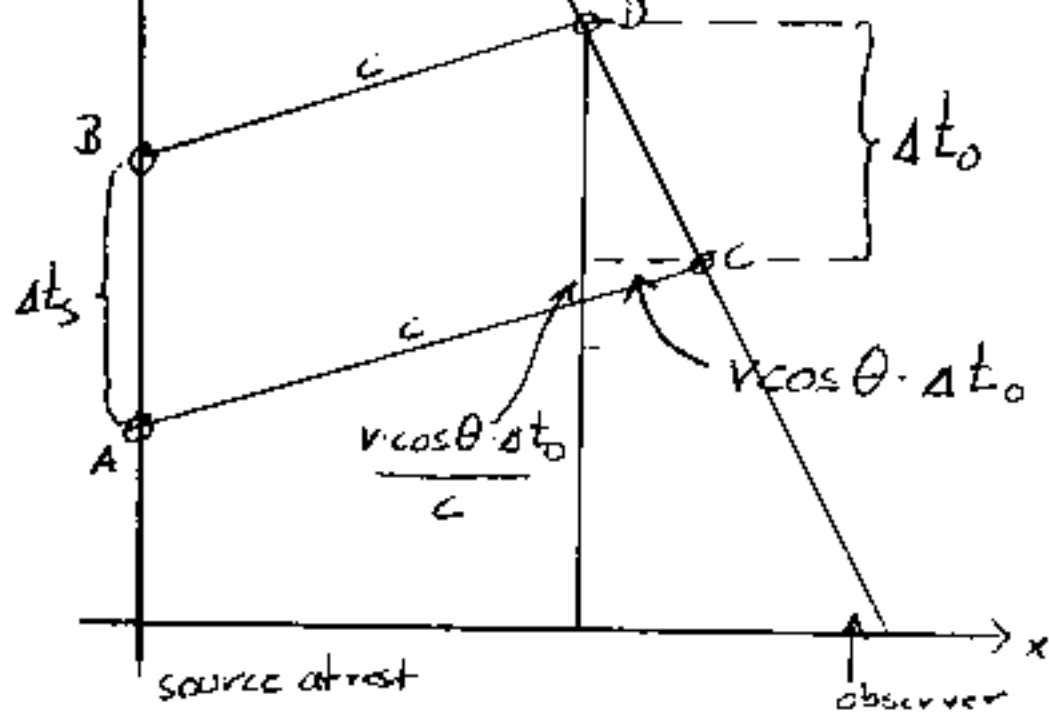
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Δt_0 is the time interval between D and C
 Δt_s is the time interval between B and A

$$\Delta t_0 = \Delta t_s - \frac{v \cdot \cos \theta \Delta t_s}{c} = \left(1 - \frac{v}{c} \cos \theta\right) \Delta t_s$$

The period between strikes corresponds to the frequency ν :

$$\underline{\underline{\nu_0 = \frac{1}{1 - \frac{v \cos \theta}{c}} \nu_s}}$$



$$\Delta t_s = \Delta t_o + \frac{v \cdot \cos \theta \Delta t_o}{c} = \left(1 + \frac{v}{c} \cos \theta\right) \Delta t_o$$

$$\Rightarrow v_s = \frac{1}{1 + \frac{v}{c} \cos \theta} v_o$$

$$\Rightarrow \underline{\underline{v_o = \left(1 + \frac{v}{c} \cos \theta\right) v_s}}$$

In our convention: $\theta < 90^\circ$ if distance between source becomes smaller.

$\theta > 90^\circ$ if distance between source becomes larger.

The observer moves with velocity \vec{v} in z-direction of the source the light ray is described by k^μ
 $k^{\mu'}$ is the light ray in the observer's frame.

$$k^{\mu'} = \Lambda \cdot k^\mu = \begin{pmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} \omega \\ k_x \\ k_y \\ k_z \end{pmatrix}$$

$$\begin{aligned} \implies \frac{\omega'}{c} &= \gamma \frac{\omega}{c} + \beta\gamma k_z = \gamma \frac{\omega}{c} + \beta\gamma \\ &= \gamma \frac{\omega'}{c} + \beta\gamma \frac{\omega}{c} \cos \theta \end{aligned}$$

where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$, $\beta = \frac{v}{c}$, θ : angle
 z-axis

$$\implies \omega' = \omega \left(\frac{1 + \frac{v}{c} \cos \theta}{\sqrt{1 - \frac{v^2}{c^2}}} \right)$$

Note: Observations made at right angles ($\theta = 90^\circ$) of motion show the non-zero transverse Doppler there should be no effect.