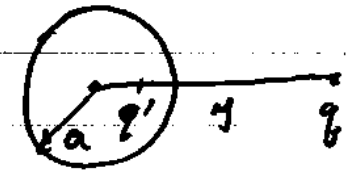
HW #5.

1.(a) By Jackson (2.9)

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{y^2}\left[1 - \frac{a^3(2y^2-a^2)}{y(y^2-a^2)^2}\right]\frac{\vec{y}}{y}$$



Attractive means $\vec{F}$ along negative $\hat{y}$, i.e.

$$1 - \frac{a^3(2y^2-a^2)}{y(y^2-a^2)^2} < 0$$

Let $y' = \frac{y}{a}$, the expression above can be rewrit to the following form,

$$(y'^2 - y' - 1)(y'^3 + y'^2 - 1) < 0$$

For $y' \geq 1$, the second term can't be Negative. so

$$y'^2 - y' - 1 < 0$$

$$\Rightarrow 1 < y' < \frac{1+\sqrt{5}}{2}$$

$$\text{i.e. } a < y < 1.6178\,a$$

$$\Rightarrow \quad 0 < y - a < 0.6178\,a$$

(b). Let $b = y - a \ll a$

$$\vec{F} = \frac{kq^2}{(a+b)^2}\left\{1 - \frac{a^3[2(a+b)^2 - a^2]}{(a+b)[(a+b)^2 - a^2]^2}\right\}$$

$$\doteq \frac{-kq^2}{a^2} \times \frac{a^5}{a \cdot 4a^2b^2}$$

$$= \frac{-kq^2}{4b^2}$$

$$= -\frac{q^2}{16\pi\varepsilon_0 b^2}$$

(c) From deviation of Jackson (2.9), one can see that the only difference from (a) is charge Q. for $Q = 2q$

The equation is

$2y'^5 - 4y'^3 - 2y'^2 + 2y' + 1 < 0$

Solve the equation by using Maple, one get the only possible solution is

$$1 < y' < 1.4276$$

$$\Rightarrow \quad 0 < y - a < 0.4276\,a$$

for $Q = \frac{q}{2}$, the equation is

$$y'^5 - 2y'^3 + y' - 4y'^2 + 2 < 0$$

solve it by using Maple. one get

$$1 < y' < 1.882$$

$$\Rightarrow \quad 0 < y - a < 0.882\,a$$

see Maple command text $\longrightarrow$ Attachment.

2. The solution is available in Jackson 2.6.

3. $I[\psi] = \frac{1}{2}\int_V \nabla\psi \cdot \nabla\psi \, d^2x - \int_V g\psi \, d^2x$

$$\nabla\psi = \hat{i}\,A(1-2x)y(1-y) + \hat{j}\,A(1-2y)x(1-x)$$

$$\Rightarrow \nabla\psi \cdot \nabla\psi = A^2(1-2x)^2y^2(1-y)^2 + A^2(1-2y)^2x^2(1-x)^2$$

$$\int \nabla\psi \cdot \nabla\psi \, dx = \int_0^1\int_0^1 dx\,dy \left[A^2(1-2x)^2y^2(1-y)^2 + A^2(1-2y)^2 \times x^2(1-x)^2\right]$$

$$= A^2 \frac{1}{3} \times \frac{1}{30} \times 2 = \frac{1}{45}A^2$$

$$\int \psi \, dx\,dy = \int A\,x(1-x)y(1-y)\,dx\,dy$$

$$= A\frac{1}{6} \times \frac{1}{6} = A\frac{1}{36}$$

$\Rightarrow I = \frac{1}{90}A^2 - \frac{1}{36}A$

$\Rightarrow \frac{\partial I}{\partial A} = \frac{A}{45} - \frac{1}{36} = 0$

$\Rightarrow A = \frac{5}{4}$

i.e. $\psi = \frac{5}{4}x(1-x)y(1-y)$

(b) → see Maple ~~context~~ command attachment.