PHY505 - Classical Electrodynamics
Hour Exam No. 2
Friday, November 15, 2002

1. A capacitor system consists of two concentric, spherical, copper shells of radii R_1 and R_2 ($R_1 < R_2$). The space inside R_1 is filled with an insulator with dielectric constant κ_1; the space between R_1 and R_2 is filled with an insulator with dielectric constant κ_2, and the space outside the system is in vacuum.

   (a) Calculate the elements of the inverse capacitance matrix.

   (b) Use the results of part (a) to calculate the energy stored in the system if the charge on the inner shell is $-Q$, and the charge on the outer shell is $2Q$?

   Extra Credit: Calculate the energy stored in the system in 1(b) by integrating the energy density of the fields,

$$\mathcal{U}(x) = \frac{1}{2}\vec{E}(x) \cdot \vec{D}(x)\,,$$

   and compare to the result from the capacitance calculation.

2. A very long cylinder of radius a and electric permittivity ϵ is placed parallel to the z-axis in a region of space where there initially existed a uniform electric field in the x-direction $\vec{E} = E_0\,\hat{e}_x$.

   (a) Find the resultant electric field inside and outside the cylinder. Neglect end effects.

   (b) Determine the polarization charge density on the cylinder.

   Extra Credit: Sketch the lines of electric field in the $x-y$ plane. Briefly explain the figure.

   Note: A solution of the two-dimensional Laplace Equation $\nabla^2\psi = 0$ with

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$$

in plane polar coordinates can be expanded in Fourier components as

$$\psi(r,\theta) = f_0(r) + \sum_{n=1}^{\infty} [f_n(r)\cos(n\theta) + g_n(r)\sin(n\theta)]$$

with

$$\begin{aligned} f_0 &= A_0 + B_0 \ln(r) \\ f_n &= A_n r^n + B_n r^{-n} \end{aligned}$$

and g_n of the same form as f_n.