PHY505 - Classical Electrodynamics
Vector Integral Theorems

1. Gauss Theorems

$$\int_V d\tau \frac{\partial V_k}{\partial x_i} = \oint_S dS\ n_i V_k$$

i)
$$\int_V d\tau\ \nabla \cdot \vec{V} = \oint_S dS\ \hat{n} \cdot \vec{V}$$

ii)
$$\int_V d\tau\ \nabla \times \vec{V} = \oint_S dS\ \hat{n} \times \vec{V}$$

iii)
$$\int_V d\tau\ \nabla^2 \phi = \oint_S dS\ \hat{n} \cdot \nabla\phi$$

iv) 1^{st} Green Identity

$$\int_V d\tau\ (\psi\, \nabla^2 \phi) + \int_V d\tau\ (\nabla\psi) \cdot (\nabla\phi) = \oint_S dS\ \psi(\hat{n} \cdot \nabla\phi)$$

v) 2^{nd} Green Identity

$$\int_V d\tau\ [\psi \nabla^2 \phi - \phi \nabla^2 \psi] = \oint_S dS\ [\psi\, \hat{n} \cdot \nabla\phi - \phi\, \hat{n} \cdot \nabla\psi]$$

2. Stokes Theorems

i)
$$\oint_C d\vec{l} \cdot \vec{V} = \int_S dS\ \hat{n} \cdot \nabla \times \vec{V}$$

ii)
$$\oint_C d\vec{l}\, \phi = \int_S dS\ \hat{n} \times \nabla\phi$$

iii)

$$\oint_C d\vec{l} \times \vec{V} = \int_S dS \; (\hat{n} \times \nabla) \times \vec{V}$$

iv)

$$\oint_C d\vec{l} \times \nabla\phi = \int_S dS \; (\hat{n} \times \nabla) \times \nabla\phi$$