PHY505 - Classical Electrodynamics
Homework No. 3
Due: Monday, September 30, 2002

1. Completeness of Spherical Harmonics.

   Consider
   $$\nabla^2 \frac{1}{|\vec{x} - \vec{x}_0|} = -4\pi\delta^{(3)}(\vec{x} - \vec{x}_0).$$
   Write the δ-function in polar coordinates. Integrate both sides of the equation over
   $$\int_{r_0-\epsilon}^{r_0+\epsilon} r^2 \, dr.$$
   Take the $\lim \epsilon \to 0$ to show
   $$\frac{1}{\sin\theta_0}\delta(\theta - \theta_0)\,\delta(\phi - \phi_0) = \sum_{l=0}^{\infty}\sum_{m=-l}^{m=l} Y_{lm}(\theta, \phi)\, Y_{lm}^*(\theta_0,\, \phi_0)\ .$$
   This is a completeness relation. Use it to show we can expand an arbitrary function of $f(\theta, \phi)$ as
   $$f(\theta, \phi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{m=l} f_{lm}\, Y_{lm}(\theta, \phi)$$
   with
   $$f_{lm} = \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \, f(\theta, \phi)\, Y_{lm}^*(\theta, \phi)$$

2. Read Jackson 1.6 (You should work out these simple examples of capacitors for yourself, but you do not have to hand this in.)

3. Consider the charge distributions shown in Fig. (a) and (b) of Problem 4.1 in Jackson.

   (a) Calculate the Cartesian dipole and quadrupole moments for both distributions .

   (b) Keeping only the lowest-order term in the expansion of set (b), plot the potential in the $x - y$ plane as a function of distance from the origin for distances greater than a.

(c) Calculate directly from Coulomb's law the exact potential for (b) in the $x - y$ plane. Plot it as a function of distance and compare with the result found in part b.

Divide out the asymptotic form in parts (b) and (c) to see the behavior at large distances more clearly.

4. (a) Derive the expression for the force given in Jackson, Problem 4.5(a).

   (b) Calculate the force between two dipoles, $\vec{p}_1$ and $\vec{p}_2$. Is $\vec{F}_{21} = -\vec{F}_{12}$? Hint: You may use the results of part (a).

5. Jackson 4.7, parts (a) and (b). To keep correct dimensions explicit, replace Jackson's density function by

$$\rho(\vec{r}) = \frac{\rho_0}{64\pi}\left(\frac{r}{r_0}\right)^2 e^{-r/r_0}\sin^2\theta \, .$$