PHY505 - Classical Electrodynamics
Homework No. 1
Due: Friday, Sept. 13, 2002

1. Elliptical cylindrical coordinates are defined by

$$\begin{aligned} x &= a\cosh u \cos v \\ y &= a \sinh u \sin v \\ z &= z \end{aligned}$$

$$0 \leq u < \infty\ , \qquad 0 \leq v < 2\pi$$

   a) Show that the surfaces, $u =$ constant are elliptic cylinders and the surfaces, $v =$ constant, are hyperbolic cylinders.
   b) Calculate the infinitesmal line element, ds^2, in elliptical cylindrical coordinates.
   c) Find $\nabla \cdot \vec{V}$, $\nabla^2\phi$ and $\nabla \times \vec{V}$ in elliptical cylindrical coordinates.

2. Prove the following variations of Stokes' Theorem

   a) $\oint d\vec{l}\, \phi(\vec{x}) = \int dS\ \ (\hat{n} \times \nabla\phi(\vec{x}))$
   b) $\oint d\vec{l} \times \vec{V}(\vec{x}) = \int dS\ (\hat{n} \times \nabla) \times \vec{V}(x)$
   c) $\oint d\vec{l} \times \nabla\phi = \int dS\ \ [(\hat{n} \cdot \nabla)\, \nabla\phi - \hat{n}\nabla^2\phi]$

3. Find the electric field for an spherically symmetric distribution of charge whose density is given in spherical polar coordinates by

$$\begin{aligned} \rho(r) &= \rho_0 e^{-r/b}\ ,\ r < b \\ &= 0\ ,\ r \geq b \end{aligned}$$