EM- HW 4

1- In this problem we just have to calculate the proper time interval in B rest frame.
The situation is symmetrical for the round trip - if we neglect accelerations, etc. at the turning point -.
We simply have

$$\Delta t_B = \Delta t_A \sqrt{1-(v^2/c^2)}$$
$$= 2 \text{ years} \times \sqrt{1-(.99)^2}$$
$$\simeq 0.282 \text{ yrs}$$

2- We'll use

$$E = E_0 \cosh y$$
$$p = E_0 \sinh y$$

, where $v = \tanh y$ (y is the rapidity, we set $c=1$)

The force is defined as

$$F = \frac{dp}{dt}$$

So in frame A, $F_A = \dfrac{dp}{dt_A}$

Using the previous relations

$$F_A = \frac{d}{dt_A}\left(E_0 \sinh y\right) = E_0 \cosh y \,\frac{dy}{dt_A} = E\,\frac{dy}{dt_A}$$

We now use the fact that rapidity is the same in both frames. We also relate it to the relative velocity

$$dv = (1-\tanh^2 y)\,dy = (1-v^2)\,dy$$

By definition, $v=0$ in B frame, so the acceleration in B frame is

$$a_B = \frac{dv}{d\tau_B} = \frac{dy}{d\tau_B}$$

Now we write F_A in terms of B-quantities

$$F_A = E\,\frac{dy}{dt_A} = E\,\frac{d\tau_B}{dt_A}\,\frac{dy}{d\tau_B} = E\,\frac{dy}{d\tau_B}\,\frac{1}{\gamma}$$

But also $E = M_0\gamma$, so using $\frac{dy}{d\tau_B} = a_B$

$$F_A = \gamma M_0 a_B \frac{1}{\gamma} = M_0 a_B$$