12/10/2001

Transformation of E-M fields

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}$$

to work out transformation properties on a vector x:

$$(\Lambda x)^0 = \gamma(ct + \vec{\beta}\cdot\vec{r})$$

$$(\Lambda x)^{\parallel} = \gamma(\vec{x}^{\parallel} + \vec{\beta}tc)$$

$$(\Lambda x)^{\perp} = \vec{x}^{\perp} \qquad \perp \text{ components do not shift}$$

Rewrite as

$$\left.\begin{aligned} \vec{E}_{\parallel}' &= \vec{E}_{\parallel} \qquad \vec{B}_{\parallel}' = \vec{B}_{\parallel} \\ \vec{E}_{\perp}' &= \gamma(\vec{E}_{\perp} + \vec{\beta}\times\vec{B}_{\perp}) \\ \vec{B}_{\perp}' &= \gamma(\vec{B}_{\perp} - \vec{\beta}\times\vec{E}_{\perp}) \end{aligned}\right\} \text{ easier to work with}$$

Poynting vector $\propto \vec{a}^2$

? $\vec{E}_{rad}$ for a slow point charge

$$\vec{E}_{rad} = \frac{q}{c^2 r}\frac{d^2}{dt^2_{retarded}}\vec{x}_{\perp}(t_{retarded})$$

only want $\vec{x}_{\perp}$ term

$$\vec{x}_{\perp} = \vec{x} - \hat{r}\cdot\vec{x}\,\hat{r}$$

differentiate ϕ and $\vec{A}$ to get $\vec{E}$ field.

if $\vec{v}$ large this gets tricky. $\vec{v}$ small get the previous form. We know $\vec{E}_{rad}$(dipole)

$$\vec{E}^{dipole}_{rad(\omega)} = \frac{e^{-i(\omega t - kr)}}{r} \left(\frac{\omega^2}{c^2} \vec{d}_\perp \right)$$

? happens w/oscillating electric charge

$$\vec{x} = -\frac{e E_0 e^{-i\omega t}}{m\omega^2} \Big\} \text{ accurate for low } \omega$$

multiply by magnitude of charge to get oscillation of dipole moment.

Helmholtz equation

$$(-\nabla^2 + k^2)\,\phi_0 = 4\pi \rho_\omega(\vec{r})$$

<u>Green's function</u>

given a differential operator L

look for an $f(x)$ that solves

$$L f(x) = y(x) \quad \Big\} \text{our problem: find } f(x)$$

<u>Solution</u>: find Green's function that solves

$$L g(x) = \delta(x)$$

then

$$f(x) = \int g(x) y(x)\, dx$$

because

$$\begin{aligned} L f(x) &= L \int g(x) y(x)\, dx \\ &= \int L g(x) y(x)\, dx \\ &= \int \delta(x) y(x)\, dx = y(x) \end{aligned}$$

you can think of the Green's function as the inverse of the differential operator 1

$$LL^{-1} = \delta(x) \doteq Lg(x) = \delta(x)$$

—courtesy C. Traiger

Employ boundary conditions to eliminate arbitrary solutions of $\nabla^2(\delta G) = 0$

free space: BC: $G \to 0$ as $|\vec{r}| \to \infty$ $\quad G = \frac{1}{|\vec{r}-\vec{r}\,'|}$

closed conductor: BC: $G = 0$ on surface

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Practice exam:

#1: $\nabla \cdot \vec{E} = \begin{cases} 4\pi\rho & \text{cgs} \\ \frac{\rho}{\varepsilon_0} & \text{SI} \end{cases}$

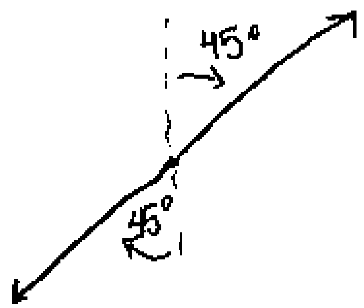
total flux $= -\frac{2e}{\varepsilon_0}$ or $-2e \times 4\pi$

#2 P is ~~not~~ a Legendre Polynomial.

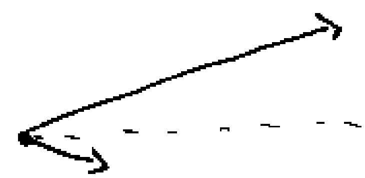
"you can make it cottage cheeze" - A.S.G.

#3

rest frame



lab frame

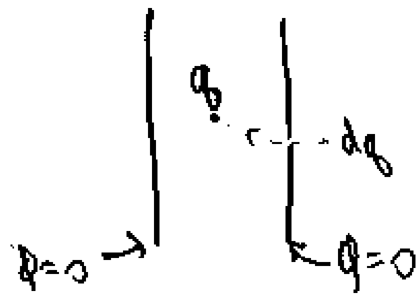


$\perp$ component does not change

$\parallel$ component does change

after boost get ⊕ tangent for top . . .
and ⊖ tangent for bottom in lab frame

#4



? How do we get q to this point?
- Bring dq at a time from ∞ through hole in 2 plates. For next dq we bring in get a bigger charge and thus a negative potential that is larger in magnitude. add all of these contributions together →

$$\text{Pot. Energy} = \tfrac{1}{2} q \phi$$

Suppose we had a spherical shell of charge

outside $\phi = \frac{q}{r}$

inside $\phi = \frac{q}{R}$

$$PE = \frac{q^2}{2R}$$

get $\frac{1}{2}$ by adding dq at a time to sphere

Memorize Legendre Polynomials & Spherical Harmonics

Remember: $\rho, \vec{J} \to \vec{E}, \vec{B}$

$$\vec{E} = -\nabla \phi - \frac{1}{c}\dot{\vec{A}}, \quad \vec{B} = \nabla \times \vec{A}$$

Lorentz Gauge

$$\partial_\mu A^\mu = 0$$

in $\lim v \to 0$ $\quad \nabla \cdot A = 0$

$$\left(\frac{1}{c^2}\partial_t^2 - \nabla^2\right) A^\mu = 4\pi \vec{J}^\mu$$